Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default*

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Abstract

The European debt crisis shares features of historical episodes of outright default on domestic public debt about which little research has been done. This paper proposes a theory of domestic sovereign default in which a government chooses debt and default optimally, responding to distributional incentives affecting the welfare of risk-averse agents who are split into debt holders and non-holders. Equilibria with debt do not exist if the government is utilitarian and default is costless. Adding exogenous default costs, the model supports equilibria with debt exposed to default risk in which debt falls as the fraction of agents who own the debt falls. Debt can also be sustained if the government’s welfare weights are biased in favor of bond holders, resulting in equilibria in which debt rises as debt ownership is more concentrated. These results are robust to adding a second asset, opening the economy to foreign investors, or introducing taxes as an alternative redistributive policy instrument.

Keywords: Public debt, sovereign default, European debt crisis.

JEL Classifications: E6, E44, F34, H63

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1 Introduction

The seminal study by Reinhart and Rogoff (2011) identified 68 episodes in which governments defaulted outright (i.e. by means other than inflation) on their domestic creditors in a cross-country database going back to 1750. These domestic defaults occurred via mechanisms such as forcible conversions, lower coupon rates, unilateral reductions of principal, and suspensions of payments. Reinhart and Rogoff also documented that domestic public debt accounts for a large fraction of total government debt in the majority of countries (about 2/3rds on average), and that domestic defaults were associated with periods of severe financial turbulence, which often included defaults on external debt, banking system collapses and full-blown economic crises. Despite of these striking features, they also found that domestic sovereign default is a “forgotten history” that remains largely unexplored in economic research.

The ongoing European debt crisis also highlights the importance of studying domestic sovereign default. In particular, four features of this crisis make it more akin to a domestic default than to the typical external default that dominates the literature on public debt default. First, countries in the Eurozone are highly integrated, with the majority of their public debt denominated in their common currency and held by European residents. Hence, from an European standpoint, default by one or more Eurozone governments means a suspension of payments to “domestic” agents, instead of external creditors. Second, as Table 1 shows, domestic public debt-GDP ratios are high in the Eurozone in general, and very large in the countries at the epicenter of the crisis (Greece, Ireland, Italy, Spain and Portugal). Third, the Eurozone’s common currency and common central bank rule out the possibility of individual governments resorting to inflation as a means to lighten their debt burden without an outright default. Fourth, and perhaps most important from the standpoint of the theory proposed in this paper, European-wide institutions such as the ECB and the European Commission are weighting the interests of both creditors and debtors in assessing the pros and cons of sovereign defaults by individual countries, and creditors and debtors are aware of these institutions’ concern and of their key role in influencing expectations and default risk. Hall and Sargent (2014) document a similar situation in the process by which the U.S. government handled the management of its debt in the aftermaths of the Revolutionary War.

Table 1 also shows that the Eurozone fiscal crisis has been characterized by rapid increases in public debt ratios and sovereign spreads that coincided with increases in government expenditure ratios. The Table also shows that wealth is unevenly distributed in the seven Eurozone countries

1The analogy with a domestic default is imperfect, however, because the Eurozone is not a single country, and in particular there is no fiscal entity with tax and debt-issuance powers over all the members. Still, the situation resembles more a domestic default than an external default in which debtors are not concerned for the interests of creditors.
listed, with mean and median Gini coefficients of around 2/3rds. In the Appendix, we present a more systematic analysis of the link between debt and inequality and show that government debt is increasing in inequality when inequality is low but decreasing for high levels of inequality.

Table 1: Euro Area: Key Fiscal Statistics and Wealth Inequality

<table>
<thead>
<tr>
<th>Moment (%)</th>
<th>Gov. Debt</th>
<th>Gov. Exp.</th>
<th>Spreads</th>
<th>Gini Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. 2011</td>
<td>France: 34.87 62.72</td>
<td>23.40 24.90</td>
<td>0.08 1.04</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>Germany: 33.34 52.16</td>
<td>18.80 20.00</td>
<td>-</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>Greece: 84.25 133.09</td>
<td>18.40 23.60</td>
<td>0.37 21.00</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Ireland: 14.07 64.97</td>
<td>16.10 20.50</td>
<td>0.11 6.99</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>Italy: 95.46 100.22</td>
<td>19.40 21.40</td>
<td>0.27 3.99</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>Portugal: 35.21 75.83</td>
<td>20.00 22.10</td>
<td>0.20 9.05</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>Spain: 39.97 45.60</td>
<td>17.60 21.40</td>
<td>0.13 4.35</td>
<td>0.57</td>
</tr>
<tr>
<td>Avg.</td>
<td>48.17 76.37</td>
<td>19.10 21.99</td>
<td>0.22 7.74</td>
<td>0.64</td>
</tr>
<tr>
<td>Median</td>
<td>35.21 64.97</td>
<td>18.80 21.40</td>
<td>0.17 5.67</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Note: Author’s calculations based on OECD Statistics, Eurostat, ECSB and Davies, Sandström, Shorrocks, and Wolff (2009). “Gov. Debt” refers to Total General Government Net Financial Liabilities (avg 90-07); “Gov. Exp.” corresponds to government purchases in National Accounts (avg 00-07); “Sov Spreads” correspond to the difference between interest rates of the given country and Germany for bonds of similar maturity (avg 00-07). For a given country $i$, they are computed as $(1 + r_i) / (1 + r_{Ger}) - 1$. “Crisis Peak” refers to the maximum value observed during years 2008-2012 (i.e. during the current financial crisis) as reported by Eurostat. Gini Wealth as reported for each country for year 2000 by Davies, Sandström, Shorrocks, and Wolff (2009) Appendix V.

Taken together, the history of domestic defaults and the risk of similar defaults in Europe pose two important questions: What explains the existence of domestic debt ratios exposed to default risk? And, is dispersion in the ownership of government debt an important determinant of domestic debt exposed to default risk?

This paper aims to answer these questions by proposing a framework for explaining domestic sovereign defaults driven by distributional incentives. This framework is motivated by the key fact that a domestic default entails substantial redistribution across domestic agents, with all of these agents presumably entering in the payoff function of the sovereign. This is in sharp contrast with what standard models of external sovereign default assume, particularly those based on the classic work of Eaton and Gersovitz (1981).

We propose an analytically tractable two-period model with heterogeneous agents and non-
insurable aggregate risk in which domestic default can be optimal for a government that responds to distributional incentives. A fraction $\gamma$ of agents start as low-wealth ($L$) agents and a fraction $1 - \gamma$ are high-wealth ($H$) agents, depending on the size of their initial holdings of government bonds. The government finances the gap between exogenous stochastic expenditures and endogenous lump-sum taxes by issuing non-state-contingent debt, retaining the option to default.

In the benchmark model we propose, the government has a utilitarian social welfare function, so it assigns the weights $\gamma$ and $1 - \gamma$ to the welfare of $L$ and $H$ agents respectively. Private agents choose optimally their bond holdings in the first period, taking as given bond prices and the probability of default in the second period. The government chooses how much debt to issue taking into account its inability to commit to repay: First, it evaluates the agents’ payoffs under repayment and default given their optimal savings plans and the government budget constraints. Second, it uses those payoffs to formulate a default decision rule for the second period that depends on how much debt is issued, the realization of government expenditures, and the concentration of public debt ownership measured by $\gamma$. Third, it chooses optimally how much debt to issue in the first period to maximize its lifetime social welfare function internalizing how the debt choice affects default incentives, default risk and bond prices.

In this model, the distribution of public debt across private agents interacts with the government’s optimal default, debt issuance and tax decisions. Default is optimal when the taxes needed to repay the debt hurt non-bond-holders more than defaulting hurts bond holders, and this happens when, for a given amount of debt and and given $\gamma$, the realization of government expenditures is high enough. This is necessary but not sufficient, however, for the model to support an equilibrium with debt subject to default risk. A second necessary condition is that the debt market in the first period can be sustained. This requires that the government finds it optimal to sell debt that is left exposed to default risk in the second period, even after internalizing this risk and the response of the equilibrium price of bonds to the amount of debt issued, and that private agents are willing to buy that debt at the market price.

If the government is utilitarian and default is costless, the model cannot support an equilibrium with debt. This is because for any given level of debt that could have been issued in the first period, the government always attains the second period’s socially efficient levels of consumption allocations and redistribution by choosing to default, and if default in period two is certain the debt market collapses in the first period. An equilibrium with debt under a utilitarian social welfare function exists if default entails an exogenous cost in terms of disposable income. When default is costly, if the amount of period-two consumption dispersion that the competitive equilibrium with repayment supports yields higher welfare than the default equilibrium net of default cost, repayment becomes optimal. Alternatively, we show that an equilibrium with debt
can be supported if we extend the benchmark model so that the government’s payoff function displays a “political” bias, even if default is costless. In this case, the government’s weight on $H$-type agents is higher than the actual fraction of these agents in the distribution of bond holdings.

Quantitative results based on a calibration to European data are used to illustrate the model’s key predictions. The model with utilitarian social welfare and default costs displays bond prices and default risk that are increasing in the concentration of debt ownership. Hence, lower public debt is sustainable as $\gamma$ rises. Most of the optimal debt positions chosen over a wide range of values of $\gamma$ are exposed to a positive probability of default, except those when $\gamma$ is very low, which have zero default risk. Debt is convex and decreasing in $\gamma$. Because of default risk, the sustainable debt is lower than what the same model supports at the same values of $\gamma$ and with the same government expenditure shocks but assuming that the government commits to repay (i.e. in the absence of default risk).

In the extension of the model with political bias, the debt is an increasing function of the concentration of debt ownership, instead of decreasing as in the utilitarian case. This is because incentives to default get weaker as the government’s weight on $L$-type agents falls increasingly below $\gamma$. The lower this weight, the higher the debt that can be supported at every value of $\gamma$. These debt amounts can easily exceed those supported in the utilitarian case by wide margins, and can be of similar magnitude as the European median with spreads again around 40 basis points and default probabilities around 1 percent.

The model with political bias also yields the interesting result that agents that do not hold public debt may prefer a government that weighs bond holders more than a utilitarian government. This is because the government with political bias has weaker default incentives, and can thus sustain higher debt at lower default probabilities, which relaxes the liquidity constraint of agents who do not hold public debt by improving tax smoothing. This finding also implies that it is possible to obtain majority-voting outcomes in which governments with political bias in favor of bond holders are elected even when the majority of voters are non-bond-holders.

We also explore other three important extensions of the model to show that the main result of the benchmark model, namely the existence of equilibria with domestic public debt exposed to default risk, is robust: Adding an additional asset as a vehicle for saving, opening the economy so that a portion of the debt is held by foreign investors, and introducing taxation as another instrument for redistributive policy.

This work is related to various strands of the large literature on public debt. First, studies on public debt as a self-insurance mechanism and a vehicle that alters consumption dispersion in heterogeneous agents models without default, such as Aiyagari and McGrattan (1998),

A second strand is the literature on external sovereign default in the line of the Eaton and Gersovitz (1981) model (e.g. Aguiar and Gopinath (2006), Arellano (2008), Pitchford and Wright (2012), Yue (2010)). Some studies in this area have examined models that include tax and expenditure policies, as well as settings with foreign and domestic lenders, but always maintaining the representative agent assumption (e.g. Cuadra, Sanchez, and Sapriza (2010)), Vasishtha (2010) and more recently Dias, Richmond, and Wright (2012) have examined the benefits of debt relief from the perspective of a global social planner with utilitarian preferences). Also in this literature, and closely related, is the work of Aguiar and Amador (2013) who analyze the interaction between public debt, taxes and default risk and Lorenzoni and Werning (2013) who study the dynamics of debt and interest rates in a model where default is driven by insolvency and debt issuance driven by a fiscal rule.

A third strand is the literature on political economy and sovereign default, which also largely focuses on external default (e.g. Amador (2003), Dixit and Londregan (2000), D’Erasmo (2011), Guembel and Sussman (2009), Hatchondo, Martinez, and Sapriza (2009) and Tabellini (1991)), but includes studies like those of Alesina and Tabellini (1990) and Aghion and Bolton (1990) that do focus on political economy aspects of government debt in a closed economy, and the work of Aguiar, Amador, Farhi, and Gopinath (2013) on optimal policy in a monetary union subject to self-fulfilling debt crises.

A fourth important strand of the literature focuses on the consequences of default on domestic agents, the role of secondary markets, discriminatory v. nondiscriminatory default and the role of domestic debt in providing liquidity to the corporate sector (see Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Gennaioli, Martin, and Rossi (2014), Basu (2009), Brutti (2011), Mengus (2014) and Di Casola and Sichlimiris (2014)). As in most of these studies, default in our setup is non-discriminatory, because the government cannot discriminate across any of its creditors when it defaults. Our analysis differs in that default is driven by the tradeoff between distributional incentives to default and endogenous

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3 See Panizza, Sturzenegger, and Zettelmeyer (2009), Aguiar and Amador (2014), and Tomz and Wright (2012) for recent reviews of the sovereign debt literature.

4 Motivated by the recent financial crisis and extending the theoretical work of Gennaioli, Martin, and Rossi (2014), there is a recent set of papers that focuses on the interaction between sovereign debt and domestic financial institutions such as Sosa-Padilla (2012), Bocola (2014), Boz, D’Erasmo, and Durdu (2014) and Perez (2015).
default costs due to the social value of debt used for self-insurance and tax-smoothing.\(^5\)

Finally, there is also a recent literature that is more related to our work in that it studies the trade-offs between distributional incentives to default and the use of public debt in infinite horizon models with domestic heterogeneity (see, in particular, D’Erasmo and Mendoza (2014) and Dovis, Golosov, and Shourideh (2014)). The studies differ in that Dovis et al. assume complete domestic asset markets, so the role of public debt in providing social insurance for domestic households is not present, while it plays a central role in endogenizing default costs in D’Erasmo and Mendoza (2014). In addition, Dovis et al. focus on the solution to a Ramsey problem that supports equilibria in which default is not observed along the equilibrium path.\(^6\)

The rest of this paper is organized as follows: Section 2 describes the payoff functions and budget constraints of households and government. Section 3 characterizes the model’s equilibrium and provides an illustration of the mechanism that drives optimal default as a policy for redistribution. Section 4 presents the benchmark calibration and the quantitative results for the utilitarian social welfare function. Section 5 discusses the extensions of the model.

2 The Model with Utilitarian Social Welfare

Consider a two-period economy inhabited by a continuum of agents with aggregate unit measure. Agents differ in their initial wealth position, which is characterized by their holdings of government debt at the beginning of the first period. This initial distribution of wealth is exogenous, but the distribution at the beginning of the second period is endogenously determined by the agents’ savings choices of the first period. The government is represented by a social planner with a utilitarian payoff who issues one-period, non-state-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy and is entirely held by domestic agents. There is explicit aggregate risk in the form of shocks to government outlays, and also implicit in the form of default risk, and there is no idiosyncratic uncertainty.

\(^5\)Also related is the work of Andreasen, Sandleris, and der Ghote (2011), Ferriere (2014) and Jeon and Kabukcuoglu (2014) who study environments in which domestic income heterogeneity plays a central role in the determination of external defaults.

\(^6\)See also Golosov and Sargent (2012) who study debt dynamics without default risk in a similar environment.
2.1 Household Preferences & Budget Constraints

All agents have the same preferences, which are given by:

\[
u(c_0) + \beta E[u(c_1)], \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma}\]

where \(\beta \in (0,1)\) is the discount factor and \(c_t\) for \(t = 0, 1\) is individual consumption. The utility function \(u(\cdot)\) takes the standard CRRA form.

All agents receive a non-stochastic endowment \(y\) each period and pay lump-sum taxes \(\tau_t\), which are uniform across agents. Taxes and newly issued government debt are used to pay for government consumption \(g_t\) and repayment of outstanding government debt. The initial supply of outstanding government bonds at \(t = 0\) is denoted \(B_0\). Given \(B_0\), the initial wealth distribution is defined by a fraction \(\gamma\) of households who are the \(L\)-type individuals with initial bond holdings \(b^L_0\), and a fraction \((1-\gamma)\) who are the \(H\)-types and hold \(b^H_0\), where \(b^H_0 = \frac{B_0 - \gamma b^L_0}{1-\gamma} \geq b^L_0 \geq 0\). Hence, \(b^H_0\) is the amount consistent with market-clearing in the government bond market at \(t = 0\).

The budget constraints of the two types of households in the first period are given by:

\[
c^i_0 + q_0 b^i_1 = y + b^i_0 - \tau_0 \quad \text{for } i = L, H. \tag{1}
\]

Agents collect the payout on their initial holdings of government debt \((b^0_i)\), receive endowment income \(y\), and pay lump-sum taxes \(\tau_0\). This net-of-tax resources are used to pay for consumption and purchases of new government bonds \(b^1_i\). Agents are not allowed to take short positions in government bonds, which is equivalent to assuming that bond purchases must satisfy the no-borrowing condition often used in heterogeneous-agents models: \(b^i_1 \geq 0\).

The budget constraints in the second period differ depending on whether the government defaults or not. If the government repays, the budget constraints take the standard form:

\[
c^i_1 = y + b^i_1 - \tau_1 \quad \text{for } i = L, H. \tag{2}
\]

If the government defaults, there is no repayment on the outstanding debt, and the agents’ budget constraints are:

\[
c^i_1 = (1 - \phi(g_1)) y - \tau_1 \quad \text{for } i = L, H. \tag{3}
\]

As is standard in the sovereign debt literature, we can allow for default to impose an exogenous cost that reduces income by a fraction \(\phi\). This cost is often modeled as a function of the realization of a stochastic endowment income, but since income is constant in this setup, we
model it as a function of the realization of government expenditures in the second period \( g_1 \). In particular, the cost is a non-increasing, step-wise function: \( \phi(g_1) \geq 0 \), with \( \phi'(g_1) \leq 0 \) for \( g_1 \leq \overline{g}_1 \), \( \phi'(g_1) = 0 \) otherwise, and \( \phi''(g_1) = 0 \). Hence, \( \overline{g}_1 \) is a threshold high value of \( g_1 \) above which the marginal cost of default is zero.\(^7\)

### 2.2 Government

At the beginning of \( t = 0 \), the government has outstanding debt \( B_0 \) and can issue one-period, non-state contingent discount bonds \( B_1 \in B \equiv [0, \infty) \) at the price \( q_0 \geq 0 \). Each period it collects lump-sum revenues \( \tau_t \) and pays for \( g_t \). Since \( g_0 \) is known at the beginning of the first period, the relevant uncertainty with respect to government expenditures is for \( g_1 \), which is characterized by a well-defined probability distribution function with mean \( \mu_g \). We do not restrict the sign of \( \tau_t \), so \( \tau_t < 0 \) represents lump-sum transfers.

At equilibrium, the price of debt issued in the first period must be such that the government bond market clears:

\[
B_t = \gamma b_t^L + (1 - \gamma) b_t^H \quad \text{for} \quad t = 0, 1.
\]

This condition is satisfied by construction in period 0. In period 1, however, the price moves endogenously to clear the market.

The government has the option to default at \( t = 1 \). The default decision is denoted by \( d_1 \in \{0, 1\} \) where \( d_1 = 0 \) implies repayment. The government evaluates the values of repayment and default as a benevolent planner with a social welfare function. In the rest of this Section we focus on the case of a standard weighted utilitarian social welfare function: \( \gamma u(c_1^L) + (1 - \gamma) u(c_1^H) \).\(^8\)

The government, however, cannot discriminate across the two types of agents when setting taxation, debt and default policies.

At \( t = 0 \), the government budget constraint is

\[
\tau_0 = g_0 + B_0 - q_0 B_1.
\]

The level of taxes in period 1 is determined after the default decision. If the government

\(^7\)This formulation is analogous to the step-wise default cost as a function of income proposed by Arellano (2008) and now widely used in the external default literature, and it also captures the idea of asymmetric costs of tax collection (see Barro (1979) and Calvo (1988)). Note, however, that for the model to support equilibria with debt under a utilitarian government all we need is \( \phi(g_1) > 0 \). The additional structure is useful for the quantitative analysis and for making it easier to compare the model with the standard external default models. In external default models, the non-linear cost makes default more costly in “good” states, which alters default incentives to make default more frequent in “bad” states, and it also contributes to support higher debt levels.

\(^8\)In Section 5.1 we consider an alternative formulation with arbitrary weights, which can be justified by political economy considerations, and the analysis could also be extended to incorporate egalitarian concerns.
repays, taxes are set to satisfy the following government budget constraint:

$$\tau_{d_1}^{d_1=0} = g_1 + B_1.$$  

(6)

Notice that, since this is a two-period model, equilibrium requires that there are no outstanding assets at the end of period 1 (i.e. \(b_2^i = B_2 = 0\) and \(q_1 = 0\)). If the government defaults, taxes are simply set to pay for government purchases:

$$\tau_{d_1}^{d_1=1} = g_1.$$  

(7)

3 Equilibrium

The analysis of the model’s equilibrium proceeds in three stages. First, we characterize the households’ optimal savings problem and determine their payoff (or value) functions, taking as given the government debt, taxes and default decision. Second, we study how optimal government taxes and the default decision are determined. Third, we examine the optimal choice of debt issuance that internalizes the outcomes of the first two stages.

3.1 Households’ Problem

Given \(B_1\) and \(\gamma\), a household with initial debt holdings \(b_0^i\) for \(i = L, H\) chooses \(b_1^i\) by solving this maximization problem:

$$v_i(B_1, \gamma) = \max_{b_1^i} \left\{ u(y + b_0^i - q_0 b_1^i - \tau_0) + \beta E_{g_1} \left[ (1 - d_1(B_1, g_1, \gamma)) u(y + b_1^i - \tau_{d_1}^{d_1=0}) + d_1(B_1, g_1, \gamma) u(y(1 - \phi(g_1)) - \tau_{d_1}^{d_1=1}) \right] \right\},$$

subject to \(b_1^i \geq 0\). The term \(E_{g_1}[^]\) represents the expected payoff across the repayment and default states in period 1.\(^9\)

The first-order condition, evaluated at the equilibrium level of taxes, yields the following Euler equation:

$$u'(c_0^i) \geq \beta(1/q_0) E_{g_1} \left[ u'(y - g_1 + b_1^i - B_1)(1 - d_1(B_1, g_1, \gamma))] \right], \quad \text{if} \quad b_1^i > 0$$  

(9)

In states in which, given \((B_1, \gamma)\), the value of \(g_1\) is such that the government chooses to default

\(^9\)Notice in particular that the payoff in case of default does not depend on the level of individual debt holdings \((b_1^i)\), reflecting the fact that the government cannot discriminate across households when it defaults.
(d_1(B_1, g_1, \gamma) = 1), the marginal benefit of an extra unit of debt is zero.\textsuperscript{10} Thus, conditional on B_1, a larger default set (i.e. a larger set of values of g_1 such that the government defaults), implies that the expected marginal benefit of an extra unit of savings decreases. This implies that, everything else equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes.\textsuperscript{11} The agents’ Euler equation has an important implication. If the Euler equation holds with equality for H-type agents (i.e. b_H^1 > 0) then L-type agents are credit constrained. This implies that H-type agents are the marginal investor and their Euler equation can be used to derive the equilibrium price.

The equilibrium bond price is the value of q_0(B_1, \gamma) for which, as long as consumption for all agents is non-negative and the default probability of the government is less than 1, the following market-clearing condition holds:

\[ B_1 = \gamma b_L^1(B_1, \gamma) + (1 - \gamma) b_H^1(B_1, \gamma), \]

where B_1 in the left-hand-side of this expression represents the public bonds supply, and the right-hand-side is the aggregate government bond demand.

It is instructive to analyze further the households’ problem assuming logarithmic utility (u(c) = log(c)) and b_L^0 = 0, because under these assumptions we can use Euler equation (9) and the bond market-clearing condition to solve for the “equilibrium” bond price in closed form, and use the solution to establish some important properties of bond prices and the associated default risk spreads. We show in the Appendix that the equilibrium bond price is:

\[ q_0(B_1, \gamma) = \beta \left( \frac{y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0}{1 + \left( \frac{\gamma}{1 - \gamma} \right) \beta B_1 \Pi(B_1, \gamma)} \right) \]

where \( \Pi(B_1, \gamma) \equiv E_{g_1} \left[ \frac{1 - d(B_1, g_1, \gamma)}{y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) B_1} \right] \) is the expected marginal utility of H-type agents for the second period, which as explained earlier weights only non-default states because the marginal benefit of debt is zero in default states. Since, as also shown in the Appendix, \( \frac{\partial \Pi(B_1, \gamma)}{\partial B_1} < 0 \), it follows that \( \frac{\partial q_0(B_1, \gamma)}{\partial B_1} < 0 \) for \( c_H^0 > 0 \). Moreover, since bond prices are decreasing in B_1, it follows that the “revenue” the government generates by selling debt, \( q_0(B_1, \gamma)B_1 \), behaves like the familiar debt Laffer curve of the Eaton-Gersovitz models derived by Arellano (2008).\textsuperscript{12} This

\textsuperscript{10} Utility in the case of default equals u(y(1 - \phi(g_1)) - g_1), and is independent of b_L^1.

\textsuperscript{11} Note also that from the households’ perspective, the individual bond decision has no marginal effect on d_1(B_1, g_1, \gamma).

\textsuperscript{12} It is straightforward to show that revenue R(B_1) = q_0(B_1, \gamma)B_1 follows a Laffer curve in the [0, B_1^{max}] interval, where B_1^{max} is the upper bound of debt such that the government chooses default for any realization of g_1 and thus q_0(B_1^{max}, \gamma) = 0. Since R(0) = 0 with R'(0) = q(0, \gamma) > 0, and R(B_1^{max}) = 0 with R'(B_1^{max}) =
Laffer curve will play a key role later in determining the government’s optimal debt choice. In particular, the government internalizes that higher debt eventually produces decreasing revenues, and that in the decreasing segment of the Laffer curve revenues fall faster as the debt increases, and much faster as default risk rises sharply. A similar expression can be obtained for the “risk-free” price (i.e., the bond price that arises in a model with full commitment), to show that the risk premium is non-negative, and it is strictly positive if there is default in equilibrium. The spread (i.e. the difference between the two prices) is increasing in $B_1$ since the default set is increasing in $B_1$. We also show that the spread is a multiple of $1/\beta \left( y - g_0 + \frac{\gamma}{1-\gamma} B_0 \right)$. As a result, the total date-0 resources available for consumption of the H-types $(y - g_0 + \frac{\gamma}{1-\gamma} B_0)$ have a first-order negative effect on the spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. Changes in $\gamma$ have a similar impact on this ratio also pushing spreads downward. However, as $\gamma$ increases default incentives also strengthens as the welfare of the wealthy is valued less. Thus, in principle the response of the spread to increases in inequality is ambiguous.

### 3.2 Government’s Problem

As explained earlier, we analyze the government’s problem following a backward induction strategy by studying first the default decision problem in the final period $t = 1$, followed by the optimal debt issuance choice at $t = 0$. 

#### 3.2.1 Government Default Decision at $t = 1$

At $t = 1$, the government chooses to default or not by solving this optimization problem:

$$\max_{d \in \{0,1\}} \left\{ W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1, \gamma) \right\},$$

where $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(B_1, g_1, \gamma)$ denote the values of the social welfare function at the beginning of period 1 in the case of repayment and default respectively. Using the government budget constraint to substitute for $\tau_1^{d=0}$ and $\tau_1^{d=1}$, the government’s utilitarian payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^H - B_1) + (1 - \gamma) u(y - g_1 + b_1^H - B_1)$$

$\frac{\partial}{\partial \gamma}(B_1^{\text{max}}, \gamma)B_1^{\text{max}} < 0$, it follows by Rolle’s theorem that $R(B_1)$ has at least one local maximum in $(0, B_1^{\text{max}})$. 

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and
\[ W^d_{1=1}(g_1, \gamma) = u(y(1 - \phi(g_1)) - g_1). \] (14)

Combining these payoff functions, it follows that the government defaults if this condition holds:
\[
\gamma \left[ u(y - g_1 + (b^L_1 - B_1)) - u(y(1 - \phi(g_1)) - g_1) \right] + \\
(1 - \gamma) \left[ u(y - g_1 + (b^H_1 - B_1)) - u(y(1 - \phi(g_1)) - g_1) \right] \leq 0 
\] (15)

Notice that all households lose \( g_1 \) of their income to government absorption regardless of the default choice. Moreover, debt repayment reduces consumption and welfare of \( L \) types and rises them for \( H \) types, whereas default implies the same consumption and utility for both types of agents.

The distributional effects of a default are implicit in condition (15). Given that debt repayment affects the cash-in-hand for consumption of low- and high-wealth agents according to \((b^L_1 - B_1) \leq 0\) and \((b^H_1 - B_1) \geq 0\) respectively, it follows that, for a given \( B_1 \), the payoff under repayment allocates (weakly) lower welfare to \( L \) agents and higher to \( H \) agents, and that the gap between the two is larger the larger is \( B_1 \). Moreover, since the default payoffs are the same for both types of agents, this is also true of the difference in welfare under repayment v. default: It is higher for \( H \) agents than for \( L \) agents and it gets larger as \( B_1 \) rises. To induce default, however, it is necessary not only that \( L \) agents have a smaller difference in the payoffs of repayment v. default, but that the difference is negative (i.e. they must attain lower welfare under repayment than under default), which requires \( B_1 > b^L_1 + y\phi(g_1) \). This also implies that taxes under repayment need to be necessarily larger than under default, since \( \tau^d_{1=0} - \tau^d_{1=1} = B_1 \).

The distributional mechanism determining the default decision can be illustrated more clearly by means of a graphical tool. To this end, it is helpful to express the values of optimal debt holdings as \( b^L_1 = B_1 - \epsilon \) and \( b^H_1(\gamma) = B_1 + \frac{\gamma}{1-\gamma}\epsilon \), for some hypothetical decentralized allocation of debt holdings given by \( \epsilon \in [0, B_1] \). Consumption allocations under repayment would therefore be \( c^L_1(\epsilon) = y - g_1 - \epsilon \) and \( c^H_1(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1-\gamma}\epsilon \), so \( \epsilon \) also determines the decentralized consumption dispersion. The efficient dispersion of consumption that the social planner would choose is characterized by the value of \( \epsilon^{SP} \) that maximizes social welfare under repayment, which satisfies this first-order condition:
\[
u' \left( y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP} \right) = u' \left( y - g_1 - \epsilon^{SP} \right). \] (16)
Hence, the efficient allocations are characterized by zero consumption dispersion, because equal marginal utilities imply \( c_{L,SP} = c_{H,SP} = y - g_1 \), which is attained with \( \epsilon_{SP} = 0 \).

Consider now the government’s default decision when default is costless \( (\phi(g_1) = 0) \). Given that the only policy instruments the government can use, other than the default decision, are non-state contingent debt and lump-sum taxes, it is straightforward to conclude that default will always be optimal. This is because default produces identical allocations in a decentralized equilibrium as the socially efficient ones, since default produces zero consumption dispersion with consumption levels \( c_L = c_H = y - g_1 \). This outcome is invariant to the values of \( B_1, g_1, \gamma \) and \( \epsilon \) (over their relevant ranges). Moreover, in this scenario default also yields the first-best outcome that attains maximum social welfare. This result also implies, however, that the model without default costs cannot support equilibria with domestic debt subject to default risk, because default is always optimal.

The above scenario is depicted in Figure 1, which plots the social welfare function under repayment as a function of \( \epsilon \) as the bell-shaped curve, and the social welfare under default (which is independent of \( \epsilon \)), as the black dashed line. Clearly, the maximum welfare under repayment is attained when \( \epsilon = 0 \) which is also the efficient amount of consumption dispersion \( \epsilon_{SP} \). Recall also that we defined the relevant range of decentralized consumption dispersion for \( \epsilon > 0 \), so welfare under repayment is decreasing in \( \epsilon \) over the relevant range.

**Figure 1: Default Decision and Consumption Dispersion**

These results can be summarized as follows:
Result 1: If \( \phi(g_1) = 0 \) for all \( g_1 \), then for any \( \gamma \in (0, 1) \) and any \( (B_1, g_1) \), the social value of repayment \( W_{d=0}^{d}(B_1, g_1, \gamma) \) is decreasing in \( \epsilon \) and attains its maximum at \( \epsilon^{SP} = 0 \) (i.e. when welfare equals \( u(y - g_1) \)). Hence, default is always optimal for any decentralized consumption dispersion \( \epsilon > 0 \).

The outcome is very different when default is costly. With \( \phi(g_1) > 0 \), default still yields zero consumption dispersion, but at lower levels of consumption and therefore utility, since consumption allocations in the default state become \( c^L = c^H = (1 - \phi(g_1))y - g_1 \). This does not alter the result that the first-best social optimum is \( \epsilon^{SP} = 0 \), but what changes is that default can no longer support the consumption allocations of the first best. Hence, there is now a threshold amount of consumption dispersion in the decentralized equilibrium, \( \tilde{\epsilon}(\gamma) \), which varies with \( \gamma \) and such that for \( \epsilon \geq \tilde{\epsilon}(\gamma) \) default is again optimal, but for lower \( \epsilon \) repayment is now optimal. This is because when \( \epsilon \) is below the threshold, repayment produces a level of social welfare higher than the one that default yields. Figure 1 also illustrates this scenario. These results can be summarized as follows:

Result 2: If \( \phi(g_1) > 0 \), then for any \( \gamma \in (0, 1) \) and any \( (B_1, g_1) \), there is a threshold value of consumption dispersion \( \tilde{\epsilon}(\gamma) \) such that the payoffs of repayment and default are equal: \( W_{d=0}^{d}(B_1, g_1, \gamma) = u(y(1 - \phi(g_1)) - g_1) \). The government repays if \( \epsilon < \tilde{\epsilon}(\gamma) \) and defaults otherwise. Moreover, \( \tilde{\epsilon}(\gamma) \) is decreasing in \( \gamma \).

3.2.2 Government Debt Decision at \( t = 0 \)

We are now in a position to study how the government chooses the optimal amount of debt to issue in the initial period. Before studying the government’s optimization problem, it is important to emphasize that in this model debt is mainly a mechanism for altering consumption dispersion across agents both within a period and across periods. In particular, consumption dispersion in each period and repayment state is given by:

\[
\begin{align*}
&c_0^H - c_0^L = \frac{1}{1 - \gamma} [B_0 - q(B_1, \gamma)B_1] \\
c_1^{H,d=0} - c_1^{L,d=0} = \frac{1}{1 - \gamma} B_1 \\
c_1^{H,d=1} - c_1^{L,d=1} = 0.
\end{align*}
\]

These expressions make it clear that, given \( B_0 \), issuing at least some debt \( (B_1 > 0) \) reduces consumption dispersion at \( t = 0 \) compared with no debt \( (B_1 = 0) \), but increases it at \( t = 1 \) if the government repays (i.e., \( d = 0 \)). Moreover, the debt Laffer curve that governs \( q_0(B_1, \gamma)B_1 \) limits the extent to which debt can reduce consumption dispersion at \( t = 0 \). Starting from \( B_1 = 0 \),
consumption dispersion in the initial period falls as \( B_1 \) increases, but there is a critical positive value of \( B_1 \) beyond which it becomes an increasing function of debt.

At \( t = 0 \), the government chooses its debt policy internalizing the above effects, including the dependence of bond prices on the debt issuance choice, and their implications for social welfare. To be precise, the government chooses \( B_1 \) so as to maximize the “indirect” social welfare function:

\[
W_0(\gamma) = \max_{B_1} \left\{ \gamma v^L(B_1, \gamma) + (1 - \gamma)v^H(B_1, \gamma) \right\}.
\]  

(17)

where \( v^L \) and \( v^H \) are the value functions obtained from solving the households’ problems defined in the Bellman equation (9) taking into account the government budget constraints and the equilibrium pricing function of bonds.

We can gain some intuition about the solution of this maximization problem by deriving its first-order condition and re-arranging it as follows (assuming that the relevant functions are differentiable):

\[
u'(c^H_0) = u'(c^L_0) + \frac{\eta}{q(B_1, \gamma) \gamma} \left\{ \beta E_{g_1} [\Delta d \Delta W_1] + \gamma \mu^L \right\}
\]  

(18)

where

\[
\eta \equiv q(B_1, \gamma) / (q'(B_1, \gamma) B_1) < 0,
\]

\[
\Delta d \equiv d(B_1 + \delta, g_1, \gamma) - d(B_1, g_1, \gamma) \geq 0, \text{ for } \delta > 0 \text{ small},
\]

\[
\Delta W_1 \equiv W^{d=1}_1(g_1, \gamma) - W^{d=0}_1(B_1, g_1, \gamma) \geq 0,
\]

\[
\mu^L \equiv q(B_1, \gamma) u'(c^L_0) - \beta E_{g_1} [(1 - d^l) u'(c^L_1)] > 0.
\]

In these expressions, \( \eta \) is the price elasticity of the demand for government bonds, \( \Delta d \Delta W_1 \) represents the marginal distributional benefit of a default, and \( \mu^L \) is the shadow value of the borrowing constraint faced by \( L \)-type agents.

If both types of agents could be unconstrained in their savings decisions, so that in particular \( \mu_L = 0 \), and if there is no change in the risk of default (or assuming commitment to remove default risk entirely), so that \( E_{g_1} [\Delta d \Delta W_1] = 0 \), then the optimality condition simplifies to:

\[
u'(c^H_0) = u'(c^L_0).
\]

Hence, in this case the social planner would want to issue debt so as to equalize marginal utilities of consumption across agents at date 0, which requires simply setting \( B_1 \) to satisfy \( q(B_1, \gamma) B_1 = B_0 \).
As explained earlier, the above scenario cannot be sustained as an equilibrium outcome, because at equilibrium if $H$-type agents are unconstrained, then $L$-types are constrained (i.e. $\mu_L > 0$). In this case, and still assuming no change in default risk or a government committed to repay, the optimality condition reduces to:

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta \mu_L}{q(B_1, \gamma)}.$$  

Since $\eta < 0$, this result implies $c_0^L < c_0^H$, because $u'(c_0^L) > u'(c_0^H)$. Thus, even with unchanged default risk or no default risk at all, the government’s debt choice sets $B_1$ as needed to maintain an optimal, positive level of consumption dispersion, which is the one that supports an excess in marginal utility of $L$-type agents relative to $H$-type agents equal to $\frac{\eta \mu_L}{q(B_1, \gamma)}$. Moreover, since optimal consumption dispersion is positive, we can also ascertain that $B_0 > q(B_1, \gamma)B_1$, which using the government budget constraint implies that the government runs a primary surplus at $t = 0$. The government borrows resources, but less than it would need in order to eliminate all consumption dispersion (which requires zero primary balance).

The intuition for the optimality of issuing debt can be presented in terms of tax smoothing and savings: Date-0 consumption dispersion without debt issuance would be $B_0/(1 - \gamma)$, but this is more dispersion than what the government finds optimal, because by choosing $B_1 > 0$ the government provides tax smoothing (i.e. reduces date-0 taxes) for everyone, which in particular cases the $L$-type agents credit constraint, and provides also a desired vehicle of savings for $H$ types. Thus, positive debt increases consumption of $L$ types (since $c_0^L = y - g_0 - B_0 + q(B_1)B_1$), and reduces consumption of $H$ types (since $c_0^H = y - g_0 + \left(\frac{\gamma}{1 - \gamma}\right)(B_0 - q(B_1)B_1)$). But issuing debt (assuming repayment) also increases consumption dispersion a $t = 1$, since debt is then paid with higher taxes on all agents, while $H$ agents collect also the debt repayment. Thus, the debt is being chosen optimally to trade off the social costs and benefits of reducing (increasing) date-0 consumption and increasing (reducing) date-1 consumption for rich (poor) agents. In doing so, the government internalizes the debt Laffer curve and the fact that additional debt lowers the price of bonds and helps reduce $\mu_L$, which in turn reduces the government’s optimal consumption dispersion.\textsuperscript{13}

In the presence of default risk and if default risk changes near the optimal debt choice, the term $E_{g_1} [\Delta d \Delta W_1]$ enters in the government’s optimality condition with a positive sign, which means the optimal gap in the date-0 marginal utilities of the two agents widens even more. Hence, the government’s optimal choice of consumption dispersion for $t = 0$ is greater than

\textsuperscript{13}Note, however, that without default risk the Laffer curve has less curvature than with default risk, because $q_0^{ND}(B_1, \gamma) \geq_0 (B_1, \gamma)$. 

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without default risk, and the expected dispersion for \( t = 1 \) is lower, because in some states of
the world the government will choose to default and consumption dispersion would then drop
to zero. This also suggests that the government chooses a lower value of \( B_1 \) than in the absence
of default risk, since date-0 consumptions are further apart. Moreover, the debt Laffer curve
now plays a central role in the government’s weakened incentives to borrow, because as default
risk rises the price of bonds drops to zero faster and the resources available to reduce date-0
consumption dispersion peak at lower debt levels. In short, default risk reduces the government’s
ability to use non-state-contingent debt in order to reduce consumption dispersion.

In summary, the more constrained the \( L \)–types agents are (higher \( \mu^L \)) or the higher the
expected distributional benefit of a default (higher \( E_{g_1}[\Delta d\Delta W_1] \)), the larger the level of debt the
government finds optimal to issue. Both of these mechanisms operate as pecuniary externalities:
They matter only because the government debt choice can alter the equilibrium price of bonds
which is taken as given by private agents.

3.3 Competitive Equilibrium with Optimal Debt & Default Policy

For a given value of \( \gamma \), a **Competitive Equilibrium with Optimal Debt and Default
Policy** is a pair of household value functions \( v^i(B_1, \gamma) \) and decision rules \( b^i(B_1, \gamma) \) for
\( i = L, H \), a government bond pricing function \( q_0(B_1, \gamma) \) and a set of government policy functions \( \tau_0(B_1, \gamma), \tau^d_1(B_1, g_1, \gamma), d(B_1, g_1, \gamma), B_1(\gamma) \) such that:

1. Given the pricing function and government policy functions, \( v^i(B_1, \gamma) \) and \( b^i_1(B_1, \gamma) \) solve
the households’ problem.

2. \( q_0(B_1, \gamma) \) satisfies the market-clearing condition of the bond market (equation (10)).

3. The government default decision \( d(B_1, g_1, \gamma) \) solves problem (12).

4. Taxes \( \tau_0(B_1, \gamma) \) and \( \tau^d_1(B_1, g_1, \gamma) \) are consistent with the government budget constraints.

5. The government debt policy \( B_1(\gamma) \) solves problem (17).

4 Quantitative Analysis

In this Section, we study the model’s quantitative predictions based on a calibration using
European data. The goal is to show whether a reasonable set of parameter values can produce an
equilibrium with debt subject to default risk, and to study how the properties of this equilibrium
change with the model’s key parameters. Since the two-period model is not well suited to account
for the time-series dynamics of the data, we see the results more as an illustration of the potential relevance of the model’s argument for explaining domestic default rather than as an evaluation of the model’s general ability to match observed public debt dynamics.\textsuperscript{14}

4.1 Calibration

The model is calibrated to annual frequency, and most of the parameter values are set so as to approximate some of the model’s predicted moments to those observed in the European data. The calibrated parameter values are summarized in Table 2.

Table 2: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta$ 0.96</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\sigma$ 1.00</td>
</tr>
<tr>
<td>Avg. Income</td>
<td>$y$ 0.79</td>
</tr>
<tr>
<td>Low household wealth</td>
<td>$b^L_0$ 0.00</td>
</tr>
<tr>
<td>Avg. Gov. Consumption</td>
<td>$\mu_g$ 0.18</td>
</tr>
<tr>
<td>Autocorrel. G</td>
<td>$\rho_g$ 0.88</td>
</tr>
<tr>
<td>Std Dev Error</td>
<td>$\sigma_e$ 0.017</td>
</tr>
<tr>
<td>Initial Gov. Debt</td>
<td>$B_0$ 0.35</td>
</tr>
<tr>
<td>Output Cost Default</td>
<td>$\phi_0$ 0.004</td>
</tr>
</tbody>
</table>

Note: Government expenditures, income and debt values are derived using data from France, Germany, Greece, Ireland, Italy, Spain and Portugal.

The preference parameters are set to standard values: $\beta = 0.96$, $\sigma = 1$. We also assume for simplicity that $L$ types start with zero wealth, $b^L_0 = 0$.\textsuperscript{15} We assume that

$$\ln(g_1) \sim N\left((1 - \rho_g) \ln(\mu_g) + \rho_g \ln(g_0), \frac{\sigma_e^2}{(1 - \rho_g^2)}\right).$$\textsuperscript{16}

\textsuperscript{14}We solve the model following a similar backward-recursive strategy as in the theoretical analysis. First, taking as given a set of values $\{B_1, \gamma\}$, we solve for the equilibrium pricing and default functions by iterating on $(q_0, b_1)$ until the date-0 bond market clears when the date-1 default decision rule solves the government’s optimal default problem (12). Then, in the second stage we complete the solution of the equilibrium by finding the optimal choice of $B_1$ that solves the government’s date-0 optimization problem (17). It is important to recall that, as explained earlier, for given values of $B_1$ and $\gamma$, an equilibrium with debt will not exist if either the government finds it optimal to default on $B_1$ for all realizations of $g_1$ or if at the given $B_1$ the consumption of $L$ types is non-positive. In these cases, there is no finite price that can clear the debt market.

\textsuperscript{15}$\sigma = 1$ and $b^L_0 = 0$ are also useful because, as we show in the Appendix, under these assumptions we can obtain closed-form solutions and establish some results analytically.

\textsuperscript{16}This specification allows us to control the correlation between $g_0$ and $g_1$ via $\rho_g$, the mean of the shock via $\mu_g$ and the variance of the unpredicted portion via $\sigma_e^2$. Note that if $g_0 = \mu_g$, $\ln(g_1) \sim N(\ln(\mu_g), \frac{\sigma_e^2}{(1 - \rho_g^2)}).$
The parameters of this process are pinned down by estimating an AR(1) model with 1995-2012 data of the government expenditures-GDP ratio (in logs) for France, Germany, Greece, Ireland, Italy, Spain and Portugal. Given the parameter estimates for each country, we set $\mu_g$, $\rho_g$ and $\sigma_e$ to the corresponding cross-country average. This results in the following values $\mu_g = 0.1812$, $\rho_g = 0.8802$ and $\sigma_e = 0.017$. Given these moments, we set $g_0 = \mu_g$ and use Tauchen (1986) quadrature method with 45 nodes in $G_1 \equiv \{g_1, \ldots, g_1\}$ to generate the realizations and transition probabilities of the Markov process that drives expectations about $g_1$.\(^{17}\)

We abstain from setting a calibrated value for $\gamma$ and instead show results for $\gamma \in [0, 1]$. Note, however, that data for the United States and Europe suggest that the empirically relevant range for $\gamma$ is $[0.55, 0.85]$, and hence when taking a stance on a particular value of $\gamma$ is useful we use $\gamma = 0.7$, which is the mid point of the plausible range.\(^{18}\)

Average income $y$ is calibrated such that the model’s aggregate resource constraint is consistent with the data when GDP is normalized to one. This implies that the value of households’ aggregate endowment must equal GDP net of fixed capital investment and net exports, since the latter two are not explicitly modeled. The average for the period 1970-2012 for the same set of countries used to estimate the $g_1$ process implies $y = 0.7883$. Note also that under this calibration of $y$ and the Markov process of $g_1$, the gap $y - g_1$ is always positive, even for $g_1 = \overline{g}_1$, which in turn guarantees $c_1^H > 0$ in all repayment states.

We set the value of $B_0$ to match the median value of government debt for Euro-zone countries reported in Table 1. As we describe in the next section, even though the optimal $B_1$ (close to 15 percent of GDP for $\gamma \in [0.55, 0.85]$) is higher than those observed in the sovereign debt literature, in this two-period model the consumption smoothing mechanism induces a reduction in the optimal $B_1$ relative to the initial condition $B_0$ even in a deterministic version of the model with stationary government purchases. Under these assumptions, the optimal debt choice is decreasing in $\gamma$ and has an upper bound of $B_1 = B_0/(1 + \beta)$ as $\gamma \to 0$.\(^{19}\) In addition, when default risk is introduced debt needs to be below the level that would lead the government to default in the second period with probability 1, and above the level at which either $c_L^0$ or $c_L^H$.

\(^{17}\)As it is standard, we assume that $g_1$ is an equally spaced grid with $\log(g_1)$ and $\log(\overline{g}_1)$ located -/+ 3 standard deviations from $\log(\mu_g)$ respectively.

\(^{18}\)In the United States, the 2010 Survey of Consumer Finances indicates that only 12% of households hold savings bonds but 50.4% have retirement accounts (which are very likely to own government bonds). These figures would suggest values of $\gamma$ ranging from 50% to 88%. In Europe, comparable statistics are not available for several countries, but recent studies show that the wealth distribution is highly concentrated and that the wealth Gini coefficient ranges between 0.55 and 0.85 depending on the country and the year of the study (see Davies, Sandstr\"{o}m, Shorrocks, and Wolff (2009)). In our model, if $b_L^0 = 0$, the Gini coefficient of wealth is equal to $\gamma$.

\(^{19}\)This occurs because as $\gamma \to 0$, the model in deterministic form collapses to a representative agent economy inhabited by H types where the optimal debt choice yields stationary consumption, $q_0 = 1/\beta$, and $B_1 = B_0/(1 + \beta)$. In contrast, an infinite horizon, stationary economy yields $B_1 = B_0$ (see Appendix for details).
become non-positive, otherwise there is no equilibrium.

The functional form of the default cost function is the following: \( \phi(g_1) = \phi_0 + (\bar{g}_1 - g_1)/y \), where \( \bar{g}_1 \) is calibrated to represent an “unusually large” realization of \( g_1 \) set equal to the largest realization in the Markov process of government expenditures, which is in turn set equal to 3 standard deviations from the mean (in the process in logs).\(^{20}\) We calibrate \( \phi_0 \) to match an estimate of the observed frequency of domestic defaults. According to Reinhart and Rogoff (2011), historically, domestic defaults are about 1/4 as frequent as external defaults (68 domestic v. 250 external in data since 1750). Since the probability of an external default has been estimated in the range of 3 to 5% (see for example Arellano (2008)), we estimate the probability of a domestic default at about 1%. The model is close to this default frequency on average when solved over the empirically relevant range of \( \gamma \)'s (\( \gamma \in [0.55, 0.85] \)) if we set \( \phi_0 = 0.004 \). Note, however, that this calibration of \( \phi_0 \) to target the default probability and the calibration of \( B_0 \) to the target described early needs to be done jointly by repeatedly solving the model until both targets are well approximated.

4.2 Results

We examine the quantitative results in the same order in which the backward solution algorithm works. We start with the second period’s utility of households under repayment and default. We then move to the first period and examine the equilibrium bond prices. Finally, we study the optimal government debt issuance \( B_1 \) for a range of values of \( \gamma \).

4.2.1 Second period default incentives for given \((B_1, g_1, \gamma)\)

Using the agents’ optimal choice of bond holdings, we compute the equilibrium utility levels they attain at \( t = 1 \) under repayment v. default for different triples \((B_1, g_1, \gamma)\). Since we are looking at the last period of a two-period model, these compensating variations reduce simply

\(^{20}\)This cost function shares a key feature of the default cost functions widely used in the external default literature to align default incentives so as to support higher debt ratios and trigger default during recessions (see Arellano (2008) and Mendoza and Yue (2012)): The default cost is an increasing function of disposable income \((y - g_1)\). In addition, this formulation ensures that households’ consumption during a default never goes above a given threshold.
to the percent changes in consumption across the default and no-default states of each agent.\footnote{These calculations are straightforward given that in the equilibria we solve for $b^L_1 = 0$, and hence $b^H_1 = B_1/(1 - \gamma)$. The same formula would apply, however, even if these conditions do not hold, using instead the policy functions $b^i_1(B_1, g_1, \gamma)$ that solve the households’ problems for any given pair of functions $d_1(B_1, g_1, \gamma)$ and $q_0(B_1, \gamma)$, including the ones that are consistent with the government’s default decision and equilibrium in the bond market.}

$$\alpha^i(B_1, g_1, \gamma) = \frac{c^i_{1,d^i=1}(B_1, g_1, \gamma)}{c^i_{1,d^i=0}(B_1, g_1, \gamma)} - 1 = \frac{(1 - \phi(g_1))y - g_1}{y - g_1 + b^i_1 - B_1} - 1$$

A positive (negative) value of $\alpha^i(B_1, g_1, \gamma)$ implies that agent $i$ prefers government default (repayment) by an amount equivalent to an increase (cut) of $\alpha^i(\cdot)$ percent in consumption.

The individual welfare gains of default are aggregated using $\gamma$ to obtain the utilitarian representation of the social welfare gain of default:

$$\overline{\alpha}(B_1, g_1, \gamma) = \gamma \alpha^L(B_1, g_1, \gamma) + (1 - \gamma) \alpha^H(B_1, g_1, \gamma).$$

A positive value indicates that default induces a social welfare gain and a negative value a loss.

Figure 2: Social Welfare Gain of Default $\overline{\alpha}(B_1, g_1, \gamma)$

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Figure 2 shows two intensity plots of the social welfare gain of default for the ranges of values of $B_1$ and $\gamma$ in the vertical and horizontal axes respectively. Panel $(i)$ is for a low value of government purchases, $g_1$, set 3 standard deviations below $\mu_g$ (in the process in logs), and panel
(ii) is for a high value \( \overline{g}_1 \) set 3 standard deviations above \( \mu_g \) (in the process in logs). The intensity of the color or shading in these plots indicates the magnitude of the welfare gain according to the legend shown to the right of the plots. The regions shown in dark blue and marked as “No Equilibrium Zone”, represent values of \((B_1, \gamma)\) for which the debt market collapses and no equilibrium exists. In this zone, there is no equilibrium because, at the given \( \gamma \), the government chooses to default on the given \( B_1 \) for all values of \( g_1 \).\(^{22}\)

The area in which the social welfare gains of default are well defined in these intensity plots illustrates two of the key mechanisms driving the government’s distributional incentives to default: First, fixing \( \gamma \), the welfare gain of default is higher at higher levels of debt, or conversely the gain of repayment is lower. Second, keeping \( B_1 \) constant, the welfare gain of default is also increasing in \( \gamma \) (i.e. higher wealth concentration increases the welfare gain of default). This implies that lower levels of wealth dispersion are sufficient to trigger default at higher levels of debt.\(^{23}\) For example, for a debt of 20% of GDP (\( B_1 = 0.20 \)) and \( g_1 = \overline{g}_1 \), social welfare is higher under repayment if \( 0 \leq \gamma \leq 0.10 \) but it becomes higher under default if \( 0.10 < \gamma \leq 0.6 \), and for higher \( \gamma \) there is no equilibrium because the government prefers default not only for \( g_1 = \overline{g}_1 \) but for all possible \( g_1 \). If instead the debt is 35% of GDP, then social welfare is higher under default for all the values of \( \gamma \) for which an equilibrium exists.

The two panels in Figure 2 differ in that panel (ii) displays a well-defined transition from a region in which repayment is socially optimal (\( \overline{\pi}(B_1, g_1, \gamma) < 0 \)) to one where default is optimal (\( \overline{\pi}(B_1, g_1, \gamma) > 0 \)) but in panel (i) the social welfare gain of default is never positive, so repayment is always optimal. This reflects the fact that higher \( g_1 \) also weakens the incentives to repay.

The default decision is directly linked to Figure 2.\(^{24}\) In particular, the repayment region \((d(B_1, g_1, \gamma) = 0)\) corresponds to \( \overline{\pi}(B_1, g_1, \gamma) < 0 \) and the default region \((d(B_1, g_1, \gamma) = 1)\) to \( \overline{\pi}(B_1, g_1, \gamma) > 0 \). Default occurs over a wider set of \((B_1, \gamma)\) pairs at higher levels of government expenditures, and for given \( g_1 \) it is more likely the higher \( B_1 \) or \( \gamma \). From the default decision, we can compute a threshold value of \( \gamma \) such that the government is indifferent between defaulting and repaying in period for a given \((B_1, g_1)\) that we denote \( \hat{\gamma}(B_1, g_1) \). For any given \((B_1, g_1)\),

\(^{22}\)There is another potential “No Equilibrium Zone” that could arise if the given \((B_1, \gamma)\) would yield \( c_0^L \leq 0 \) at the price that induces market clearing, and so the government would not supply that particular \( B_1 \). This happens for low levels of \( B_1 \) relative to \( B_0 \). To determine if \( c_0^L \leq 0 \) at some \((B_1, \gamma)\) we need \( q_0(B_1, \gamma) \), since combining the budget constraints of the \( L \) types and the government yields \( c_0^L = y - g_0 - B_0 + q_0 B_1 \). Hence, to evaluate this condition we take the given \( B_1 \) and use the \( H \) types Euler equation and the market clearing condition to solve for \( q_0(B_1, \gamma) \), and then determine if \( y - g_0 - B_0 + q_0 B_1 \leq 0 \), if this is true, then \((B_1, \gamma)\) is in the lower no equilibrium zone.

\(^{23}\)Note that the cross-sectional variance of initial debt holdings is given by \( Var(b) = B^2 \frac{1}{1-\gamma} \) when \( b_0^L = 0 \). This implies that the cross-sectional coefficient of variation is equal to \( CV(b) = \frac{1}{1-\gamma} \), which is increasing in \( \gamma \) for \( \gamma < 1/2 \).

\(^{24}\)See the corresponding Figure in the Appendix.
the government chooses to default if $\gamma \geq \hat{\gamma}$. Default incentives depicted in Figure 2 result in a default threshold that is decreasing in $B_1$ and $g_1$. This default threshold determines whether debt is exposed to default risk or if the equilibrium exists at all. If, for a given value of $B_1$, $\gamma$ is higher than the curve representing $\hat{\gamma}(B_1, g_1)$, the government defaults for sure and there is no equilibrium. Alternatively, if $\gamma$ is lower than $\hat{\gamma}(B_1, g_1)$, the government repays for sure and debt would be issued effectively without default risk. Thus, for the model to support equilibria with debt subject to default risk, the optimal debt chosen by the government in the first period for a given $\gamma$ must lie between these two extreme threshold curves.

4.2.2 Bond prices for given $(B_1, \gamma)$

Figure 3 shows equilibrium bond prices and a comparison with the prices from the model with the government committed to repay. This figure presents $q_0(B_1, \gamma)$ as a function of $\gamma$ for three values of $B_1$ ($B_L < B_M < B_H$ with $B_M$ set to the value that we show later corresponds to the optimal debt choice of the government when $\gamma = 0.5$). The bond price functions are truncated when the equilibrium does not exist.

Figure 3 illustrates three key features of public debt prices discussed in Section 3: (i) The equilibrium price is decreasing in $B_1$ for given $\gamma$ (the pricing functions shift downward as $B_1$ rises). This follows from a standard demand-and-supply argument: For a given $\gamma$, as the
government borrows more, the price at which the $H$ types are willing to demand the additional debt falls and the interest rate rises. The equilibrium price must be consistent with the demand for bonds implied by their Euler equation, which implies that, for any $b_0^H$, as $B_1$ increases, a lower $q_0$ is needed to induce the rise in $b_1^H$ that clears the bond market. This effect is present whether there is risk of default or not, and even without uncertainty.\footnote{See the Appendix for proofs showing that $q'(B_1) < 0$ in all these scenarios for the log utility case.}

(ii) Default risk reduces the price of bonds below the risk-free price and thus induces a risk premium. Prices are either identical or nearly identical, and the spreads zero or negligible, for all three levels of debt at levels of inequality such that $\gamma \leq 0.50$. This corresponds to combinations of $\{B_1, \gamma\}$ such that the probability of default is either zero or very close to zero. As $\gamma$ increases above 0.50, however, default risk becomes nontrivial and bond prices subject to default risk fall sharply below the risk free prices.

(iii) Bond prices are a non-monotonic function of wealth dispersion: When default risk is sufficiently low, bond prices are increasing in $\gamma$, but eventually they become a steep decreasing function of $\gamma$. Whether default risk is increasing or decreasing in $\gamma$ depends on the relative strength of the composition effect mentioned earlier. When $\gamma$ increases $H$-type agents become a smaller fraction of the population and wealthier and a higher $q_0(B_1, \gamma)$ is needed to clear the market. At the same time, default risk rises (since higher wealth dispersion strengthens default incentives), the growing risk premium becomes the dominating force (at about $\gamma > 0.5$) and produces bond prices that fall sharply as $\gamma$ increases.\footnote{In the Appendix, we show the bond demand decision rules (for given $B_1$) of the $H$ types that validates the intuition provided here.}

4.2.3 Optimal Debt Choice

Given the solutions for household decision rules, tax policies, bond pricing function and default decision rule, we finally solve for the government’s optimal choice of debt issuance in the first period (i.e. the optimal $B_1$ that solves problem (17)) for a range of values of $\gamma$. Moreover, given this optimal debt, we go back and identify the equilibrium values of the rest of the model’s endogenous variables that are associated with optimal debt choices.
Figure 4: Equilibrium Optimal Government Debt Policy

Figure 4 shows the four main components of the equilibrium: Panel (i) plots the optimal first-period debt issuance in the model with default risk, $B_1^*(\gamma)$, and in the case when the government is committed to repay so that the debt is risk free, $B_1^{RF}(\gamma)$; Panel (ii) shows the equilibrium debt prices that correspond to the optimal debt of the same two economies; Panel (iii) shows the default spread (the difference in the inverses of the bond prices); and Panel (iv) shows the probability of default. Since the government that has the option to default can still choose a debt level for which it prefers to repay in all realizations of $g_1$, we identify with a square in Panel (i) the equilibria in which $B_1^*(\gamma)$ has a positive default probability. This is the case for all but the smallest value of gamma considered ($\gamma = 0.05$), in which the government sets $B_1^*(\gamma)$ at 20% of GDP with zero default probability.

Panel (i) shows that optimal debt falls as $\gamma$ increases in both the economy with default risk and the economy with a government committed to repay. This occurs because, as explained in Section 3, in both cases the government seeks to reallocate consumption across agents and across periods by altering the product $q(B_1)B_1$ optimally, and in doing this it internalizes the
response of bond prices to its choice of debt. As \( \gamma \) rises, this response is influenced by stronger default incentives and a stronger demand composition effect. The latter dominates in this quantitative experiment, because panel (ii) shows that the equilibrium bond prices always rise with \( \gamma \). Hence, the government internalizes that as \( \gamma \) rises the demand composition effect strengthens demand for bonds, pushing bond prices higher, and as a result it can actually attain a higher \( q(B_1)B_1 \) by choosing lower \( B_1 \). This is a standard Laffer curve argument: In the upward sloping segment of this curve, increasing debt increases the amount of resources the government acquires by borrowing in the first period.

Although the Laffer curve argument and the demand composition effect explain why both \( B^*_1(\gamma) \) and \( B^{RF}_1(\gamma) \) are decreasing in \( \gamma \), default risk is not innocuous. As Panel (i) shows, the optimal \( B_1 \) choices of the government that cannot commit to repay are lower than those of the government that can. This reflects the fact that the government optimally chooses smaller debt levels once it internalizes the effect of default risk on the debt Laffer curve and its distributional implications.

The negative relationship between \( B_1 \) and \( \gamma \) is in line with the empirical evidence on the negative relationship between public debt ratios and wealth Gini coefficients at relatively high levels of inequality documented in the Introduction. The model cannot, however, account for the positive relationship observed at lower levels of inequality.\(^{27}\) In the range of empirically relevant values of \( \gamma \), optimal debt ratios range from 5 to 15% of GDP with default risk. Since the median in the European data was 35%, these ratios are relatively low, but still they are notable given the simplicity of the two-period setup, with the inertia to reduce debt levels and constraints on feasible equilibrium debt positions discussed earlier.

Panels (iii) and (iv) show that, in contrast with standard models of external default, in this model the default spread is neither similar to the probability of default nor does it have a monotonic relationship with it.\(^{28}\) Both the spread and the default probability start at zero for \( \gamma = 0.05 \) because \( B^*_1(0.05) \) has zero default probability. As \( \gamma \) increases up to 0.5, both the spread and the default probability of the optimal debt choice are similar in magnitude and increase together, but for the regions where the default probability is constant (for \( \gamma > 0.5 \)) the spread falls with \( \gamma \). These results are in line with the findings of the theoretical analysis in Section 3.

Why is the default probability of the optimal debt position show constant regions as \( \gamma \) rises above 0.5, even though \( B^*_1(\gamma) \) falls sharply? The answer is illustrated in Figure 5. This figure shows the optimal debt choice \( B^*_1(\gamma) \) plotted in Figure 4 (with the axis inverted) together with

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\(^{27}\)In this exercise the relationship is conditional on the structure of the model and all the parameters held constant as \( \gamma \) varies in Figure 4, while in the empirical analysis we conditioned the relationship in a non-structural fashion by introducing country fixed effects and the size of the government as control variables.

\(^{28}\)In the standard models, the two are similar and a monotonic function of each other because of the arbitrage condition of a representative risk-neutral
curves for the wealth concentration default thresholds \( \hat{\gamma}(B_1, g_1) \) corresponding to three values of \( g_1 \) (the lowest and highest realizations, and the mean of \( g_1 \) respectively).

Figure 5: Default Threshold, Debt Policy and Equilibrium Default

![Graph showing default threshold and debt policy](image)

Note: \( g \) and \( \bar{g} \) are the smallest and largest possible realizations of \( g_1 \) in the Markov process of government expenditures, which are set to \(-/+/\) 3 standard deviations off the mean respectively. The dotted lines correspond to a set of selected thresholds for different values of \( g_1 \) (the plot shows only 1/3 of the total values of \( g_1 \) we use to solve the model).

As Figure 5 shows, for \( \gamma \leq 0.5 \) the optimal debt level is relatively vertical and goes from below the threshold for \( \bar{g} \) (implying risk-free debt) to a region with default risk that increases with gamma (i.e. crosses vertically the thresholds for lower values of \( g_1 \)). As it reaches a value of \( \gamma = 0.50 \) the optimal level of debt becomes parallel to the threshold \( \hat{\gamma} \) and results in a flat default probability (as it occurs, for example, when \( \gamma \) is higher than 0.75 in panel (iv) of Figure 4).

The government is choosing debt responding to its incentives to reallocate consumption across agents and across periods internalizing the dependence of the debt Laffer curve on its borrowing decision. In fact, whenever \( B_1^*(\gamma) \) is exposed to nontrivial risk of default at equilibrium (i.e. values of \( \gamma ge 0.50 \)), \( B_1^*(\gamma) \) yields the maximum resources to the government that the Laffer curve allows. Figure 6 illustrates this result. It shows debt Laffer curves for five values of \( \gamma \) in the \([0.05,0.95]\) range.\(^{29}\)

\(^{29}\)Each curve is truncated at values of \( B_1 \) in the horizontal axis that are either low enough for \( c_0^t \leq 0 \) or high enough for default to be chosen for all realizations of \( g_1 \), because as noted before in these cases there is no equilibrium.
When $\gamma \geq 0.50$, $B^*_1(\gamma)$ is located at the maximum of the corresponding Laffer curve. In these cases, setting debt higher than at the maximum of the Laffer curve is suboptimal because default risk reduces bond prices sharply, moving the government to the downward sloping region of the Laffer curve, and setting it lower is also suboptimal because then default risk is low and extra borrowing generates more resources since bond prices change little, leaving the government in the upward sloping region of the Laffer curve. Thus, if the optimal debt has a nontrivial probability of default, the government’s debt choice exhausts its ability to raise resources by borrowing. For lower values of $\gamma$, the government’s optimal debt is to the left of the maximum of the Laffer curve, and thus the debt choice does not exhaust the government’s ability to raise resources by borrowing. This happens when the default probability is either zero or positive but small. For example, when $\gamma = 0.25$ the default probability is close to zero and the optimal debt choice is to the left of the maximum of the corresponding Laffer curve.

5 Extensions

This Section presents the results of four important extensions of the model. First, the political bias case in which the social welfare function assigns weighs to agents that deviate from the
fraction of $L$ and $H$ types observed in the economy. Second, the case in which agents have access to a second asset as vehicle for saving. Third, an economy in which risk-neutral foreign investors can buy some of the government’s debt. Fourth, a case in which proportional distortionary taxes on consumption can be used as an alternative tool for redistributive policy.

5.1 Biased Welfare Weights

Assume now that the weights of the government’s payoff function differ from the utilitarian weights $\gamma$ and $1 - \gamma$. This can be viewed as a situation in which, for political reasons or related factors, the government’s welfare weights are biased in favor of one group of agents. The government’s welfare weights on L- and H-type households are denoted $\omega$ and $(1 - \omega)$ respectively, and we refer to $\omega$ as the government’s political bias.

The government’s default decision at $t = 1$ is determined by following the following optimization problem:

$$\max_{d \in \{0, 1\}} \left\{ W_{d^=0}(B_1, g_1, \gamma, \omega), W_{d^=1}(g_1) \right\},$$

(19)

where $W_{d^=0}(B_1, g_1, \gamma, \omega)$ and $W_{d^=1}(g_1)$ denote the government’s payoffs in the cases of no-default and default respectively. Using the government budget constraints to substitute for $\tau_{d^=0}$ and $\tau_{d^=1}$, the government payoffs can be expressed as:

$$W_{d^=0}(B_1, g_1, \gamma, \omega) = \omega u(y - g_1 + b_L^L - B_1) + (1 - \omega)u(y - g_1 + b_H^L - B_1)$$

(20)

and

$$W_{d^=1}(g_1) = u(y(1 - \phi(g_1)) - g_1).$$

(21)

We follow the same approach as before to characterize the optimal default decision graphically. The parameter $\epsilon$ is used again to represent the dispersion of hypothetical decentralized consumption allocations under repayment: $c^L(\epsilon) = y - g_1 - \epsilon$ and $c^R(\gamma, \epsilon) = y - g_1 + \frac{\gamma}{1 - \gamma} \epsilon$. Under default the consumption allocations are again $c^L = c^R = y(1 - \phi(g_1)) - g_1$. Recall that under repayment, the dispersion of consumption across agents increases with $\epsilon$, and under default there is zero consumption dispersion. The repayment government payoff can now be rewritten as:

$$W_{d^=0}(\epsilon, g_1, \gamma, \omega) = \omega u(y - g_1 + \epsilon) + (1 - \omega)u \left( y - g_1 + \frac{\gamma}{1 - \gamma} \epsilon \right).$$

The socially efficient planner chooses its optimal consumption dispersion $\epsilon^{SP}$ as the value of $\epsilon$ that maximizes the above expression. Since as of $t = 1$ the only instrument the government can use to manage consumption dispersion relative to what the decentralized allocations support is
the default decision, it will repay only if doing so allows it to get closer to $\epsilon^{SP}$ than by defaulting.

The planner’s optimality condition is now:

$$\frac{u'(c^H_1)}{u'(c^L_1)} = \frac{u'(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP})}{u'(y - g_1 - \epsilon^{SP})} = \frac{(\omega)}{(1-\gamma)\gamma} \left(1 - \frac{\gamma}{1-\omega}\right).$$ (22)

This condition implies that optimal consumption dispersion for the planner is zero only if $\omega = \gamma$. For $\omega > \gamma$ the planner likes consumption dispersion to favor $L$ types so that $c^L_1 > c^H_1$, and the opposite holds for $\omega < \gamma$.

The key difference with political bias v. the model with a utilitarian government is that the former can support equilibria with debt subject to default risk even without default costs. Assuming $\phi(g_1) = 0$, there are two possible scenarios depending on the relative size of $\gamma$ and $\omega$. First, if $\omega \geq \gamma$, the planner again always chooses default as in the setup of Section 2. This is because for any decentralized consumption dispersion $\epsilon > 0$, the consumption allocations feature $c^H > c^L$, while the planner’s optimal consumption dispersion requires $c^H \leq c^L$, and hence $\epsilon^{SP}$ cannot be implemented. Default brings the planner the closest it can get to the payoff associated with $\epsilon^{SP}$ and hence it is always chosen.

In the second scenario $\omega < \gamma$ (i.e. the political bias assigns more (less) weight to $H$ ($L$) types than the fraction of each type of agents that actually exists). In this case, the model can support equilibria with debt even without default costs. In particular, there is a threshold consumption dispersion $\hat{\epsilon}$ such that default is optimal for $\epsilon \geq \hat{\epsilon}$, where $\hat{\epsilon}$ is the value of $\epsilon$ at which $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ and $W_1^{d=1}(g_1)$ intersect. For $\epsilon < \hat{\epsilon}$, repayment is preferable because $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W_1^{d=0}(g_1)$. Thus, without default costs, equilibria for which repayment is optimal require two conditions: (a) that the government’s political bias favors bond holders ($\omega < \gamma$), and (b) that the debt holdings chosen by private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Figure 7 illustrates the outcomes described above. This Figure plots $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ for $\omega \leq \gamma$. The planner’s default payoff and the values of $\epsilon^{SP}$ for $\omega \leq \gamma$ are also identified in the plot. The vertical intercept of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ is always $W_1^{d=1}(g_1)$ for any values of $\omega$ and $\gamma$, because when $\epsilon = 0$ there is zero consumption dispersion and that is also the outcome under default. In addition, the bell-shaped form of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ follows from $u'(.) > 0, u''(.) < 0$.\[30\]

\[30\]Note in particular that $\frac{\partial W_1^{d=0}(\epsilon, g_1, \gamma, \omega)}{\partial \epsilon} \geq 0 \iff \frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))} \geq \left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right)$. Hence, the planner’s payoff is increasing (decreasing) at values of $\epsilon$ that support sufficiently low (high) consumption dispersion so that $\frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))}$ is above (below) $\left(\frac{\omega}{\gamma}\right)\left(\frac{1-\gamma}{1-\omega}\right)$.\]
Take first a scenario with $\omega > \gamma$. In this case, the planner’s payoff under repayment is the dotted bell curve shown in green. Here, $\epsilon^{SP} < 0$, because condition (22) implies that the planner’s optimal choice features $c^L > c^H$. Since default is the only instrument available to the government, however, these consumption allocations are not feasible, and by choosing default the government attains $W_d=1$, which is the highest feasible government payoff for any $\epsilon \geq 0$. In contrast, in a scenario with $\omega = \gamma$, for which the planner’s payoff function is the red, dashed bell curve, the planner would choose $\epsilon^{SP} = 0$, and default attains exactly the same payoff, so default is chosen. In short, if $\omega \geq \gamma$, the government always defaults for any decentralized distribution of debt holdings represented by $\epsilon > 0$, and thus equilibria with debt cannot be supported.

When $\omega < \gamma$, the planner’s payoff is the blue curve. The intersection of the downward-sloping segment of $W_1^{d=0}(\epsilon, g_1, \gamma, \omega)$ with $W_d=1$ determines the default threshold $\hat{\epsilon}$ such that default is optimal only in the default zone where $\epsilon \geq \hat{\epsilon}$. Default is still a second-best policy for the planner, because with it the planner cannot attain $W_d=0(\epsilon^{SP})$, it just gets the closest it can get. In contrast, the choice of repayment is preferable in the repayment zone where $\epsilon < \hat{\epsilon}$, because in this zone $W_1^{d=0}(\epsilon, g_1, \gamma, \omega) > W_d=1(g_1)$.

Adding default costs to this political bias setup ($\phi(g_1) > 0$) makes it possible to support repayment equilibria even when $\omega \geq \gamma$. With default costs there are threshold values of consumption dispersion, $\hat{\epsilon}$, separating repayment from default zones for $\omega \geq \gamma$. The range of values of $\epsilon$ for which repayment is chosen widens as $\gamma$ rises relative to $\omega$. Thus, when default
is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in \( \epsilon \), do not produce consumption dispersion larger than the value of \( \hat{\epsilon} \) associated with a given \((\omega, \gamma)\) pair. Intuitively, the consumption of \( H \)-type agents must not exceed that of \( L \)-type agents by more than what \( \hat{\epsilon} \) allows, because otherwise default is optimal.

Figures 8 and Figure 9 illustrate the main quantitative predictions of the model with political bias. Figure 8 shows how the planner’s welfare gain of default varies with \( \omega \) and \( \gamma \) for two different levels of government debt \((B_{1,L} = 0.085 \text{ and } B_{1,H} = 0.145)\). The no-equilibrium region, which exists for the same reasons as before, is shown in dark blue. In line with the earlier discussion, within the region where the equilibrium is well defined, the planner’s value of default increases monotonically as its preference for redistribution \((\omega)\) increases, keeping \( \gamma \) constant, and falls as actual wealth concentration \((\gamma)\) rises, keeping \( \omega \) constant. Because of this, the north-west and south-east corners in each of the panels present cases that are at very different positions on the preference-for-default spectrum. When \( \omega \) is low, even for very high values of \( \gamma \), the government prefers to repay (north-west corner), because the government puts relatively small weight on \( L \)-type agents. On the contrary, when \( \omega \) is high, even for relatively low levels of \( \gamma \), a default is preferred. It is also interesting to note that as we move from Panel \((i)\) to Panel \((ii)\), so that government debt raises, the set of \( \gamma \)'s and \( \omega \)'s such that the equilibrium exists or repayment is preferred (i.e. a negative \( \alpha(B_1, g_1, \gamma, \omega) \)) expands. This is because as we increase the level of debt \( B_1 \), as long as the government does not choose to default for all \( g_1 \), the higher level of debt allows low-wealth households to attain positive levels of consumption (since initial taxes are lower).
Figure 8: Planner’s welfare gain of default $\overline{\alpha}(B_1, g_1, \gamma, \omega)$

Panels (i) − (iv) in Figure 9 display the model’s equilibrium outcomes for the optimal debt chosen by the government in the first period and the associated equilibrium bond prices, spreads and default probabilities under three possible values of $\omega$, all plotted as functions of $\gamma$. The scenario with $\omega = \gamma$, shown in blue corresponds to the utilitarian case of Section 4, and the other two scenarios correspond to high and low values of $\omega$ ($\omega_L = 0.25$ and $\omega_H = 0.45$ respectively).\(^{31}\)

\(^{31}\)Note that along the blue curve of the utilitarian case both $\omega$ and $\gamma$ effectively vary together because they are always equal to each other, while in the other two plots $\omega$ is fixed and $\gamma$ varies. For this reason, the line corresponding to the $\omega_L$ case intersects the benchmark solution when $\gamma = 0.25$, and the one for $\omega_H$ intersects the benchmark when $\gamma = 0.45$.  

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Figure 9 shows that the optimal debt level is increasing in $\gamma$. This is because, following the analysis illustrated in Figure 7, the incentives to default grow weaker and the repayment zone widens as $\gamma$ increases for a fixed value of $\omega$. Moreover, the demand composition effect of higher $\gamma$ is still present, so along with the lower default incentives we still have the increasing per capita demand for bonds of H types. These two effects combined drive the increase in the optimal debt choice of the government. It is also interesting to note that in the $\omega_L$ and $\omega_H$ cases the equilibrium exists for all values of $\gamma$ (even those that are lower than $\omega$). Without default costs each curve would be truncated exactly where $\gamma$ equals either $\omega_L$ or $\omega_H$, but since these simulations retain the default costs used in the utilitarian case, there can still be equilibria with debt for lower values of $\gamma$ (as explained earlier).

In this model with political bias, the government is still aiming to optimize debt by focusing on the resources it can reallocate across periods and agents, which are still determined by the debt Laffer curve $q(B_1)B_1$, and internalizing the response of bond prices to debt choices.$^{32}$ This

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$^{32}$When choosing $B_1$, the government takes into account that higher debt increases disposable income for L-type agents in the initial period but it also implies higher taxes in the second period (as long as default is not optimal). Thus, the government is willing to take on more debt when $\omega$ is lower.
relationship, however, behaves very differently than in the benchmark model, because now higher optimal debt is carried at decreasing default probabilities, which leads the planner internalizing the price response to choose higher debt, whereas in the benchmark model lower optimal debt was carried at increasing equilibrium default probabilities, which led the planner internalizing the price response to choose lower debt.

In the empirically relevant range of $\gamma$, and for values of $\omega$ lower than that range (since $\omega_L = 0.25$ and $\omega_H = 0.45$, while the relevant range of $\gamma$ is $[0.55 - 0.85]$), this model can sustain significantly higher debt ratios than the model with utilitarian payoff, and those ratios are close to the observed European median. At the lower end of that range of $\gamma$, a government with $\omega_H$ chooses a debt ratio of about 25 percent, while a government with $\omega_L$ chooses a debt ratio of about 35 percent.

The behavior of equilibrium bond prices (panel (ii)) with either $\omega_L = 0.25$ or $\omega_H = 0.45$ differs markedly from the utilitarian case. In particular, the prices no longer display an increasing, convex shape, instead they are (for most values of $\gamma$) a decreasing function of $\gamma$. This occurs because the higher supply of bonds that the government finds optimal to provide offsets the demand composition effect that increases individual demand for bonds as $\gamma$ rises.

At low values of $\gamma$ the government chooses lower debt levels (panel (i)) in part because the default probability is higher (panel (iv)), which also results in higher spreads (panel (iii)). But as $\gamma$ rises and repayment incentives strengthen (because $\omega$ becomes relatively smaller than $\gamma$), the probability of default falls to zero, the spreads vanish, and debt levels increase. The price remains relatively flat because, again, the higher debt supply offsets the demand composition effect.

The political bias extension yields an additional interesting result: For a sufficiently concentrated distribution of bond holdings (high $\gamma$), $L-$type agents prefer that the government weighs the bond holders more than a utilitarian government (i.e, there are values of $\gamma$ and $\omega$ for which, comparing equilibrium payoffs under a government with political bias v. a utilitarian government, $v^L(B_1, \omega, \gamma) > v^L(B_1, \gamma)$). To illustrate this result, Figure 10 plots the equilibrium payoffs in the political-bias model for the two types of agents as $\omega$ varies for two values of $\gamma$ ($\gamma_L = 0.15$ and $\gamma_H = 0.85$). The payoffs for the $L-$ and $H-$ types are in Panels (i) and (ii), respectively. The vertical lines identify the payoffs that would be attained with a utilitarian government (which by construction coincide with equilibria for the political bias model in which $\omega=\gamma$).
Figure 10: Welfare as a function of Political Bias for different values of $\gamma$

Panel (i): $v^L_0(\gamma, \omega)$
Panel (ii): $v^H_0(\gamma, \omega)$

Panel (ii) shows that the payoff of $H$-types is monotonically decreasing in $\omega$ (their welfare always falls as their weigh in the government’s payoff function falls). This is due to the fact that $H$ types always prefer the higher debt levels attained by low $\omega$ governments, because it enhances their ability to smooth consumption at a lower risk of default. In contrast, Panel (i) shows that the payoff of $L$-types is non-monotonic in $\omega$, and has a well-defined maximum. In the case with $\gamma = \gamma_L$, the maximum point is at $\omega = \gamma_L$, which corresponds to the equilibrium under the utilitarian government, but when $\gamma = \gamma_H$ the maximum point is at about $\omega = 0.75$, which is smaller than $\gamma_H$. Thus, in this case, ownership of public debt is sufficiently concentrated for the agents that do not hold it to prefer a government that chooses $B_1$ weighting the welfare of bond holders by more than the utilitarian government. This occurs because with $\gamma_H$ the utilitarian government has strong incentives to default, and thus the equilibrium supports low debt, but $L-$type agents would be better off if the government could sustain more debt, which the government with $\omega = 0.75$ can do because it weighs the welfare of bond holders more, and thus has weaker default incentives. The $L-$ type agents desire more debt because they are liquidity-constrained (i.e. $\mu^L > 0$) and higher debt improves the smoothing of taxation and thus makes this constraint less tight.

The above results yield an important political economy implication: Under a majority voting electoral system, in which candidates are represented by values of $\omega$, it can be the case that majorities of either $L$ or $H$ types elect governments with political bias $\omega < \gamma$. This possibility
is captured in Figure 10. When the actual distribution of bond holdings is given by $\gamma_H$, the majoritiy of voters are $L$ types, and thus it follows from Panel (i) that the government represented by the $\omega$ at the maximum point (around 0.75) is elected. In this case, agents that do not hold government bonds vote for a government that favors bond holders (i.e. $L$ types are weighed at 0.75 instead of 0.85 in the government’s payoff function). When the distribution of bond holdings is given by $\gamma_L$, the majority of voters are $H$ types and the electoral outcome is determined in Panel (ii). Since the payoff of $H$ types is decreasing in $\omega$, they elect the government at the lower bound of $\omega$. Hence, under both $\gamma_L$ and $\gamma_H$, a candidate with political bias beats the utilitarian candidate. This result is not general, however, because we cannot rule out the possibility that there could be a $\gamma \geq 0.5$ such that the maximum point of the $L$-types payoff is where $\omega = \gamma$, and hence the utilitarian government is elected under majority voting.

### 5.2 A Second Asset

We now extend the baseline model by allowing domestic agents to save using a risk-free asset, in addition to government bonds. In particular, agents have access to a non-stochastic production technology $y_i^t = z(k_i^t)^\theta$ with $0 < \theta < 1$, where $y_i^t$ is total output and $k_i^t$ is capital for agent of type $i$ in period $t$, respectively. The initial aggregate level of capital is denoted by $K_0$. As before, there is a fraction $\gamma$ of $L$-type agents that are endowed with $b_{L0}$ and $k_{L0}$ units of domestic sovereign debt and capital respectively. A fraction $(1 - \gamma)$ of agents are of the $H$-type and have endowments given by $b_{H0} = B_0 - \gamma b_{L0}$ and $k_{H0} = K_0 - \gamma k_{L0}$. We assume that capital depreciates at rate $\delta$. At period 0, agents choose how much of their savings they want to allocate to public bonds $b_i^0 \geq 0$ and capital $k_i^0 \geq 0$.

The assumptions we made above to introduce the second asset, particularly the curvature of the production function, serve the purpose of supporting a well-defined portfolio choice for private agents (i.e. the fraction of wealth to allocate to bonds and capital), which is in turn useful to solve the government’s problem. This formulation is also helpful because it approximates the cases in which agents could buy a foreign risk-free asset or a domestic asset that is in zero net supply (and is also risk free). If, for example, agents could buy a foreign risk-free asset, we could obtain well defined portfolios by introducing adjustment costs.

The agents’ budget constraint in the initial period is:

$$c_0^i + q_0 b_1^i + k_1^i = z(k_0^i)^\theta + k_0^i(1 - \delta) + b_0^i - \tau_0 \quad \text{for } i = L, H. \quad (23)$$

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33 This statement is of course conditional on parameters supporting the interesting case in which at least some agents do hold both assets at equilibrium.
The budget constraints in period 1 for the case of no-default and default are:

\[ c_{i,d=0}^i = z(k_1^i)\theta + k_1^i(1 - \delta) + b_1^i - \tau_1 \quad \text{for } i = L, H. \]  

(24)

\[ c_{i,d=1}^i = (1 - \phi(g_1))z(k_1^i)\theta + k_1^i(1 - \delta) - \tau_1 \quad \text{for } i = L, H. \]  

(25)

The first-order condition with respect to \( b_1^i \) is

\[ u'(c_{0,i}^i) \geq \left( \frac{\beta}{q_0} \right) E_{g_1} \left[ u'(c_{i,d=0}^i)(1 - d_1) \right], \quad \text{if } b_1^i > 0. \]

As in the benchmark, the marginal benefit of an extra unit of domestic bond is positive only in those states where the government chooses to repay. Similarly, the first-order condition with respect to capital is given by

\[ u'(c_{0,i}^i) \geq \beta E_{g_1} \left[ u'(c_{i,d=0}^i)(1 - d_1) + u'(c_{i,d=1}^i)d_1 \right] \left[ \theta z(k_1^i)\theta - 1 + 1 - \delta \right], \quad \text{if } k_1^i > 0. \]

Since the asset is risk-free, the marginal benefit of an extra unit of capital is positive in all possible future states of the world. However, it will be more valuable in those states where the government chooses to default since the marginal utility is higher. These first-order conditions also show that the level of government debt \( B_1 \) (as well as the initial wealth dispersion \( \gamma \)) influence the individual asset demand through the default decision directly, and indirectly through the bond price.

When the borrowing constraint on bonds binds, this introduces a wedge between the agents’ expected marginal product of capital and the expected return on the bond. However, if both decisions are interior (which is likely to happen for \( H \)-type agents), agents equalize the expected return across assets, which implies:

\[ \frac{q_0}{[\theta z(k_1^i)\theta - 1 + 1 - \delta]^{-1}} = \frac{E_{g_1} \left[ u'(c_{i,d=0}^i)(1 - d_1) \right]}{E_{g_1} \left[ u'(c_{i,d=0}^i)(1 - d_1) + u'(c_{i,d=1}^i)d_1 \right]} . \]  

(26)

This equation shows that when capital and bond decisions are interior, the wedge between the price of the bond and the marginal productivity of capital is driven uniquely by default risk (weighted by marginal utility in each state of the world). In the particular case in which in addition we are in a combination of \( B_1 \) and \( \gamma \) where there is no default risk, the optimal
individual choice of capital is given by:

$$(1/q_0) = (\alpha z(k^i_0)^{\alpha -1} + 1 - \delta)$$

$$\Rightarrow k^i_1 = \left[\frac{\alpha zq_0}{1 - q_0(1 - \delta)}\right]^{1/\alpha}.$$ 

The demand for capital increases with the price of the bond, or falls with the interest rate, as in standard investment models.

In this environment, the government faces similar budget constraints to those presented in the benchmark model. However, when choosing the optimal level of debt and whether to default or not, it takes into account not only the distribution of bond holdings across agents but also the distribution of capital. In particular, at $t = 1$, the repayment value is given by:

$$W^{d=0}_{1}(B_1, g_1, \gamma) = \gamma u(z(k^L_1)^\theta + (1-\delta)k^L_1-g_1+b^L_1-B_1)+(1-\gamma)u(z(k^H_1)^\theta + (1-\delta)k^H_1-g_1+b^H_1-B_1)$$

and the value of default is

$$W^{d=1}_{1}(g_1, \gamma) = \gamma u(z(k^L_1)^\theta (1-\phi(g_1))) + (1-\gamma)u(z(k^H_1)^\theta (1-\phi(g_1))) + (1-\delta)k^H_1-g_1).$$

Note that in each case, the individual capital and bond decisions $\{b^i_1, k^i_1\}$ are a function of $B_1$ and $\gamma$. This implies that the variance of consumption, a key moment to determine the differential value of repayment v. default, is a function of the amount of debt the government has issued in period 0.

The model seems simple, but its solution needs to be obtained numerically. To make this case as close as possible to the benchmark model, we assume that $k^i_0 = K_0$ (i.e. all initial heterogeneity is in initial bond holdings). Still, this results in heterogeneous bond and capital holdings in the second period. We set $z$ to normalize GDP to 1, $\theta$ equal to 0.33 (a standard value), $\delta = 0.10$ and set $K_0$ so the capital to output ratio is equal to 2. All other parameters are set equal to those in the benchmark.

Figure 11 presents the decision rules of the model. The top panels correspond to those for the $H$-type and the bottom panels to those for the $L$-type. They are truncated for combinations of $B_1$ and $\gamma$ where the equilibrium does not exist because the government defaults for all possible realizations of $g_1$. 

40
As in the benchmark model, $L$–type agents are at the borrowing constraint for all values of $B_1$. Their capital choice is determined completely by the first-order condition for capital. As we get close to a region with default risk, their demand for capital decreases since the marginal benefit of an extra unit of savings decreases, because they expect to receive a transfer in the case of a default. On the other hand, $H$–type agents are never at the borrowing constraint, so their choice of capital is given by equation (26). As in the benchmark economy with one asset, while default risk is low ($B_1 = B_{1,L}$), the bond price increases with $\gamma$ to induce the $H$–type to demand a decreasing fraction of their initial bond position (but still increasing in $\gamma$). This increase in bond prices drives the increasing demand for capital for the $H$–type when $B_1 = B_{1,L}$. As we move to higher levels of $B_1$, similar factors are at play but since default risk is not trivially small, bond prices do not rise as fast, inducing a relatively flat demand for capital with slightly increasing demand for domestic bonds for $H$–types with a decreasing demand for capital for $L$–types.
Figure 12: Equilibrium Model with Two Assets

Panel (i): Debt Choice $B_t^*(\gamma)$
Panel (ii): Bond Price $q(B_t^*(\gamma), \gamma)$
Panel (iii): Spread (%)
Panel (iv): Def. Prob. $p(B_t^*(\gamma), \gamma)$

Note: $mpk^H$ denotes the marginal product of capital for the $H$–type and is given by
$\theta z(k^H)^{\theta-1} + 1 - \delta$. The spread is computed as $1/q(B_1, \gamma) - mpk^H$.

Figure 12 presents the equilibrium functions as they vary with $\gamma$. As before, we present the optimal debt choice of the government, and the associated bond prices, spreads (measured against the marginal product of capital of the $H$–type) and the default probability.\(^{34}\) This figure shows that the results of the benchmark model are robust to the introduction of an additional asset.

Panel (i) shows that optimal debt falls as $\gamma$ increases and Panel (iv) that default risk is not zero for $\gamma > 0.20$ (non negligible for $\gamma > 0.50$). Importantly, the probability of a default event is increasing in the initial level of wealth inequality. As default risk increases, the wedge between the marginal product of capital and the equilibrium bond price increases driving up the spreads (see Panels (ii) and (iii)). Different from our baseline model, bond prices remain below 1, implying positive real interest payments for all values of $\gamma$.

\(^{34}\)We measure spreads against the marginal product of capital of the $H$–type since the wedge is only driven by default risk. In the case of the $L$–type agents there is an additional component that arises from the multiplier on the non negativity constraint on domestic bonds.
Figure 13 compares the equilibrium functions of the benchmark model with those of the economy with two assets.

The model with two assets sustains higher levels of debt, but as default risk increases the government chooses to reduce debt issuance faster in the economy with capital than in the benchmark. Even though we observe a lower default probability and spreads, the previous result derives from the fact that lower levels of debt are sustainable when agents have access to another asset. The cost of a default is not as high for the \( H \)-types and a default reduces consumption dispersion that increases faster with \( \gamma \) in the economy with capital than in the benchmark since initial dispersion is amplified via capital.

5.3 International Investors

While a large fraction of sovereign debt in Europe is in hands of domestic households (for the countries in Table 1, 60 percent is the median and 75 percent the average), the fraction in hands of foreign investors is not negligible. For this reason, we extend the benchmark model
to incorporate foreign investors and move from a closed to an open economy. In particular, we assume that there is a pool of international investors modeled in the same way as in the Eaton-Gersovitz class of external default models: risk-neutral lenders with an opportunity cost of funds equal to an exogenous, world-determined real interest rate $\bar{r}$. As is common practice, we assume that these bonds are pari passu contracts, which rules out the possibility for the government to discriminate among borrowers when choosing to default. This also maintains the symmetry with the baseline model, in which the government was not allowed to default on a particular set of domestic households.

Since foreign lenders are the marginal investor of sovereign debt, in this model the price of the bond is given by

$$q(B_1, \gamma) = \frac{1 - p(B_1, \gamma)}{(1 + \bar{r})}$$

where $p(B_1, \gamma)$ is the default probability. More precisely, $p(B_1, \gamma) = E_{g_1}[d(B_1, \gamma, g_1)]$. While the arbitrage condition is functionally identical to the one of the Eaton-Gersovitz models, they embody different mechanisms. The two are similar in indicating that, because of risk neutrality, risk premia are equal to default probabilities. But there is a critical difference in how these probabilities are determined. In Eaton-Gersovitz models they follow from the values of continuation v. default of a representative agent, while in our model they are determined by comparing those values for a utilitarian social welfare function, which in turn depend on the dispersion of individual payoffs of default v. repayment (and on the welfare weights). Hence, inequality affects default probabilities via changes in the relative magnitudes of individual payoffs of default v. repayment.

We let the position of foreign investors be denoted $B^f_t$, which defines also the economy’s net foreign asset position. We assume that a fraction $\phi^f$ of the initial stock of debt is in their hands. That is $B^f_0 = \phi^f B_0$. A fraction $(1 - \phi^f)$ is distributed among domestic households according to $\gamma$. We denote the domestic demand by $B^d_t = \gamma b^L_t + (1 - \gamma) b^H_t$. The debt market clearing condition is

$$B^d_t + B^f_t = B_t \tag{27}$$

We do not restrict the value of $[\gamma b^L_t + (1 - \gamma) b^H_t]$ to be less than or equal to $B_1$, so $B^f_t$ could be positive or negative. When $B^f_t > 0$ the country is a net external borrower, because the bonds issued by the government are less than the domestic demand for them, and when $B^f_t < 0$ the country is a net external saver.

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35This assumption is made only to approximate the quantitative predictions of the model with the data but is not crucial for any of the results presented below.
The problems of the household and the government remain identical to the described in the benchmark. Of course, agents understand that there is a new pricing equation and that market-clearing conditions incorporate the foreign demand. We solve the model numerically using the same parameter values of the benchmark model. We set $\phi^f = 0.25$ to match the average fraction of foreign debt observed in our sample of European countries and $\pi$ to 2 percent to match the average real interest rate in Germany in the 2000-2007 period.

Figure 14: Planner’s welfare gain of default $\varpi(B_1, g_1, \gamma)$

Figure 14 shows how the planner’s welfare gain of default varies with $\gamma$ and $B_1$ for different levels of government expenditures ($g_1 = \underline{g}_1$ and $g_1 = \overline{g}_1$). The no-equilibrium region, which exists for the same reasons as before, is shown in dark blue. In line with the characteristics of default incentives of the benchmark model, within the region where the equilibrium is well defined, for a given $\gamma$, the planner’s value of default increases monotonically with the level of debt $B_1$. However, we observe that, contrary to the benchmark case, in the economy with foreign lenders, conditional on $B_1$, the welfare gain of default has an inverted-U shape in the $\gamma$ dimension. That is, for a given $B_1$ the value of $\alpha$ decreases with $\gamma$ for low $\gamma$, reaches a minimum point and then increases with $\gamma$. This also determines the bell-shaped “No Equilibrium Zone”.

45
The intuition for this result is simple and derives from the decision rules of domestic agents (see the Appendix for the corresponding plot). For a given level of $B_1$, when $\gamma$ is below ($\gamma < 0.25$ for $B_1 = B_M$ for example), the country is on average a foreign borrower (i.e. $B^f > 0$). That implies that a default generates a direct increase in domestic resources equal to the forgone debt payments to foreign lenders. In this region, both $L$–type and $H$–type agents are at the borrowing limit. As $\gamma$ increases, the country becomes a net saver in foreign markets. Increases in $\gamma$ are associated with an increasing portion of domestic debt in hands of $H$–types. This reduces the benefit of a default on foreign lenders. However, as in the model without foreign lenders, domestic consumption dispersion increases. For mid-range $\gamma$’s the first effect dominates the second and repayment is the preferred option. As $\gamma$ increases even further the dispersion in domestic consumption increases to points where default is again the optimal alternative for the government. In this region, the main driver of domestic default is redistribution among domestic agents as in our benchmark economy.

Figure 15 shows the comparison of the equilibrium functions in the economy with foreign lenders v. the benchmark economy. As we described before, the introduction of foreign lenders and the possibility of a “foreign” default constraint debt values and result in lower debt levels than in the benchmark, for values of $\gamma$ lower than 0.75 (see Panel (i)). This is also reflected in higher default probabilities and spreads in the economy with foreign lenders than in the benchmark (see Panel (iii) and (iv)). By construction, the upper bound on the price of the economy with foreign lenders is $(1+r)^{-1}$, so the the distributive effect that negative real interest rates have in the benchmark economy dissipate (see Panel (ii)). This induces the government to take on more risk and redistribute via debt issuance in the economy with foreign lenders than in the benchmark.
5.4 Redistributive Taxation (Partial Default)

In the benchmark model, the fact that taxes are lump-sum (i.e. the same for all agents) implies that issuing debt in the first period and defaulting in the second are the only tools the government can use to reduce consumption dispersion each period. Hence, we now examine how the models predictions change if we introduce taxes as an alternative tool for redistribution. In particular, we introduce a proportional consumption tax that is invariant across periods and realizations of $g_1$.\footnote{Using this tax as opposed to other taxes simplifies the solution of the model, because aggregate consumption (as a function of $y$ and $g_t$) is pinned down by the resource constraint, and thus known before solving the agents problem. As a result, the algorithm does not need to iterate on the level of transfers to close the government budget constraint, while introducing a wealth tax would require it. An income tax would require not only iterating on aggregate transfers but also extending the model to consider endogenous labor supply, because the benchmark model assumes no income heterogeneity, which makes proportional income tax very close to a lump-sum tax.}

With consumption taxes, the cost of purchasing goods in left-hand-side of the agents’ budget constraints becomes $(1 + \tau^e)c^i_t$ for $i = L, H$ and $t = 0, 1$. Since aggregate consumption in each
period is still given by $y - g_t$, the government budget constraints simplify to the following expressions: At date 0, the constraint is:

$$\tau_0 + \tau^c y = g_0(1 + \tau^c) + B_0 - q_0 B_1. \quad (28)$$

In the second period, under repayment, the constraint is:

$$\tau_{1}^{d_i=0} + \tau^c y = g_1(1 + \tau^c) + B_1. \quad (29)$$

In the second period, under default the budget constraint is:

$$\tau_{1}^{d_i=1} + \tau^c y(1 - \phi(g_1)) = g_1(1 + \tau^c). \quad (30)$$

Using the budget constraints of agents and the government, we can proceed as with the benchmark model and derive expressions for the difference in consumption of the two agents in the two periods and default scenarios.

$$c^H_0 - c^L_0 = \frac{1}{1 - \gamma} \left[\frac{B_0 - q(B_1, \gamma) B_1}{(1 + \tau^c)}\right]$$

$$c^H_{1, d=0} - c^L_{1, d=0} = \frac{1}{1 - \gamma} \frac{B_1}{(1 + \tau^c)}$$

$$c^H_{1, d=1} - c^L_{1, d=1} = 0.$$}

These expressions are similar to those in (17), except that the terms that include public debt in the date-0 difference and the date-1 difference under repayment are divided by $(1 + \tau^c)$. Consumption taxes reduce consumption dispersion in both instances. Intuitively, the consumption tax plays a role akin to inflation in reducing the real value of public debt. Hence, this tax always reduces consumption dispersion in the first period (as long as there is a primary deficit), and also always reduces dispersion in the second period under repayment. In fact, assuming repayment, this tax is a better mechanism for redistribution because debt can only reduce dispersion at date 0 at the expense of increasing it a date 1. Moreover, the tax’s ability to redistribute is not hampered by the debt Laffer curve that hampers the ability to redistribute with debt because of default risk. The tax acts in a fact as a de-facto partial default in both periods. As a result, if we allow it to be chosen optimally, letting $\tau^c$ go to infinity would be optimal because it completely removes consumption dispersion in all periods at no cost. The purpose of this analysis, however, is to see how the existence of an alternative redistribution tool affects the results we have obtained for debt and default risk, rather than focus on the optimal use of the
consumption tax.\textsuperscript{37}

We solve the model numerically using the parameters of the benchmark calibration. For the value of the consumption tax, we use $\tau^c = 0.16$ which is the value estimated for the Eurozone in Mendoza, Tesar, and Zhang (2014). We also solve the model for $\tau^c = 0.32$ and $\tau^c = 0.48$, which are two and three times larger than the data proxy.\textsuperscript{38}

Figure 16 presents the comparison of the equilibrium of the benchmark economy with the three economies with taxes. This figure shows that the main result of the paper (the ability to support the existence of public debt with positive default risk) is robust to incorporating distributive taxes.

Figure 16: Comparison Equilibrium Benchmark vs Proportional Consumption Taxes

As consumption taxes increase, default incentives grow weaker (see lower default probabili-

\textsuperscript{37}The consumption tax as modeled lacks important trade-offs that would make it suboptimal to set it infinitely large. In particular, it is non-distortionary, because there is no labor supply choice, and because, since the tax is constant, it does not distort savings decisions. The tax is also assumed to be known with perfect foresight, whereas uncertainty about inflation would have an effect similar to a partial risk of default that would affect demand for bonds and bond prices.

\textsuperscript{38}In the tax rate estimates in Mendoza, Tesar, and Zhang (2014), the highest consumption tax is observed in Finland at 24 percent.
ties in Panel (iv)), which is natural because the tax reduces consumption dispersion, and hence reduces the need for the government to use default to lower dispersion. As a result, the government can sustain higher debt levels (see Panel (iv)), although the effect is not very large (debt increases by at most 5 percentage points for $\gamma = 0.6$ and a change in the consumption tax from 0 to 48 percent). This effect is negligible when default incentives are weak to start with, which occurs at low values of $\gamma$. The properties of the price function observed in the benchmark case (i.e. increasing in $\gamma$ for a given level of default risk) are sustained in the economy with consumption taxes (see Panel (ii)). In line with the reduction in default probabilities, the model with consumption taxes also displays lower spreads than the benchmark (see Panel (iii)).

We acknowledge that the time- and state-invariant consumption tax introduces an alternative tool for redistribution in a highly stylized, but also highly tractable, manner. Still, the quantitative results do show that the predictions of the optimal domestic default framework we proposed are robust to introducing consumption tax rates of up to almost 50 percent, three times the highest consumption tax observed in Europe. A 50 percent tax implies reducing the real value of debt by 2/3rds, which is a sizable redistribution taking place via taxation rather than default, and yet the distributional incentives tempered by default costs still produce equilibria with debt exposed to default risk. These findings also suggest that the existence of other tools for redistribution contribute to support higher debt levels at lower default frequencies that what would exist without those alternative tools. Moreover, the results are also interesting as an illustration of what the model could look like if we were to combine de-jure outright default with partial, de-facto default (the latter commonly takes place via inflation, but as noted earlier the consumption tax in this setup plays the same role as inflation in reducing the real value of public debt).

6 Conclusions

This paper proposes a framework in which domestic sovereign default and public debt subject to default risk emerge as an equilibrium outcome. In contrast with standard models of sovereign default on external debt, this framework highlights the role of wealth heterogeneity and the distributional effects of default across domestic agents in shaping the government’s default incentives. These are features common to both the historical episodes of outright domestic default documented by Reinhart and Rogoff (2011) and the ongoing European debt crisis.

The framework we developed consists of a two-period model with high- and low-wealth agents, non-insurable aggregate uncertainty in the form of shocks to government expenditures and default risk, and a utilitarian government who sets debt, transfers and the default decision
responding to distributional incentives. The government is aware of its inability to commit, and chooses how much debt to issue seeking to allocate resources optimally across agents and across periods. In choosing the optimal debt, the government takes into account how the debt chosen in the first period influences the second period’s default incentives and default probability, and the feedback of these two into the first period’s equilibrium price of government bonds. This feedback affects the resources that can be raised by borrowing, which are driven by a variant of the well-known debt Laffer curve of the external default literature.

In this environment, the distribution of public debt across private agents interacts with the government’s optimal default, debt issuance and tax decisions. Distributional incentives alone cannot support equilibria with debt, because default is always optimal in the second period, and hence the debt market cannot function in the first period. We also showed that equilibria with debt can exist if we introduce either exogenous default costs or government preferences biased in favor of bond holders.

Quantitative results based on a calibration to European data show that sustainable debt falls and default risk rises as the fraction of low-wealth individuals rises. Because of default risk, sustainable debt is much lower than when the government is committed to repay (at the same levels of wealth inequality). In an extension of the model in which the government’s welfare weights are biased in favor of bond holders, sustainable debt becomes an increasing function of wealth inequality, instead of decreasing as in the utilitarian benchmark. This is because incentives to default grow weaker as the government’s weight on agents that do not hold government debt falls increasingly below the actual fraction of the population that they represent. In this setup, optimal debt levels chosen in the empirically relevant range of wealth inequality estimates exceed those supported in the utilitarian benchmark by wide margins, and can be of similar magnitude as the European median.

The results from model with political bias also show that agents that do not hold public debt may prefer that the government weighs the welfare of bond holders by more than a utilitarian government. This allows the government to sustain higher debt at lower default probabilities, which relaxes the liquidity constraint of agents without government bonds by improving tax smoothing. Moreover, this finding implies that it is possible for majority voting outcomes to result in governments with political bias in favor of bond holders being elected even when the majority of voters are non-bond-holders.

The main finding of the paper (i.e. that distributional incentives to default tempered by default costs can support equilibria with debt exposed to default risk) is robust to other three important extensions of the model. First, adding an additional asset in the form of risk-free physical capital. Second, introducing risk-neutral foreign investors who can hold a portion of
the debt issued by the government. Third, introducing consumption taxes as an alternative tool for redistributing resources across agents.

This model can be viewed as a blueprint for further research into models of domestic sovereign default driven by distributional incentives, and their interaction with agent heterogeneity and incomplete insurance markets. The two-period environment is a useful starting point because of the ease with which it can be analyzed and solved, but it also imposes limitations on attempts to bring the model to the data. In particular, self-insurance motives, which the literature shows can produce significant welfare benefits for the existence of public debt markets (see Aiyagari and McGrattan (1998)), are minimized by the two-period life horizon. In an infinite horizon model, this mechanism could produce a large endogenous cost of default that may support the existence of public debt subject to default risk without exogenous costs of default and/or political biases in the government’s preferences. In related work (see D’Erasmo and Mendoza (2014)) we are looking into this possibility.
References


A1 Appendix (NOT FOR PUBLICATION)

A1.1 Empirical Analysis Debt and Inequality

Further and more systematic empirical analysis of the relationship between wealth inequality and public debt shows a statistically significant, bell-shaped relationship: Debt is increasing in wealth inequality when inequality is low, until the Gini coefficient reaches about 0.75, and then becomes decreasing in wealth inequality. This finding follows from estimating a cross-country panel regression using the international database of Gini coefficients for wealth produced by Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012), together with data on public debt-output ratios and control variables (i.e. the size of the government, proxied by the ratio of government expenditures to GDP, and country fixed effects). The results of panel regressions of the debt ratio on the Gini coefficient, the square of this coefficient, and the controls (using the observations for 2000 and 2012) are shown in Table 3, and Figure 17 shows a plot of the 2012 data and the fitted regression curve. The regressions only explain about a tenth of the cross-country variations on debt ratios, but the linear and quadratic effects of the wealth Gini coefficient are statistically significant and opposite in sign.

Table 3: Panel Regressions of Public Debt Ratios and Wealth Inequality

<table>
<thead>
<tr>
<th></th>
<th>Panel Country Fixed Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dep. Var: Debt to GDP</td>
</tr>
<tr>
<td>Wealth Gini</td>
<td>17.65**</td>
</tr>
<tr>
<td>s.e.</td>
<td>7.06</td>
</tr>
<tr>
<td>Wealth Gini(^2)</td>
<td>-17.65**</td>
</tr>
<tr>
<td>s.e.</td>
<td>4.76</td>
</tr>
<tr>
<td>G/Y</td>
<td>-0.010</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.012</td>
</tr>
<tr>
<td>avg. country FE</td>
<td>-5.92**</td>
</tr>
<tr>
<td>s.e.</td>
<td>2.60</td>
</tr>
<tr>
<td>years</td>
<td>2000-2012</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.118</td>
</tr>
<tr>
<td># obs.</td>
<td>139</td>
</tr>
</tbody>
</table>

Note: Author’s calculations based on WDI, Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012). Debt to GDP refers to Central Government Debt as share of GDP (source WDI). Wealth Gini as reported for each country for year 2000 and 2012 by Davies, Sandstrm, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012). G/Y corresponds to government final consumption as a share of GDP (source WDI). Coefficients and standard errors reported are derived from a panel linear regression with Country Fixed Effects. The sample contains 87 countries with data for years 2000 and 2012. If debt to GDP or government expenditure ratio is not available for year 2000 or 2012 data for year 1999 or 2011 is used.
Figure 17: Public Debt Ratios and Wealth Inequality

Note: Author’s calculations based on WDI, Davies, Sandström, Shorrocks, and Wolff (2009) and Davies, Lluberas, and Shorrocks (2012).
Fitted line from results presented in Table 3

A1.2 Deterministic Model without Default Risk and Log Utility

This Appendix derives solutions for a version of the model in which low-wealth (L) types do not hold any bonds and high-wealth (H) types buy all the debt. We cover first the fully deterministic case, without any shocks to income or government policies, and no default risk, but government expenditures may be deterministically different across periods. Government wants to use debt to relocate consumption across agents and across periods optimally given a utilitarian welfare function. Ruling out default on initial outstanding debt, the planner trades off the desire to use debt to smooth taxation for L types (reduce date-0 taxes by issuing debt) against the cost of the postponement of consumption this induces on H types who save to buy the debt. Log utility provides closed form solutions. The goal is to illustrate the mechanisms that are driving the model when default risk and stochastic government purchases are taken out. Later in the Appendix we derive some results for the model with stochastic government purchases, and make some inferences for the case with default risk.

A1.2.1 Households

A fraction $\gamma$ of agents are L types, and $1 - \gamma$ are H types. Preferences are:

$$\ln(c^0_i) + \beta \ln(c^1_i) \quad for \quad i = L, H$$

(A.1)
Budget constraints are:
\[ c^L_0 = y - \tau_0, \quad c^H_0 = y - \tau_0 + b^H_0 - q b^H_1 \quad (A.2) \]
\[ c^L_1 = y - \tau_1, \quad c^H_1 = y - \tau_1 + b^H_1 \quad (A.3) \]

Since L types do not save, the solution to their problem is trivial: they can only consume what their budget constraints allow. This is important because altering taxes affects disposable income, which will in turn affect the optimal debt choice of the government. For H types, the Euler equation is:
\[ q = \beta \frac{c^H_0}{c^H_1} \quad (A.4) \]

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, it must be the case that they are credit constrained (i.e. they would want to hold negative assets). That is, at the equilibrium price of debt their Euler equation for bonds would satisfy:
\[ q > \beta \frac{c^L_0}{c^L_1} \quad (A.5) \]

### A1.2.2 Government

The government budget constraints are:
\[ \tau_0 = g_0 + B_0 - q B_1 \quad (A.6) \]
\[ \tau_1 = g_1 + B_1 \quad (A.7) \]

The initial debt \( B_0 \geq 0 \) is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:
\[ \gamma (\ln(c^L_0) + \beta \ln(c^L_1)) + (1 - \gamma) (\ln(c^H_0) + \beta \ln(c^H_1)) \quad (A.8) \]

### A1.2.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt \( B_1 \) is given by a price \( q \) that satisfies the market-clearing condition of the bond market: \( b^H_1 = B_1 / (1 - \gamma) \). By construction the same condition is assumed to hold for the initial conditions \( b^H_0 \) and \( B_0 \). This implies that the initial wealth of H-types is given by \( b^H_0 = B_0 / (1 - \gamma) \).

Rewriting the Euler equation of H types using the budget constraint, the government budget constraints and the bond market-clearing conditions yields:
\[ q = \beta \frac{y - g_0 + \left(\frac{\gamma}{1 - \gamma}\right) B_0 - q \left(\frac{\gamma}{1 - \gamma}\right) B_1}{y - g_1 + \left(\frac{\gamma}{1 - \gamma}\right) B_1} \quad (A.9) \]
Hence, the equilibrium price of bonds for a given government supply is:

\[
q(B_1) = \beta \frac{y - g_0 + \left(\frac{\gamma}{1 - \gamma}\right) B_0}{y - g_1 + \left(\frac{\gamma}{1 - \gamma}\right) (1 + \beta) B_1}
\]  
(A.10)

Note that this price is not restricted to be lower than 1 (i.e. \(q(B_1) > 1\) which implies a negative real rate of return on government debt can be an equilibrium outcome). In particular, as \(\gamma\) rises the per capita bond demand of H-types increases and this puts upward pressure on bond prices, and even more so if the government finds it optimal to offer less debt than the initial debt, as we showed numerically and explain further below. As \(\gamma \to 1\), the limit of the equilibrium price goes to \(q(B_1) = \frac{\beta y - g_0}{y - g_1}\) even tough market-clearing requires the demand of the infinitesimal small H type to rise to infinity.

After some simplification, the derivative of this price is given by:

\[
q'(B_1) = \frac{-q(B_1) \left(\frac{\gamma}{1 - \gamma}\right) (1 + \beta)}{y - g_1 + \left(\frac{\gamma}{1 - \gamma}\right) (1 + \beta) B_1}
\]  
(A.11)

which at any equilibrium with a positive bond price satisfies \(q'(B_1) < 0\) (notice \(c^H > 0\) implies that the denominator of this expression must be positive).

Consider now what happens to this equilibrium as the fraction of L-types vanishes. As \(\gamma \to 0\), the economy converges to a case where there is only an H type representative agent, and the price is simply \(q(B_1) = \frac{\beta y - g_0}{y - g_1}\), which is in fact independent of \(B_1\) and reduces to \(\beta\) if government purchases are stationary. Trivially, in this case the planner solves the same problem as the representative agent and the equilibrium is efficient. Also, for an exogenously given \(B_0\) and stationary \(g\), the competitive equilibrium is stationary at this consumption level:

\[
c^h = y - g + \left(\frac{\gamma}{1 - \gamma}\right) \frac{B_0}{1 + \beta}
\]  
(A.12)

and the optimal debt is:

\[
B_1 = \frac{B_0}{1 + \beta}
\]  
(A.13)

Hence, in this case consumption and disposable income each period are fully stationary, yet the optimal debt policy is always to reduce the initial debt by the fraction \(1/(1 + \beta)\). This is only because of the two-period nature of the model. With an infinite horizon, the same bond price would imply that an equilibrium with stationary consumption and an optimal policy that is simply \(B_1 = B_0\). It also follows trivially that carrying no initial debt to start with would be first-best, using lump-sum taxation to pay for \(g\).
### A1.2.4 Optimal debt choice

The government’s optimal choice of $B_1$ in the first period solves this maximization problem:

$$
\max_{B_1} \left\{ \gamma \left[ \ln \left( y - g_0 + q(B_1) B_1 \right) + \beta \ln \left( y - g_1 - B_1 \right) \right] \right. \\
\left. + (1 - \gamma) \left[ \ln \left( y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0 - q(B_1) \left( \frac{\gamma}{1 - \gamma} \right) B_1 \right) + \beta \ln \left( y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) B_1 \right) \right] \right\}
$$

(A.14)

where $q(B_1) = \beta \frac{y - g_0 + \left( \frac{\gamma}{1 - \gamma} \right) B_0}{y - g_1 + \left( \frac{\gamma}{1 - \gamma} \right) (1 + \beta) B_1}$.

The first-order condition is:

$$
\gamma \left[ u'(c^L_0) \left[ q'(B_1) B_1 + q(B_1) \right] - \beta u'(c^H_L) \right] \\
+ (1 - \gamma) \left( \frac{\gamma}{1 - \gamma} \right) \left[ -u'(c^H_0) \left[ q'(B_1) B_1 + q(B_1) \right] + \beta u'(c^H_L) \right] = 0
$$

(A.15)

Using the Euler equation of the H types and simplifying:

$$
u'(c^L_0) + \left[ \frac{u'(c^L_0) q'(B_1) - \beta u'(c^H_L)}{q'(B_1) B_1} \right] = u'(c^H_0)
$$

(A.16)

This expression is important, because it defines a wedge between equating the two agents’ marginal utility of consumption that the planner finds optimal to maintain, given that the only instrument that it has to reallocate consumption across agents is the debt. Notice that, since as noted earlier for L types to find it optimal to hold zero assets it must be that they are credit constrained,” their Euler equation implies that at the equilibrium price: $u'(c^L_0) q(B_1) - \beta u'(c^H_L) > 0$. Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports $u'(c^L_0) > u'(c^H_L)$ or $c^H_0 > c^L_0$, and notice that from the budget constraints this also implies $B_0 - q(B_1) B_1 > 0$, which implies $B_1/B_0 < 1/q(B_1)$. Furthermore, the latter implies that the optimal debt must be lower than any initial $B_0$ for any $q(B_1) \geq 1$, and also for “sufficiently high” $q(B_1)$.

**Comparison with no-debt equilibrium:** Notice that since $B_0 - q(B_1) B_1 > 0$, the planner is allocating less utility to L type agents than those agents would attain without any debt. Without debt, and a tax policy $\tau_t = g_t$, all agents consume $y - g_t$ every period, but with debt L-types consume less each period given that $B_1 > 0$ and $B_0 - q(B_1) B_1 > 0$. Compared with these allocations, when the planner finds optimal to choose $B_1 > 0$ is because he is trading off the pain of imposing higher taxes in both periods, which hurts L types, against the benefit the H types get of having the ability to smooth using government bonds. Also, $B_0 - q(B_1) B_1 > 0$ highlights that there is a nontrivial role to the value of $B_0$, because if $B_0$ were zero $B_1$ would need to be negative which is not possible by construction. Hence, the model only has a sensible solution if there is already enough outstanding debt (and wealth owned by H type agents) that gives the government room to be able to improve the H type’s ability to smooth across the two periods, which they desire to do more the higher is $B_0$.

**Comparison with sub-optimal debt equilibrium:** By choosing positive debt, the government provides tax smoothing for L types. Given $B_0$ and the fact that $B_0 - q(B_1) B_1 > 0$, positive debt allows to lower date-0 taxes, which increases consumption of L types (since
\[ c_L^0 = y - g_0 - B_0 + q(B_1)B_1. \]
The same policy lowers the consumption of H types \( (\text{since } c_H^0 = y - g_0 + \left( \frac{\tau_1}{1-\tau} \right)(B_0 - q(B_1)B_1)). \) Hence, debt serves to redistribute consumption across the two agents within the period. This also changes inter-temporal consumption allocations, with the debt reducing L types consumption in the second period and increasing H types consumption. Hence, with commitment to repay \( B_0 \), the debt will be chosen optimally to trade off these social costs and benefits of issuing debt to reallocate consumption atemporally across agents and intertemporally.

It is also useful to notice that the demand elasticity of bonds is given by
\[ \eta \equiv \frac{q(B_1)}{q'(B_1)B_1}, \]
so the marginal utility wedge can be expressed as \( \eta \left[ \frac{u'(c^L_0) - \beta u'(c^H_1)}{u'(c^L_0)} \right] \) and the planner’s optimality condition reduces to:
\[ 1 + \eta \left[ 1 - \frac{\beta u'(c^L_1)}{q(B_1)u'(c^L_0)} \right] = \frac{u'(c^H_1)}{u'(c^L_0)} \] (A.17)

Hence, the planner’s marginal utility wedge is the product of the demand elasticity of bonds and the L-type agents shadow value of being credit constrained (the difference \( 1 - \frac{\beta u'(c^L_1)}{q(B_1)u'(c^L_0)} > 0 \), which can be interpreted as an effective real interest rate faced by L-type agents that is higher than the return on bonds). The planner wants to use positive debt to support an optimal wedge in marginal utilities only when the demand for bonds is elastic AND L-type agents are constrained.

### A1.3 Extension to Include Government Expenditure Shocks

Now consider the same model but government expenditures are stochastic. In particular, realizations of government purchases in the second period are given by the set \([g^1_1 < g^2_1 < ... < g^M_1]\) with transition probabilities denoted by \( \pi(g^i_1|g_0) \) for \( i = 1, ..., M \) with \( \sum_{i=1}^{M} \pi(g^i_1|g_0) = 1. \)

#### A1.3.1 Households

Preferences are now:
\[ \ln(c^i_0) + \beta \left( \sum_{i=1}^{M} \pi(g^i_1|g_0) \ln(c^i_1) \right) \] for \( i = L, H \) (A.18)

Budget constraints are unchanged:
\[ \begin{align*}
  c_L^0 &= y - \tau_0, \quad c_H^0 = y - \tau_0 + b_H^0 - qb_1^H \\
  c_L^1 &= y - \tau_1, \quad c_H^1 = y - \tau_1 + b_1^H
\end{align*} \] (A.19) (A.20)

We still assume that L types do not save, so they can only consume what their budget
constraints allow. For H types, the Euler equation becomes:

\[ q = \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \left( \frac{c^H_i}{c^H_{i-1}} \right) \]  

(A.21)

For L types, in order to make the assumption that they hold no assets consistent at equilibrium, their Euler equation for bonds must satisfy:

\[ q > \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \left( \frac{c^L_0}{c^L_{i-1}} \right) \]  

(A.22)

A1.3.2 Government

The government budget constraints are unchanged:

\[ \tau_0 = g_0 + B_0 - qB_1 \]
\[ \tau_1 = g_1 + B_1 \]

The initial debt \( B_0 \geq 0 \) is taken as given and the government is assumed to be committed to repay it.

The social planner seeks to maximize this utilitarian social welfare function:

\[ \gamma \left( \ln(c^L_0) + \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \ln(c^L_1) \right) + (1 - \gamma) \left( \ln(c^H_0) + \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \ln(c^H_1) \right) \]  

(A.23)

A1.3.3 Competitive equilibrium in the bond market

A competitive equilibrium in the bond market for a given supply of government debt \( B_1 \) is given by a price \( q \) that satisfies the market-clearing condition of the bond market: \( b^H_1 = B_1/(1 - \gamma) \) and the H-types Euler equation.

We can solve the model in the same steps as before. First, rewrite the Euler equation of H types using their budget constraints, the government budget constraints and the market-clearing conditions:

\[ q = \beta \sum_{i=1}^{M} \pi(g^i_1|g_0) \left[ \frac{y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 - q \left( \frac{\gamma}{1-\gamma} \right) B_1}{y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1} \right] \]  

(A.24)

From here, we can solve again for the equilibrium price at a given supply of bonds:

\[ q(B_1) = \beta \left( \frac{y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0}{1 + \left( \frac{\gamma}{1-\gamma} \right) \beta B_1} \right) \left( \frac{\sum_{i=1}^{M} \pi(g^i_1|g_0)}{\sum_{i=1}^{M} y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1} \right) \]  

(A.25)
As $\gamma \to 0$ we converge again to the world where there is only an H type representative agent, but now the pricing formula reduces to the standard formula for the pricing of a non-state-contingent asset $q(B_1) = \beta \left( \sum_{i=1}^{M} \pi(g_i^0) \frac{y}{y-g_0} \right)$.

As $\gamma \to 1$ the equilibrium degenerates again into a situation where market clearing requires the demand of the infinitesimal small H type to rise to infinity. The derivative of the price at any equilibrium with a positive bond price satisfies $q'(B_1) < 0$.

To show this, define $\Pi(B_1) \equiv \sum_{i=1}^{M} \frac{\pi(g_i^0)}{y-g_i^0} - \ln(B_1)$ which yields $\Pi'(B_1) = -\sum_{i=1}^{M} \frac{\pi(g_i^0)\gamma}{(y-g_i^0)B_1}$.

Then taking the derivative $q'(B_1)$ and simplifying we get:

$$q'(B_1) = \frac{\beta \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 \right) \left[ \Pi'(B_1) - \beta \left( \frac{\gamma}{1-\gamma} \right) (\Pi(B_1))^2 \right]}{\left( 1 + \beta \left( \frac{\gamma}{1-\gamma} \right) B_1 \Pi(B_1) \right)^2} \quad \text{(A.26)}$$

Since $\Pi'(B_1) < 0$ and positive $c_H^0$ implies $y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 > 0$, it follows that $q'(B_1) < 0$.

We can also gain some insight into the implicit risk premium (the ratio $q(B_1)/\beta$) and the related question of why the asset price can exceed 1 in this setup. Recall that in fact the latter was already possible without uncertainty when $\gamma$ is large enough, because of the demand composition effect (higher $\gamma$ implies by market clearing that the fewer H type agents need to demand more bonds per capita, so the bond price is increasing in $\gamma$ and can exceed 1). The issue now is that, as numerical experiments show, an increase in the variance of $q_1$ also results in higher bond prices, and higher than in the absence of uncertainty, and again for $\gamma$ large enough we get both $q(B_1) > 1$ and $q(B_1)/\beta > 1$. The reason bond prices increase with the variability of government purchases is precautionary savings. Government bonds are the only vehicle of saving, and the incentive for this gets stronger the larger the variability of $q_1$. Hence, the price of bonds is higher in this stochastic model than in the analogous deterministic model because of precautionary demand for bonds, which adds to the effect of demand composition (i.e. the price is higher with uncertainty than without at a given $\gamma$).

### A1.3.4 Optimal debt choice

The government’s optimal choice of $B_1$ solves again a standard maximization problem:

$$\max_{B_1} \left\{ \gamma \left[ \ln \left( y - g_0 - B_0 + q(B_1)B_1 \right) + \beta \sum_{i=1}^{M} \pi(g_i^0) \ln \left( y - g_1 - B_1 \right) \right] + (1-\gamma) \left[ \ln \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 - q(B_1) \left( \frac{\gamma}{1-\gamma} \right) B_1 \right) \right] + \beta \sum_{i=1}^{M} \pi(g_i^0) \ln \left( y - g_1 + \left( \frac{\gamma}{1-\gamma} \right) B_1 \right) \right\} \quad \text{(A.27)}$$

where $q(B_1)$ is given by the expression solved for in the competitive equilibrium.
The first-order condition is:

\[
\gamma \left[ u'(c_0^L) [q'(B_1)B_1 + q(B_1)] - \beta \sum_{i=1}^{M} \pi(g_i^1 | g_0) u'(c_i^L) \right] + (1 - \gamma) \left( \frac{\gamma}{1 - \gamma} \right) \left[ -u'(c_0^H) [q'(B_1)B_1 + q(B_1)] + \beta \sum_{i=1}^{M} \pi(g_i^1 | g_0) u'(c_i^H) \right] = 0
\]  

Using the stochastic Euler equation of the H types and simplifying:

\[
u'(c_0^L) [q'(B_1)B_1 + q(B_1)] - \beta \sum_{i=1}^{M} \pi(g_i^1 | g_0) u'(c_i^L) = u'(c_0^H) q'(B_1)B_1  \quad \text{(A.29)}
\]

\[
u'(c_0^L) + \left[ \frac{u'(c_0^L)q'(B_1) - \beta \sum_{i=1}^{M} \pi(g_i^1 | g_0) u'(c_i^L)}{q'(B_1)B_1} \right] = u'(c_0^H)  \quad \text{(A.30)}
\]

This last expression, compared with the similar expression of the planner without uncertainty, implies that in the planner’s view, the government expenditure shocks only matter to the extent that they affect the shadow price of the binding credit constraint of the L types. As before, since for L types to find it optimal to hold zero assets it must be that they are “credit constrained,” their Euler equation would imply that at the equilibrium price: \( u'(c_0^L)q(B_1) - \beta \sum_{i=1}^{M} \pi(g_i^1 | g_0) u'(c_i^L) > 0 \). Hence, the above optimality condition for the planner together with this condition imply that the optimal debt choice supports \( u'(c_0^L) > u'(c_0^H) \) or \( c_0^H > c_0^L \), and notice that from the budget constraints this implies again \( B_0 - q(B_1)B_1 > 0 \), which implies \( B_1/B_0 < 1/q(B_1) \). Furthermore, the latter implies that the optimal debt must be lower than any initial \( B_0 \) for any \( q(B_1) \geq 1 \), and also for ”sufficiently high” \( q(B_1) \). Thus the optimal debt choice again has an incentive to be lower than the initial debt.

### A1.4 Pricing Function Stochastic Model with Default

We can also make an inference about what the pricing function looks like in the model with default risk, because with default we have a similar Euler equation, except that the summation that defines the term \( \Pi(B_1) \) above will exclude all the states of \( g_1 \) for which the government chooses to default on a given \( B_1 \) (and also at a given value of \( \gamma \)). That is, the term in question becomes \( \Pi^D(B_1) \equiv \sum_{\{i:d(B_1, g_1^i, \gamma) = 0\}} \frac{\pi(g_i^1 | g_0)}{y-g_1^i + (\gamma/1-\gamma)B_1} \leq \Pi(B_1) \), and the pricing function with default risk is:

\[
q^D(B_1) = \beta \left( \frac{y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0}{1 + \left( \frac{\gamma}{1-\gamma} \right) \beta B_1 \Pi^D(B_1)} \right) \leq q(B_1)  \quad \text{(A.31)}
\]
Moreover, it follows from the previous analysis that this pricing function is also decreasing in \( B_1 \) (\( q^{D}(B_1) < 0 \)), and \( \Pi^{D'}(B_1) = -\sum_{\{i: d(B_1, g_i, \gamma) = 1\}} \frac{\pi(g_i\mid g_0)(\gamma)}{(y-g_i+(\frac{\gamma}{1-\gamma})B_0)} \) is negative but such that \( \Pi'(B_1) \leq \Pi^{D'}(B_1) < 0 \). Also, it is clear from the above pricing functions that if the probability of default is small, so that are only a few values of \( i \) for which \( d(B_1, g_i, \gamma) = 1 \) and/or the associated probability \( \pi(g_i\mid g_0) \) is very low, the default pricing function will be very similar to the no-default pricing function.

If we define the default risk spread as \( S(B_1, \gamma) \equiv [1/q^{D}(B_1, \gamma)] - [1/q(B_1, \gamma)] \), where \( \gamma \) has been added as an argument of the price functions because those prices also depend on variations in inequality, the spread reduces to the following expression:

\[
S(B_1, \gamma) = \left( \frac{1}{\beta \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 \right)} \right) \left[ \frac{1}{\Pi^{D}(B_1, \gamma)} - \frac{1}{\Pi(B_1, \gamma)} \right]
\]

Clearly since \( \Pi^{D}(B_1, \gamma) \leq \Pi(B_1, \gamma) \) the spread is non-negative, and it is strictly positive if there is default at equilibrium. The spread is increasing in \( B_1 \), because as the debt rises default is chosen optimally in more of the possible realizations of \( g_i \) and hence \( \Pi^{D}(B_1, \gamma) \) falls further below \( \Pi(B_1, \gamma) \), so that the gap between the reciprocals of these two terms widens. Note also that the spread is a multiple of the gap between these reciprocals, with the multiple given by \( 1/\beta \left( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 \right) \). As a result, the total date-0 resources available for consumption of the H-types \( y - g_0 + \left( \frac{\gamma}{1-\gamma} \right) B_0 \) have a first-order negative effect on the spreads. This is because, as this measure of income rises, the marginal utility of date-0 consumption of H types falls, which pushes up bond prices. The are also second order effects, because the equilibrium allocation of \( B_1 \) also depends on that income measure, and thus \( \Pi^{D}(B_1, \gamma) \) and \( \Pi(B_1, \gamma) \) vary with it as well, but these are not considered here.

In terms of the effect of changes in \( \gamma \) on \( S \), notice that there are two effects. First, there is a negative effect because higher \( \gamma \) means that, for a given \( B_0 \), the resources available for date-0 consumption of H types increase, since fewer H type agents need to demand enough initial bonds to clear the bond market, which means that per-capita each of the H types hold more date-0 bonds and have more bond income. Second, there is a positive effect because rising \( \gamma \) weakens default incentives as the welfare of the wealthy is valued more, and hence default is optimally chosen in more states, which increases \( [1/\Pi^{D}(B_1, \gamma) - 1/\Pi(B_1, \gamma)] \). Thus, in principle the response of the spread to increases in inequality is ambiguous. The weaker the response of the default probability to changes in \( \gamma \), however, the more likely it is that the first effect will dominate and the spreads will be a decreasing function of inequality.

### A1.5 Additional Figures Benchmark Model

Figure 18 shows two panels with the optimal default decision organized in the same way as the panels of Figure 2. The plots separate the regions where the government chooses to repay \( (d(B_1, g_i, \gamma) = 0 \) shown in white), where it chooses to default \( (d(B_1, g_i, \gamma) = 1 \) in green) and where the equilibrium does not exist (in blue).
These plots illustrate the implications of the mechanisms highlighted in Figure 2 for the default decision. The repayment region \((d(B_1, g_1, \gamma) = 0)\) corresponds to the region with \(\bar{\alpha}(B_1, g_1, \gamma) < 0\). Hence, the government defaults at higher \(B_1\) the lower a given \(\gamma\), or at higher \(\gamma\) the lower a given \(B_1\). Moreover, the two plots show that when \(g_1 = \bar{g}_1\) the government defaults for combinations of \(\gamma\) and \(B_1\) for which it repays when \(g_1 = g_1\). Thus, default occurs over a wider set of \((B_1, \gamma)\) pairs at higher levels of government expenditures, and thus it is also more likely to occur.

We examine further the behavior of the default decision by computing the threshold value of \(\gamma\) such that the government is indifferent between defaulting and repaying in period for a given \((B_1, g_1)\). These indifference thresholds \((\hat{\gamma}(B_1, g_1))\) are plotted in Figure 19 against debt levels ranging from 0 to 0.4 for three values of government expenditures \(\{g_1, \mu_g, \bar{g}_1\}\). For any given \((B_1, g_1)\), the government chooses to default if \(\gamma \geq \hat{\gamma}\).
Figure 19: Default Threshold $\hat{\gamma}(B_1, g_1)$

Figure 19 shows that the default threshold is decreasing in $B_1$. Hence, the government tolerates higher debt ratios without defaulting only if wealth concentration is sufficiently low. Also, default thresholds are decreasing in $g_1$, because the government has stronger incentives to default when government expenditures are higher (i.e., the threshold curves shift inward)\textsuperscript{39}. This last feature of $\hat{\gamma}$ is very important to determine equilibria with debt subject to default risk. If, for a given value of $B_1$, $\gamma$ is higher than the curve representing $\hat{\gamma}$ for the lowest realization in the Markov process of $g_1$ (which is also the value of $g_1$), the government defaults for sure and, as explained earlier, there is no equilibrium. Alternatively, if for a given value of $B_1$, $\gamma$ is lower than the curve representing $\hat{\gamma}$ for the highest realization of $g_1$ (which is the value of $\overline{g}_1$), the government repays for sure and debt would be issued effectively without default risk. Thus, for the model to support equilibria with debt subject to default risk, the optimal debt chosen by the government in the first period for a given $\gamma$ must lie between these two extreme threshold curves. We show that this is the case later in this Section.

Figure 20 shows intensity plots of the equilibrium tax functions in period 1 organized in the same way as in Figure 2.

\textsuperscript{39}$\hat{\gamma}$ approaches zero for $B_1$ sufficiently large, but in Figure 19 $B_1$ reaches 0.40 only for exposition purposes.
Putting together these plots with those of the default decision in Figure 18 illustrates the model’s distributional incentives to default from the perspective of tax policy. If the government defaults, the tax is \( \tau_1 = g_1 \), but if it repays the tax is \( \tau_1 = g_1 + B_1 \). Since all agents pay the same taxes but \( L \) types do not collect bond repayments, the lower taxes under default provide a distributional incentive to default that is larger when wealth is more concentrated. Figure 20 shows that, for given \( g_1 \), the repayment scenarios with higher taxes are more likely when a large fraction of households hold debt (low \( \gamma \)) and thus benefit from a repayment, or when the debt is low so that the distributional incentives to default are weak. Moreover, equilibria with higher taxes are more likely to be observed at low than at high levels of government expenditures, because default is far likely with the latter.

Figure 21 plots the bond demand decision rules (for given \( B_1 \)) of the \( H \) types in the same layout as the bond prices (i.e. as functions of \( \gamma \) for three values of \( B_1 \)).\(^{40}\) This plot validates the intuition provided above about the properties of these agents’ bond demand function. In particular, as \( \gamma \) increases, the demand for bonds of \( H \) types grows at an increasing rate, reflecting the combined effects of higher per-capita demand by a smaller fraction of \( H \)-type agents and a rising default risk premium. Thus, the convexity of these bond decision rules reflects the effects of wealth dispersion on demand composition and default risk explained earlier.

\(^{40}\)We do not include the bond decision rules for \( L \) types because they are credit constrained (i.e. their Euler condition holds with inequality) and choose \( b_L^0 = b_L^* = 0 \).
A1.6 Sensitivity Analysis Benchmark Model

This Section presents the results of a set of counterfactuals that shed more light on the workings of the model and also show some results for the case in which the social welfare function has biased weights that do not correspond to the fractions of $L$ and $H$ types in the economy.

A1.7 Sensitivity Analysis

The sensitivity analysis studies how the main results discussed in the previous Section are affected by changes in the initial debt $B_0$, initial government expenditures $g_0$, and the constant in the default cost function $\phi_0$.

A1.7.1 Lower Initial Debt Level $B_0$

Figure 22 uses the same layout of Figure 4 to compare the optimal government debt and associated equilibrium bond prices, spreads and default probability under the original calibration with $B_0 = 0.35$ and a value that is 20% lower ($B_0,L = 0.28$).

Panel (i) shows that the optimal debt choice is always lower for the lower $B_0$, but the difference narrows as $\gamma$ rises. This occurs because at low $\gamma$ default risk plays a negligible role, and the demand composition effect implies lower per-capita demand for bonds from $H$ type agents and a smaller fraction of credit-constrained $L$ types, so the government wants to sell less debt. But once default risk becomes relevant, the optimal debt choice is about the same.

With the lower $B_0$, bond prices are uniformly lower albeit slightly (panel (ii)), spreads are increasing as $\gamma$ rises for a wider range of $\gamma$ and attain a lower maximum (panel (iii)), and
the same is true for default probabilities (panel (iv)). Bond prices are lower with lower initial debt because for a given $\gamma$ this implies lower initial wealth of H types ($b_H^{\gamma}$), and in turn this requires a lower bond price to clear the market. This effect is stronger than two other effects that push bond prices in the opposite direction: First, the slightly lower debt $B_1$ that the government finds optimal to supply at lower levels of $B_0$. Second, the higher disposable income of households resulting from the lower date-0 taxes needed to repay lower levels of $B_0$, which increases demand for bonds.

Figure 22: Changes in Initial Government Debt $B_0$

In terms of the implications for the empirically relevant range of $\gamma$ in the European data, these results show that at the lower $B_0$ the model continues to predict debt ratios of about 8 to 12%, but now at spreads that are only half (40 v. 100 basis points) and at default probabilities below 1 percent instead of 1 to 1.5 percent. Moreover, these results also show that spreads and default probabilities can display richer patterns than the ones found in the initial calibration.
A1.7.2 Lower Initial Government Expenditures $g_0$

Figure 23 compares the model’s equilibrium outcomes under the calibrated value of initial government expenditures, $g_0 = \mu_g = 0.181$, and a scenario in which $g_0$ is 1.5 standard deviations below the mean, $g_{0,L} = 0.171$.

Like the reduction in $B_0$, the reduction in $g_0$ increases date-0 disposable income via lower date-0 taxes. They differ, however, in two key respects: First, changes in $g_0$ affect the expected level of government expenditures for $t = 1$, as reflected in changes in the transition probabilities which are conditional on $g_0$. Second, changes in $g_0$ do not affect the aggregate wealth of the economy and the initial bond holdings of $H$ types.

Panel (i) shows that the optimally debt is slightly higher with lower $g_0$, unless $\gamma$ is below 0.35. This reflects the fact that the lower $g_0$ allows the government to issue more debt in the initial period, because the likelihood of hitting states with sufficiently high $g_1$ for optimal default to occur in the second period is lower. This also explains why in panels (ii)-(iv), despite the higher optimal debt with the lower $g_0$, bond prices, default probabilities and spreads are lower. Moreover, optimal debt is about the same with lower $g_0$ as in the initial calibration for $\gamma \leq 0.35$ because at this low level of wealth concentration default risk is not an issue and the mechanism that we just described is irrelevant.
A1.7.3 Fixed Cost of Default $\phi_0$

Panels $(i) - (iv)$ in Figure 24 compare the equilibrium outcomes for the initial calibration of the default cost parameter ($\phi_0 = 0.004$) with an scenario with a higher values ($\phi_{0,H} = 0.02$).

Figure 24: Changes in Cost of Default $\phi_0$

Panel (i): Debt. Choice $B_1^*(\gamma)$

Panel (ii): Bond Price $q(B_1^*(\gamma), \gamma)$

Panel (iii): Spread (%)

Panel (iv): Def. Prob. $p(B_1^*(\gamma), \gamma)$

Qualitatively, the changes in the equilibrium are in the direction that would be expected. Higher default costs increase the optimal debt and reduce bond prices, spreads and default probabilities. Quantitatively, however, the changes in optimal debt and bond prices are small, while the reduction in spreads and default probabilities are significantly larger.

A1.8 Additional Figures Model with Biased Welfare Weights

Adding default costs to this political bias setup ($\phi(g_1) > 0$) makes it possible to support repayment equilibria even when $\omega \geq \gamma$. As Figure 25 shows, with default costs there are threshold values of consumption dispersion, $\hat{\epsilon}$, separating repayment from default zones for $\omega \geq < \gamma$. 

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Figure 25: Default decision with non-utilitarian planner when $\phi(g_1) > 0$

It is also evident in Figure 25 that the range of values of $\epsilon$ for which repayment is chosen widens as $\gamma$ rises relative to $\omega$. Thus, when default is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in $\epsilon$, do not produce consumption dispersion larger than the value of $\hat{\epsilon}$ associated with a given $(\omega, \gamma)$ pair. Intuitively, the consumption of $H$-type agents must not exceed that of $L$-type agents by more than what $\hat{\epsilon}$ allows, because otherwise default is optimal.

Figure 26 shows the default decision rule induced by the planner’s welfare gains of default, again as a function of $\omega$ and $\gamma$ for the same two values of $B_1$. The region in white corresponds to cases where $d(B_1, g_1, \gamma, \omega) = 0$, the green region corresponds to $d(B_1, g_1, \gamma, \omega) = 1$ and the blue region corresponds to cases in which there is no equilibrium.
In line with the pattern of the government’s welfare gains of default presented in Figure 8, this Figure shows that when the $\omega$ is low enough, the government chooses default, and for a given $\omega$ the default region is larger the lower is $\gamma$. Taxes and prices for given values of $B_1$ and $\omega$ are linked to the default decision and $\gamma$ as in the benchmark model and the intuition behind their behavior is straightforward.

**A1.9 Additional Figures Model with Foreign Investors**

Figure 27 shows the bond decision rules for domestic and foreign investors as the value of $\gamma$ and $B_1$ change.
Figure 27: Decision Rule Domestic and Foreign Agents

Figure 28 presents the optimal domestic and foreign demand (Panel (i)) and the optimal debt issuance of the government (Panel (ii)). This Panel also marks with red squares the debt levels where there is positive risk of default and shows that the feature of equilibrium domestic default is also robust to the introduction of foreign lenders.
Panel (i) of Figure 28 shows that domestic (foreign) demand is an increasing (decreasing) function of $\gamma$. Panel (ii) of Figure 28 shows that the debt choice of the government is a concave function of $\gamma$. This is a direct result of default incentives. When $\gamma$ is low the government has strong incentives to default of foreign lenders, so debt issuance remains low. As $\gamma$ increases and domestic households increase their demand for sovereign debt, forces for a “foreign” default decrease and higher debt levels are attainable. As higher levels of $\gamma$, “domestic” default incentives constraint the government to increase the debt further and the optimal choice decreases.