Belief Regimes and Sovereign Debt Crises*

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Abstract

Sovereign debt spreads occasionally exhibit sharp, large spikes in spreads over risk-free bonds. We document that these movements are only weakly correlated with movements in domestic output and are frequently followed by reductions in the face value of debt outstanding. Motivated by this evidence, we propose a quantitative model with long-term bonds and three sources of risk: fluctuations in the growth of domestic income; movements in the risk premia associated with default risk; and shifts in creditor “beliefs” regarding the actions of other creditors. We show that the shifts in creditor beliefs directly play an important role in generating default risk, but also amplify the impact of shocks to fundamentals. Interestingly, persistent changes to risk premia have a negligible impact on spreads, and an increase in risk premium may even lead to a decline in spreads. The latter reflects that a higher risk premium provides discipline regarding future debt issuances. More generally, the sovereign borrowing decisions are quantitatively sensitive to equilibrium bond prices. Even large, relatively unexpected shocks to creditor beliefs have only a modest effect on spreads as the government responds by aggressively deleveraging.

*The views expressed here are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System. We thank Manuel Amador for numerous helpful comments. We also thank our discussant Luigi Paciello.*
1 Introduction

In this paper, we study an environment in which sovereign debtors face risk due to shocks to the endowment, shocks to the risk premia charged in loan markets, and shocks to creditor “beliefs” regarding the actions of other creditors. We motivate this environment by documenting a number of facts regarding emerging market bonds. First, as is well known, emerging market spreads over benchmark risk-free bonds are volatile. Second, while large spikes in spreads are correlated with declines in output, the correlation is relatively weak. In fact, a sizable proportion of such spikes occur when growth is positive and in line with historical means. Third, we show that while some of these unexplained movements in spreads can be explained by shifts in measures of global risk premia, there remains a large and time-varying residual component to movements in spreads. One possible interpretation of this residual source of risk is that the likelihood of a self-fulfilling debt crisis is time varying. It is this latter notion that we embed in our quantitative model to study the associated equilibrium outcomes.

Specifically, we consider a small open economy that trades a non-contingent (but defaultable) bond with a pool of creditors and a constant world risk-free interest rate, as in Eaton and Gersovitz (1981) and the recent quantitative literature starting with Aguiar and Gopinath (2006) and Arellano (2008). The economy’s endowment process is subject to shocks to both transitory and trend output, as in (Aguiar and Gopinath, 2006, 2007). The creditors involved in sovereign lending are risk-averse with finite wealth, and hence the sovereign pays a risk premium. We allow the pool of creditors, and the associated stock of wealth to be invested, to vary over time exogenously, generating another source of risk. Finally, we build on the timing introduced by Cole and Kehoe (2000) to support multiple Markov equilibria which may involve rollover crises. We consider an equilibrium such that creditor beliefs regarding the probability of a rollover crisis is time varying. To the best of our knowledge, this is the first quantitative model that incorporates all three sources of risk.\footnote{A related recent effort is Bocola and Dovis 2015, which attempts to decompose the relative contribution of each factor in the recent crisis in Italy.}

The process for creditor beliefs follows a three-state Markov process. One belief regime is a rollover crisis a la Cole and Kehoe (2000), in which the government is forced to default by a run on its debt. A second belief regime is a “tranquil” regime, in which the probability of a run in the next quarter is negligible. The middle regime is a “vulnerable” regime, in which creditors do not run in the current quarter, but consider the likelihood of a run in the
next quarter to be relatively high.

We calibrate the model to Mexico and explore the role of the multiple sources of risk. We find that roughly half of the simulated debt crises, defined as a jump in spread in the upper 5th percentile of the distribution, occur when creditors believe a rollover crisis is imminent. The other half occur when creditor beliefs are relatively “tranquil,” despite the fact that we calibrate beliefs to be tranquil for over 80 percent of the periods. Debt crises in the tranquil-belief regime are associated with boom-bust cycles in output. The preceding boom is necessary to generate large debt positions, which makes default probabilities sensitive to the subsequent bust. Shifts in creditor beliefs can generate sharp spikes in spreads in the absence of declines in fundamentals, bringing us closer to the patterns documented in the data.

Interestingly, shocks to creditor wealth are relatively unimportant in generating large spikes in spreads. For high levels of debt, a drop in creditor wealth raises the risk premium charged on sovereign bonds and generates an increase in spreads. However, for moderate levels of debt, spreads may actually fall with a drop in creditor wealth. This is because wealth shocks are persistent and provide a disincentive to issue more debt in the future, mitigating the dilution problem associated with long-maturity bonds.

A switch from tranquil to vulnerable beliefs generates a sharp increase in spreads. The sovereign either immediately defaults, or begins to reduce debt quickly. Along the path, it may be hit with a rollover crisis, but conditional on survival it reduces its level of debt. These dynamics are reminiscent of the theoretical insights of Cole and Kehoe (2000).

We calibrate the model so that defaults occur on average twice every hundred years. Only about half of the outright defaults in the simulation occur when creditor beliefs are in the tranquil regime, despite the fact that this regime accounts for the vast majority of the periods. These are triggered by large declines in endowment growth, as in the standard Eaton-Gersovitz framework. Approximately 10 percent of the defaults occur in the vulnerable regime and another 40 percent are due to outright rollover crises. The defaults when beliefs are tranquil are not unrelated to the probability of a rollover crisis at some point in the future. We compute an alternative model in which rollover crises never occur and evaluate default decisions using these alternative policy functions. All of the simulated defaults in the benchmark model occur in parts of the state space in which the sovereign would repay debt if rollover crises are always probability zero events. This reflects that the tradeoff between defaulting versus retaining access to credit markets is fundamentally changed when creditor
beliefs are time varying.

2 Motivating Facts

2.1 Data for Emerging Markets

We start with a set of facts that guides our model of sovereign debt crises. Our sample spans the period 1993Q4 through 2014Q4, and includes data from 20 emerging markets: Argentina, Brazil, Bulgaria, Chile, Columbia, Hungary, India, Indonesia, Latvia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Turkey, and Ukraine. For each of these economies, we have data on GDP in US dollars measured in 2005 domestic prices and exchange rates (real GDP), GDP in US dollars measured in current prices and exchange rates (nominal GDP), gross external debt in US dollars (debt), and on market spreads on sovereign debt.\(^2\)

Tables 1 and 2 report summary statistics for the sample.\(^3\) Table 1 document the high and volatile spreads that characterized emerging market sovereign bonds during this period. The standard deviation of the level and quarterly change in spreads are 6.76 and 2.29 percentage points, respectively. Table 2 reports an average external debt-to-(annualized)GDP ratio of 0.46. This level is low relative to the public debt levels observed in developed economies. The fact that emerging markets generate high spreads at relatively low levels of debt-to-GDP reflects one aspect of the “debt intolerance” of these economies documented by Reinhart, Rogoff, and Savastano (2003).

While many of the countries in our sample have very high spreads, only two - Russia in 1998 and Argentina in 2001 - ended up defaulting on their external debt, while a third, Ukraine, defaulted on its internal debt (in 1998). This reflects that defaults are relatively rare events; Tomz and Wright (2013) document over a larger sample that default occurs with an unconditional probability of roughly 2 percent.

\(^2\)Data source for GDP and debt is Haver Analytics’ Emerge database. The source of the spread data is JP Morgan’s Emerging Market Bond Index (EMBI).

\(^3\)Note that Russia defaulted in 1998 and Argentina in 2001, and while secondary market spreads continued to be recorded post default, these do not shed light on the cost of new borrowing as the governments were shut out of international bond markets until they reached a settlement with creditors. Similarly, the face value of debt is carried throughout the default period for these economies.
Table 1: Sovereign Spreads: Summary Statistics

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean $r - r^*$</th>
<th>Std Dev $r - r^*$</th>
<th>Std Dev $\Delta(r - r^*)$</th>
<th>95th pct $\Delta(r - r^*)$</th>
<th>Frequency Crisis</th>
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<td>Corr $(r - r^*, %\Delta B)$</td>
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2.2 Debt Crises

One of the first facts that we want to document is that there is a relatively weak link between a country’s fundamentals and whether or not it has a debt crisis. We consider a debt crisis to be a large spike in spreads, regardless of whether the country defaults or not. From Table 1, we see that the 95th percentile of the quarterly change in spread is 158 basis points. We define a debt crisis to be a quarterly change in spreads that exceeds this threshold. By definition, 5 percent of the sample involves a debt crisis. However, this is not equally distributed across countries. From Table 1, we see that five countries never have experience a crisis, while nearly 20 percent of Argentina’s quarterly observations are above the crisis threshold.

An important question is whether debt crises are associated with domestic fundamentals. Table 2 reports a generally negative correlation between the change in spreads and output. However, this statistic masks surprising heterogeneity across crisis episodes. Figure 1 depicts the density of GDP growth in crisis and non-crisis quarters. Specifically, each quarter is assigned to a crisis or non-crisis category, depending on whether the change in EMBI spread from last quarter exceeds the 95th percentile threshold. We then plot the estimated kernel density of quarterly GDP growth for each subsample. In Panel (a), the solid line depicts the kernel density of quarterly growth in real GDP in quarters defined as non-crisis, while the dashed line depicts the corresponding histogram for crisis quarters. Panel (b) redefines growth as the average growth over the preceding four quarters; that is, if period \(t\) is used to define crisis status (that is, \(r_t - r_{t-1} > 1.58\)), then growth is averaged over quarters \(t - 5\) and \(t - 1\) (that is, \((\ln Y_{t-1} - \ln Y_{t-5})/4\)). Panel (c) looks at subsequent growth; that is, average growth between period \(t\) and \(t + 4\).

By comparing the two densities in Panel (a), one can see from the increase in the frequency of negative growth rates for countries experiencing crises relative to the overall distribution that there is a higher tendency for negative growth rates to be associated with crises. However, what is striking is the extent to which countries experiencing positive growth also experienced crises. Overall, the graph indicates some association between negative growth and debt crises, but the association is not strong. Specifically, while mean contemporaneous (quarterly) growth during a crisis is -1.2, as opposed to 1.0 during non-crisis quarters, nearly half (46 percent) of the crisis periods have strictly positive growth.

Not only do contemporaneous fundamentals have a hard time accounting for debt crises, they do an even worse job of forecasting debt crises. Specifically, in Panel (b) we see that
Figure 1: Sovereign Debt Crises and Growth

(a) Contemporaneous Growth
(b) Preceding-Year Average Growth
(c) Subsequent-Year Average Growth

Note: Each panel overlays two histograms (kernel densities) of GDP growth. The sample consists of 23 emerging markets between 1993Q4 and 2014Q4 (see text). The histogram labelled “No Crisis” refers to periods in which the quarterly change in sovereign bond spreads is less than 158 basis points. The histogram labelled “Crisis” refers to periods in which the change in spread is greater than or equal to 158 basis points. This threshold is chosen so that 5 percent of the periods are defined as “Crisis.” In Panel (a), growth is defined as the quarterly change in log GDP contemporaneous with the quarterly change in spreads used to define a crisis. Specifically, if a crisis occurs in quarter \( t \) due to \( r_t - r_{t-1} > 185 \), where \( r_t \) is the spread over the risk-free rate observed in quarter \( t \), then growth is \( y_t - y_{t-1} \), where \( y_t \) is log GDP in quarter \( t \). In Panel (b), growth is averaged over the year preceding the quarter used to define a crisis; that is \( (y_{t-1} - y_{t-5})/4 \). In Panel (c), growth is averaged over the subsequent year; that is \( (y_{t+4} - y_t)/4 \).
growth is often quite positive in the year prior to the jump in spreads. Here the association between growth and crises is substantially weaker than with contemporaneous growth rates.

The lack of contemporaneous and lagged association between growth debt crises leaves open the possibility that it is the anticipation of future bad fundamentals that lead to a rise in spreads and debt crises. If such news shocks played an important role, then one would expect to see high spreads forecast negative future fundamentals. In the third panel we illustrate the lack of a tight connection between news about fundamentals and spreads by again graphing growth rate histograms, but now the density is for growth rates in the year following a debt crises. The number of crises followed by positive growth is remarkable given that there are many reasons to believe high spreads should suppress economic activity (see, for example, Neumeyer and Perri, 2005).

This evidence suggests that while negative GDP growth is correlated with increases in spreads, the association is rather weak. This fact complements the finding of (Tomz and Wright, 2007) that a significant fraction (roughly 40%) of defaults occur when GDP is above trend. While shocks to output are a natural starting point for understanding sovereign debt crises, the data suggest there is much more to the story.

2.3 Deleveraging

The data from emerging markets can also shed light on debt dynamics during a crisis. Table 2 documents that periods of above-average spreads are associated with reductions in the face value of gross external debt. The pooled correlation of spreads at time $t$ and the percentage change in debt between $t-1$ and $t$ is $-0.19$. The correlation of the change in spread and debt is roughly zero. However, a large change in spread (that is, a crisis period) is associated with a subsequent decline in debt. In particular, regressing the percent change in debt between $t$ and $t+1$ on the indicator for a crisis in period $t$ generates a coefficient of $-1.6$ and a t-stat of nearly 3. This relationship is robust to the inclusion of country fixed effects. This implies that a sharp spike in spreads is associated with a subsequent decline in the face value of debt. Note that the decline in the level of outstanding debt is not always associated with a decline in the debt-to-GDP ratio. From Figure 1 we know that many crises are associated with declines in GDP, which frequently are larger than the percentage declines in debt.
2.4 Excess Returns

Another possible source of volatility in sovereign debt spreads is shifts in the appetite for risk. Sovereign bonds appear to earn an excess return, and the extent of this return is correlated with such global risk factors as US stock returns and volatility indices (Pan and Singleton, 2008; Longstaff, Pan, Pedersen, and Singleton, 2011; Borri and Verdelhan, 2011). A similar pattern emerges from our sample of emerging markets. In particular, we compute the realized returns on a value-weighted portfolio of actively traded, emerging country bonds, specifically the EMBI+ index constructed by JP Morgan. Table 3 reports the return for the full sample period the index is available, as well as two sub-periods. The table also reports the returns to portfolio of 2-year and 5-year maturity U.S. Treasury debt. We offer two risk-free references as the EMBI+ does not have a fixed maturity structure and typically ranges between 2 and 5 years.

The EMBI+ index paid a return in excess of the risk-free portfolio of five to six percent. This excess return is roughly stable across our two sub-periods as well. The index reported in Table 3 represents a fairly diversified portfolio of emerging economy debt. The underlying country-level returns range from 1.3 percent for Argentina to over 10 percent for Brazil, Bulgaria, and Peru. Whether the realized return reflects the ex ante expected return depends on whether our sample accurately reflects the population distribution of default and repayment. The assumption is that by pooling a portfolio of bonds, the EMBI+ followed over 20 years of data provides a fair indication of the expected return on a typical emerging market bond. Of course, we cannot rule out the possibility that this sample is not representative. Nevertheless, the observed returns are consistent with a fairly substantial risk premia charged to sovereign borrowers.

We explore the impact of global risk factors on country-level spreads in Table 4. In the first two columns of panel (a), we report the results of regressing the level of spreads (at the close of each quarter) on the contemporaneous level of the P/E ratio or the VIX as well as country fixed effects. The last two columns perform the same regressions with the level of the log bond price index as the dependent variable. Panel (b) reports the results of the same regressions using quarterly first differences. For the price index, we compute quarterly log changes; for all other variables the differences are in levels. In levels, an increase in the P/E ratio, which implies a decrease in the associated risk premium, is associated with an increase in spreads, and a decline in bond prices. However, this pattern is reversed if the VIX is used as the measure of risk.
Table 3: Realized Bond Returns

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<tr>
<th>Period</th>
<th>EMBI+</th>
<th>2-Year Treasury</th>
<th>5-Year Treasury</th>
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<td>1993Q1–2014Q4</td>
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</table>

Panel (b) suggests that in first differences, an increase in risk raises spreads and lowers prices, regardless of which measure we use as a proxy for global appetite for risk. Contrasting the two panels suggests that emerging market spreads co-move with global risk premia at high frequencies, but this relationship may be weaker or reversed at lower frequencies. Given the relatively short time frame of the exercise, the latter statement is speculative. Nevertheless, we will see in the model that a low frequency reduction in creditors’ appetite for risk may indeed generate higher prices and lower spreads.

2.5 The European Crisis

The above facts concerned emerging markets, which have generated most of the post-war debt crises. However, the recent crisis in Europe has renewed interest in debt crises in more advanced economies. [TBA]

2.6 Taking Stock

The empirical facts documented above suggest sovereign debt crises are more than just the result of poor fundamentals, which is the natural starting point of the quickly growing literature on sovereign default. A successful quantitative model should address the following empirical regularities:

1. Crises, and particularly defaults, are low probability events;
2. Crises are not tightly connected to poor fundamentals;
3. Spreads are highly volatile;
Table 4: Global Risk Factors

(a) Levels

<table>
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<tr>
<th></th>
<th>$r_t - r_t^*$</th>
<th>$r_t - r_t^*$</th>
<th>$\ln p_t$</th>
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<td>VIX</td>
<td>14.81</td>
<td>-0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.41)</td>
<td>(0.002)</td>
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</tr>
<tr>
<td>$R^2$</td>
<td>0.28</td>
<td>0.29</td>
<td>0.41</td>
<td>0.31</td>
</tr>
<tr>
<td>$N$</td>
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<td>1,250</td>
<td>1,282</td>
<td>1,282</td>
</tr>
<tr>
<td>Countries</td>
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<td>22</td>
<td>22</td>
<td>22</td>
</tr>
</tbody>
</table>

(b) Differences

<table>
<thead>
<tr>
<th></th>
<th>$\Delta (r_t - r_t^*)$</th>
<th>$\Delta (r_t - r_t^*)$</th>
<th>$\Delta \ln p_t$</th>
<th>$\Delta \ln p_t$</th>
</tr>
</thead>
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<tr>
<td>$\Delta$ P/E Ratio</td>
<td>-30.06</td>
<td>0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.05)</td>
<td>(0.002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta$ VIX</td>
<td>10.68</td>
<td>0.08</td>
<td>-0.0036</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.04)</td>
<td>(0.0008)</td>
<td></td>
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<tr>
<td>$R^2$</td>
<td>0.04</td>
<td>0.08</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>$N$</td>
<td>1,228</td>
<td>1,228</td>
<td>1,260</td>
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<td>22</td>
</tr>
</tbody>
</table>
4. Rising spreads are associated with de-leveraging by the sovereign; and

5. Risk premia are an important component of sovereign spreads.

In considering what features of real-world economies are important in generating these patterns, the first thing to recognize is that sovereign debt lacks a direct enforcement mechanism: most countries default despite having the physical capacity to repay. Yet, countries seem perfectly willing to service significant amounts of debts most of the time (rescheduling of debts and outright default are relatively rare events). Without any deadweight costs of default, the level of debt that a sovereign would be willing to repay is constrained by the worst punishment lenders can inflict on the sovereign, namely, permanent exclusion from all forms of future credit. It is well known that this punishment is generally too weak, quantitatively speaking, to sustain much debt (this is spelled out in a numerical example in Aguiar and Gopinath, 2006). Thus, we need to posit deadweight costs of default.

Second, defaults actually occurring in equilibrium reflect that debt contracts are not fully state-contingent, and default provides an implicit form of insurance. However, even with rational risk-neutral lenders who break-even on average for every loan extended to sovereigns, the deadweight cost of default (which do not accrue to lenders) makes default an actuarially unfair form of insurance against bad states of the world. This insurance-through-default becomes even more actuarially unfair with risk-averse lenders. Moreover, sovereigns can self-insure by accumulating reserves as precautionary savings, which is an attractive option for smoothing transitory shocks. Given that this suggests fairly substantial deadweight costs of default and substantial risk aversion on the part of lenders, the insurance offered by the possibility of default appears to be quite costly in practice. The fact that countries carry large external debt positions despite the costs suggests that sovereigns are fairly impatient, perhaps reflecting political economy frictions.

However, while myopia can explain in part why sovereigns borrow, it does not necessarily explain why they default. As noted already, default is a very costly form of insurance against bad states of the world. This fact – via equilibrium prices – can be expected to encourage the sovereign to stay away from debt levels for which the probability of default is significant. This has two implications. First, when crises/defaults do materialize, they come as a surprise, which is consistent with these events being low probability events. Unfortunately, the other side of this coin is that getting the mean and volatility of spreads right is a challenge for quantitative models, especially once we incorporate that many shifts in spreads occur in
relatively good economic times. Getting high and variable spreads means getting periods of high default risk as well as substantial variation in expected future default risk. This will be difficult to achieve when the borrower has a strong incentive to adjust its debt-to-output level to the point where the probability of future default is (uniformly) low.

This discussion highlights the fact that getting both high and time-varying spreads, high debt-to-output and infrequent crisis and defaults poses a challenge. One way to attack the problem is to leverage the fact that an important element of sovereign borrowing is the sovereign’s need to roll over its debts (the time horizon of investors is typically shorter than the time horizon of the sovereign). The inability to rollover one’s debt can induce default in circumstance in which the borrower would have repaid with the rollover. From the lender’s perspective a rollover default represents a coordination failure if repayment would have occurred with rollover. This channel has been thought to be important for explaining the sorts of debt crises we see (e.g. Calvo, 1988; Cole and Kehoe, 2000). A contribution of the current study is to quantify how the possibility of rollover default impacts the level and the time variation in spreads, and whether that matches the data.

3 Environment

We consider a single-good, discrete-time environment, with time indexed by \( t = 0, 1, \ldots \). The focus of the analysis is a small open economy that receives a stochastic endowment. The economy is small relative to the rest of the world in the sense that its endowment realizations and decisions do not affect the world risk-free interest rate. However, financial markets are segmented in the sense that the economy can borrow from a set of potential lenders with limited wealth. Consumption and saving decisions are made on behalf of the domestic economy by a sovereign government. In this section, we proceed by characterizing the domestic economy and the sovereign’s problem, then turn to the lenders’ problem, and conclude by defining an equilibrium in the sovereign debt market.
3.1 The Domestic Economy

3.1.1 Technology

The economy receives a stochastic endowment $Y_t > 0$ each period. The endowment process is characterized by:

$$Y_t = G_t e^{z_t},$$

where

$$\ln G_t \equiv \sum_{s=1}^{t} g_s,$$

is the cumulation of period growth rates $g_t$, and $z_t$ represents fluctuations around trend growth. We assume that $g_t$ and $z_t$ follow finite-state first-order Markov processes. The relevant state vector for the current endowment and its probability distribution going forward is $(Y_t, g_t, z_t)$.

3.1.2 Preferences

The domestic economy is run by an infinitely lived sovereign government, which enjoys preferences over the sequence of aggregate consumption $\{C_t\}_{t=0}^{\infty}$ given by:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(C_t),$$

with $\beta \in (0, 1)$ and

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

with $\sigma \neq 1$. We assume that the sovereign has enough instruments to implement any feasible consumption sequence as a domestic competitive equilibrium, and therefore abstract from the problem of individual residents of the domestic economy. This does not mean that the government necessarily shares the preferences of its constituents, but rather that it is the relevant decision maker viz a viz international financial markets.

To ensure that the government’s problem is well behaved, we require:

$$\max_g \mathbb{E}_g \left\{ \beta e^{(1-\sigma)g'} \right\} < 1,$$
where the max is taken over elements of the Markov process for $g_t$.

### 3.1.3 Financial Markets

The sovereign issues non-contingent bonds to a competitive pool of lenders (described below). Bonds pay a coupon every period up to and including the period of maturity, which, without loss of generality, we normalize to $r^*$ per unit of face value, where $r^*$ is the (constant) international risk-free rate. With this normalization, a risk-free bond will have an equilibrium price of one. For tractability, we consider a bond with random maturity, as in Leland (1994). In particular, each bond matures next period with a constant hazard rate $\lambda \in [0, 1]$. We let the unit of a bond be infinitesimally small, and let maturity be $iid$ across individual bonds, such that with probability one a fraction $\lambda$ of any non-degenerate portfolio of bonds matures each period. The constant hazard of maturity implies that all bonds are symmetric before the realization of maturity at the start of the period, regardless of when they were purchased. Note as well the expected maturity of a bond is $1/\lambda$ periods, and so $\lambda = 0$ is a console and $\lambda = 1$ is one-period debt. While we vary $\lambda$ across quantitative exercises, within any specific environment there is only one maturity traded. With these conventions, a portfolio of sovereign bonds of measure $x$ receives a payment (absent default) of $(r^* + \lambda)x$, and has a continuation face value of $(1 - \lambda)x$.

We denote the outstanding stock of debt at the start of period $t$ by $B_t$. We do not restrict the sign of $B_t$, which allows the government to be either a net creditor ($B < 0$) or debtor ($B > 0$). Net issuances of new debt in period $t$ is given by $B_{t+1} - (1 - \lambda)B_t$, where $(1 - \lambda)$ is the fraction of debt that does not mature in the current period. If $B_{t+1} < (1 - \lambda)B_t$, then the government is repurchasing its outstanding debt rather than issuing new debt. We denote the debt-to-income ratio by $b_t \equiv \frac{B_t}{Y_t}$. To rule out Ponzi schemes, we place an upper bound on the debt-to-income ratio: $b_t \leq b_0$, \forall t.

### 3.1.4 Timing

The timing of a period is depicted in Figure 2. Let $s$ denote the aggregate state after the period’s realization of random variables but before the period’s consumption and debt-issuance decisions have been made. Specifically, $s = (Y, g, z, b, w, \rho)$, where $(Y, g, z)$ characterize the

---

4See also Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012) and Arellano and Ramnarayan (2012).

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current period endowment state; \( b \) is the inherited stock of debt normalized by \( Y \); \( w \) is the stock of lender wealth also normalized by \( Y \) and discussed in Section 3.3; and \( \rho \) is a random variable that coordinates equilibrium beliefs and is discussed in Section 3.5. As \( Y \) is an element of \( s \), the normalization of the other variables is without loss of generality. Let \( S \) denote the set of possible states \( s \). Other than \( b \), the elements of \( s \) follow an exogenous Markov process.

![Figure 2: Timing within a Period](image)

After observing the period’s realized \( s \), the government decides to auction \( B' - (1 - \lambda)B \) units of debt, where \( B' \) represents the face value of debt at the start of the next period. There is one auction per period. While this assumption is standard, it does allow the government to commit to the amount auctioned within a period.\(^5\) Let \( b' \) denote next period’s face value of debt normalized by the current endowment: \( b'_t \equiv \frac{B_{t+1}}{Y_t} \). The evolution of debt-to-income is then:

\[
b_{t+1} = b'_t \frac{Y_t}{Y_{t+1}} = b'_t e^{-g_{t+1} - z_{t+1} + z_t}.
\]

We postpone the formal definition of equilibrium until Section 3.4, but to anticipate we shall consider equilibria in which endogenous variables are functions of the state \( s \in S \). Let \( q(s, b') \) denote the equilibrium price schedule. The government is large in its own debt market and internalizes the fact that it faces different prices depending on how much debt it auctions; hence in choosing, \( b' \) the government internalizes the entire price schedule as a function of \( b' \). Given outstanding debt \( B \), the government is contractually obligated to pay \( \lambda B \) in principal and \( r^*B \) in interest payments. These payments are financed through

\(^5\)For an exploration of an environment in which the government cannot commit to a single auction, see Lorenzoni and Werning (2013).
current endowment as well as new debt issuances, and are made in a sub-period labeled “settlement” in figure 2. If the government makes its contracted payments, it consumes \( C = Y \left[ 1 - (r^* + \lambda) b + q(s, b')(b' - (1 - \lambda) b) \right] \), and continues on to the next period with the new debt state governed by \( b' \).

If the government fails to make the contracted payment, the government enters a default state the next period. While in default, the government has no access to foreign financial markets. Moreover, in the default state the government loses part of its endowment, which proxies for economic disruptions that are a consequence of default in practice. Let \( \phi(g, z) \) denote the proportional loss of output, so that in the default state the government receives \((1 - \phi(g, z))Y\) when the non-default state endowment is \( Y \). The proportion lost is a function of the realized endowment shocks \((g, z)\), but is otherwise independent of the level of income \( Y \). While in default, the government has a constant hazard \( \xi \in [0, 1] \) of exiting the default state. Having exited default, the government no longer suffers an endowment penalty and regains access to foreign financial markets. The debt outstanding at the time of default is forgiven, implying that creditors are paid zero in a default.

If the government defaults at settlement, it does not receive the proceeds from that period’s auction. In particular, it consumes its endowment (minus the cost of default). The amount raised via auction, if any, is disbursed to outstanding bondholders in proportion to the face value of their bond positions. An important implication of this convention is that holders of newly issued bonds are not fully compensated if the government defaults immediately after the auction. That is, each unit of debt is treated equally and receives \( q(s, b')(B' - (1 - \lambda) B)/B' \), which is strictly less than the price paid as long as \( B > 0 \), and therefore implies an immediate loss for new bondholders. In this regard, our timing deviates from that of Eaton and Gersovitz (1981), which has become standard in the quantitative sovereign debt literature. In the standard timing, the bond auction occurs after that period’s default decision has been made. Thus newly auctioned bonds do not face within-period default risk. Our timing expands the set of equilibria relative to the Eaton-Gersovitz timing, and in particular allows a tractable way of introducing self-fulfilling debt crises.\(^6\)

\(^6\)The timing in Figure 2 is adapted from Aguiar and Amador (2014b), which in turn is a modification of Cole and Kehoe (2000). The difference relative to Cole and Kehoe is that we do not allow the government to consume the proceeds of an auction if it defaults. This simplifies the off-equilibrium analysis without materially changing the results. See Auclert and Rognlie (2014) for a discussion of how the Eaton-Gersovitz timing in some standard environments has a unique Markov equilibrium, thus ruling out self-fulfilling crises.
3.2 The Government’s Problem

Let $V(s)$ denote the start-of-period value for the government, conditional on the state $s$ and the equilibrium price schedule $q$ (which we suppress in the notation). Working backwards through a period, at the time of settlement the government has issued $B'(1-\lambda)B$ units of new debt at price $q(s,b')$ and owes $(r^*+\lambda)B$.

If the government honors its obligations at settlement, its payoff is:

$$V^R(s,b') = u(C) + \beta\mathbb{E}[V(s')|s,b'],$$

with

$$C = Y - (r^* + \lambda)B + q(s,b')(B'^* - (1-\lambda)B$$

$$= Y [1 - (r^* + \lambda)b + q(s,b')(b'^* - (1-\lambda)b].$$

Note that consumption is pinned down at settlement by the budget constraint; if the required consumption is negative, we define $V^R(s,b') = -\infty$, which is always dominated by default.

If the government defaults at settlement, its payoff is:

$$V^D(s) = u(C) + \beta(1-\xi)\mathbb{E}[V^D(s')|s] + \beta\xi\mathbb{E}[V(s')|s,b' = 0],$$

with

$$C = (1-\phi(s))Y,$$

and where we recall that $\xi$ is the probability of exiting the default state. The output cost of default is governed by the function $\phi$. Note that $s'$ includes $b'$ as an element, and $b' = 0$ while in the default state by definition. The amount of new debt implied by $b'$ is not relevant for the default payoff as the government does not receive the auction proceeds if it defaults at settlement.

The start-of-period value function is:

$$V(s) = \max \left\{ \max_{b' \leq \tilde{b}} V^R(s,b'(s)), V^D(s) \right\}, \forall s \in S.$$ (5)

Let $B : S \rightarrow (-\infty, \tilde{b}]$ denote the policy function for $b'$ generated by the government’s
problem. We denote the policy function for default at settlement conditional on \( b' \) by \( D(s, b') \). For technical reasons, we allow the government to randomize over default and repayment when indifferent; that is, when \( V^R(s, b') = V^D(s) \). Therefore, \( D : S \times (-\infty, \bar{b}] \rightarrow [0, 1] \) is the probability the government defaults at settlement, conditional on \((s, b')\). The policy function of consumption is implied by those for debt and default.

### 3.3 Lenders

We assume financial markets are segmented and only a subset of foreign agents participate in the sovereign debt market. This assumption allows us to introduce risk premia on sovereign bonds as well as explore how shocks to foreign lenders’ wealth influence the domestic economy; all the while treating the risk-free rate as parametric. For tractability, we assume that a set of lenders has access to the sovereign bond market for one period, and then exits, to be replaced by a new set of lenders. The short horizon of the specialist lenders is for tractability, avoiding the need to solve an infinite horizon portfolio problem and carry another endogenous state variable. Nevertheless, it allows us to capture the possibility that exogenous shocks to lenders’ wealth affect the price of sovereign bonds.\(^7\)

Specifically, each period a unit measure of identical lenders enter the sovereign debt market. Let \( W_t \) denote the aggregate wealth of the agents that can participate in period \( t \)'s bond market, and \( W_{i,t} \) represent the wealth of a (representative) individual agent \( i \in [0, 1] \). Lenders allocate their wealth across sovereign bonds and a risk free asset that yields \( 1 + r^* \).

As noted above, the risk-free rate is pinned down by the larger world financial market, and specialists in the sovereign bond market can freely borrow and lend at this rate.

The one-period return on sovereign bonds depends on the default decision within the period as well as next period’s default decision. Let \( \delta_t \) and \( \delta_{t+1} \) denote the government’s realized default decisions in period \( t \) and at period \( t+1 \)'s settlement, respectively. The return also depends on period \( t + 1 \)'s bond prices, as only a fraction \( \lambda \) of the bond portfolio matures next period. Let \( q_t \) and \( q_{t+1} \) denote the equilibrium price in \( t \) and \( t + 1 \), respectively. A lender which invests a fraction \( \mu \) of wealth in sovereign bonds at price \( q_t \) has an end-of-auction bond

\(^7\)An alternative is to specify an exogenous process for the stochastic discount factor. We view our approach and this alternative as two ways to capture the same phenomenon.
position $\frac{\mu W_{i,t}}{q}$, and a next period wealth given by:

$$W'_{i,t} = (1 - \mu)W_{i,t}(1 + r^*) + \mu W_{i,t} \frac{(1 - \delta_t)(1 - \delta_{t+1})[r^* + \lambda + (1 - \lambda)q_{t+1}]}{q_t}.$$  (6)

Note that $W'_{i,t}$ is used to denote the wealth of a period-$t$ lender in $t + 1$; as $W_{i,t+1}$ represents the wealth of a new entrant in $t + 1$.

The representative lender’s decision is how much sovereign debt to purchase at auction conditional on an equilibrium price and end-of-period debt $b'$. Dropping time subscripts, a representative lender $i$ with individual wealth $W_i$ at the start of a period solves the following problem:

$$L(W_i, s, b') = \max_{\mu} E \left[ v(W'_i) \bigg| s, b' \right],$$

subject to (6), where

$$v(W) = \frac{W^{1-\gamma}}{1 - \gamma},$$

with $\gamma \neq 1$. In forming expectations over $W'_i$, the lender uses the equilibrium policy functions of the government:

$$\delta_t = 1 \text{ with probability } \mathcal{D}(s_t, b'_t)$$
$$\delta_{t+1} = 1 \text{ with probability } \mathcal{D}(s_{t+1}, \mathcal{B}(s_{t+1})) \text{ in state } s_{t+1}$$
$$q_{t+1} = q(s_{t+1}, \mathcal{B}(s_{t+1})).$$

This problem implies an optimal amount of debt purchased, conditional on $(W_i, s, b')$ and an equilibrium price schedule $q$. Evaluating this policy at $W_i = W$, let $\mathcal{L}(s, b')$ denote the aggregate demand for sovereign debt normalized by $Y \in s$. Market clearing for sovereign bonds is therefore:

$$\mathcal{L}(s, b') = b'.$$  (7)

---

Note that in (6) we omit possible proceeds from an auction followed immediately by default. Recall from the timing depicted in Figure 2 that in such an event, the proceeds are rebated proportionally to all (legacy and new) bondholders. As this amount is always zero along the equilibrium path, we omit it from (6) to minimize clutter.

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The FOC for $\mathcal{L}(s, b')$ implies:

$$q(s, b') = \frac{\mathbb{E}[w'^{-\gamma}(1 - \delta(s, b'))(1 - \delta(s', b')(r^* + \lambda + (1 - \lambda)q(s', b''(s')))]}{(1 + r^*)\mathbb{E}[w'^{-\gamma}]}, \tag{8}$$

where $w'$ is representative lender’s next period wealth normalized by $Y$ and $b''(s')$ represents the government’s choice of debt next period conditional on $s'$. Equation (8) encompasses well known special cases: If lenders are risk neutral and debt is short term ($\gamma = 0$ and $\lambda = 1$), $q(y, b')$ is simply the probability of repayment on the debt; if lenders are risk neutral but the debt is long-term ($\gamma = 0$ and $\lambda > 0$)

$$q(s, b') = \frac{\mathbb{E}(1 - \delta(s, b'))(1 - \delta(s', b'))(r^* + \lambda + (1 - \lambda)q(s', b''(s')))]}{(1 + r^*)}. \tag{9}$$

### 3.4 Definition of Equilibrium

**Definition 1** (Equilibrium). An equilibrium consists of a price schedule $q : S \times (-\infty, \bar{b}] \to [0, 1]$; government policy functions $B : S \to (-\infty, \bar{b}]$ and $D : S \times (\infty, \bar{b}] \to [0, 1]$; and a lender policy function $\mathcal{L} : S \times (-\infty, \bar{b}] \to \mathbb{R}$; such that: (i) $B$ and $D$ solve the government’s problem from Section 3.2, conditional on $q$ and $\mathcal{L}$; (ii) $\mathcal{L}$ solves the representative lender’s problem from Section 3.3 conditional on $q$ and the government’s policy functions; and (iii) market clearing: equation (7) holds for all $s \in S$ and $b' \in (-\infty, \bar{b}]$.

Note that the market clearing condition requires that the price schedule clears the market at all potential $b'$, even for debt position that occur off equilibrium. This is a perfection requirement which ensures that if the government were to deviate from $B$ and issue a sub-optimal amount of debt, these bonds would be priced in a manner consistent with equilibrium behavior going forward.

### 3.5 Equilibrium Selection

A primary target of the analysis is to explore quantitatively the role of self-fulfilling debt crises. We do so by exploiting the multiplicity of equilibria. The multiplicity we focus on is static and involves the current price schedule offered to the sovereign. With an adverse price schedule, the sovereign can be induced to default today while with a generous price schedule it would not default for the same debt and output state variables. Moreover, the
current and future default behavior of the sovereign in these two circumstances is consistent with the lenders optimally choosing to lend the prescribed amount at the prescribed price. We refer to a default with such an adverse price schedule as a rollover crisis. We refer to the case when the lender chooses to default with the generous schedule as a solvency default.

Note that both rollover crisis and solvency defaults will feature failed auctions, defined as when the government is unable to auction any positive amount of debt. This is because no lender will lend at a positive price \( q \) if default is certain. Formally, this means that \( q(s, b') = 0 \) for all \( b' > (1 - \lambda)b \). In this event, the government must meet its debt obligations exclusively out of current endowment without recourse to new debt issuances.

To see how this can occur in equilibrium, consider the government’s problem at settlement following a failed auction. In the case that \( b' = (1 - \lambda)b \), which implies that the government did not issue new bonds when faced with a price of zero, then:

\[
V^R(s, (1 - \lambda)b) = u \left( Y [1 - (r^* + \lambda)b] \right) + \beta \mathbb{E} \left[ V(s') \mid s, b' = (1 - \lambda)b \right].
\]

If \( V^R(s, (1 - \lambda)b) \leq V^D(s) \) then the government will find it weakly optimal to default at settlement. Moreover, in the equilibrium we consider, \( V^R(s, b') \) is decreasing in its second argument, and so default is weakly optimal for any \( b' > (1 - \lambda)b \) when \( q(s, b') = 0 \). On the other hand, for a given stock of debt and endowment, there may be a positive price schedule that can also be supported in equilibrium. That is, if \( q(s, b') > 0 \) for some \( b' > (1 - \lambda)b \) then the government may decide to issue new bonds to help pay off maturing debt and find it optimal to repay at settlement.

We incorporate these possibilities in a tractable manner by introducing an additional (sunspot) random variable \( \rho \) which coordinates equilibrium behavior. We let \( \rho \) take three values: \( \rho \in \{r_C, r_T, r_V\} \). State \( r_C \) is the crisis regime, which implies that there is a rollover crisis if one can be supported in equilibrium. That is, if the aggregate endowment is low enough and stock of inherited debt high enough such that \( V^R(s, (1 - \lambda)b) < V^D(s) \), then a crisis occurs with \( q(s, b') = 0 \) for all \( b' > (1 - \lambda)b \). If \( s \) is such that a crisis cannot be supported in equilibrium, there is no crisis when \( r_C \) is realized. In states \( r_T \) and \( r_V \), there is no rollover crisis this period, but the states differ in the probability of a crisis next period. Regime \( \rho = r_T \) is the tranquil regime, in which the probability that next period \( \rho = r_C \) is very low. That is, \( \Pr(\rho' = r_C \mid \rho = r_T) \) is close to or equal to zero. On the other hand, \( \rho = r_V \) is the vulnerable regime, characterized by the fact that a crisis is much more likely next period. That is, \( \Pr(\rho' = r_C \mid \rho = r_V) \) is sufficiently larger than 0 that a crisis is a real threat.
We can think of $r_T$ and $r_V$ as two regimes, one in which the government is relatively safe from self-fulfilling crises, and one in which a crisis is relatively likely. More precisely, $\rho = r_T$ implies a very probability of a crisis next period, and then, assuming positive autocorrelation in the regimes, relative safety going forward. Regime $\rho = r_V$ implies danger next period, and given positive autocorrelation, continued danger into the future. The third regime, $r_C$, is when a crisis occurs as long as debt is high enough relative to output. The random variable $\rho$ follows a first-order Markov chain.

One restriction we impose on equilibria is that the price schedule $q$ is homogenous of degree zero in $Y$. This implies that prices are functions of the ratios of debt and lenders’ wealth to trend endowment, but not of the level of endowment itself (other than through $g$ and $z$ as conditioning variables). One could conceivably construct equilibria where this is not the case by allowing creditor beliefs to vary with the level of trend endowment, conditional on these ratios. We rule these out.

Lastly, with failed auctions and long-term debt the government may have an incentive to buy back its debt if the price is low enough. This incentive would be quite strong if $q = 0$ when it does so. In this case purchasing the outstanding long-term debt costs nothing and the default penalty is avoided. However, if it buys back virtually all of its debt then its incentive to default will be gone since the continuation payoff under repayment will have risen; i.e. $V^R(s, 0) > V^R(s, (1 - \lambda)b)$. Hence, a lender should be willing to pay the risk-free price for this last piece of debt and thereby out bid the government for it. To deal with this conundrum we follow Aguiar and Amador (2014b) and assume that in the case of a failed auction, when $b' \leq (1 - \lambda)b$ the price of the debt, $q(s, b')$, is high enough to make the government just indifferent defaulting and not when it both pays off the maturing debt and buys back some or all of the remaining outstanding debt. With this indifference, we then assume that the government mixes between defaulting, and not defaulting. The mixing probability is set to rationalize the assumed price in the case of a buy-back in a failed auction. With this indifference we can also assume that the government never buys back its debt in the case of a failed auction. Hence, this discussion concerns out-of-equilibrium outcomes.

4 Calibration

The quantitative model’s parameters are presented in Tables 5 and 6. We now describe how we calibrate these parameters using Mexico as our reference economy.
Technology

For the endowment process, we assume the growth rate process is governed by

\[ g_{t+1} = (1 - \rho_g)\bar{g} + \rho_g g_t + \varepsilon_{t+1} \]

where \( \varepsilon \sim N(0, \sigma^2_\varepsilon) \). The transitory component of output \( z_t \) is assumed to be iid, orthogonal to \( \varepsilon_t \) and to have mean zero and variance \( \sigma^2_z \). The implied growth rate of log output is

\[ y_{t+1} - y_t = g_{t+1} + z_{t+1} - z_t + \varepsilon_{t+1}. \]

We estimate this model using quarterly Mexican GDP expressed in constant US dollars for the period 1980Q1 through 2015Q1.\(^9\) The estimated parameter vector is reported in Table 5.

With these parameters in hand, we discretize the process for \( g \) using Tauchen’s method using 50 grid points spanning \( \pm 3\sigma_g / \sqrt{1 - \rho_g^2} \). The iid \( z \) shock is drawn from a continuous Normal distribution truncated at \( \pm 3\sigma_z \). When taking expectations, we numerically integrate over \( z \)’s continuous distribution by evaluating at 11 grid points.

Preferences

The coefficient of relative risk aversion for the sovereign and the creditors is set to 2. The government’s discount factor is set through the moment matching procedure described below.

Financial Markets

We set the risk-free interest rate at 1 percent quarterly (hence, 4 percent annually). The average maturity length is set to 8 quarters, that is \( \lambda = 1/8 \), which implies a Macaulay duration of 6.4 quarters. This is shorter than the average maturity (or duration) observed in many emerging markets. However, maturity length is not constant over time and tends to shorten when the probability of a crisis is high (Broner, Lorenzoni, and Schmukler, 2013; Arellano and Ramanarayanan, 2012). Moreover, much of a country’s short-term debt is

\(^9\)We use the dollar-value of GDP to capture the fact that a depreciation of the peso makes servicing dollar-denominated debt more costly in terms of domestic goods, and hence this additional source of variation is relevant for the sovereign’s borrowing and default decisions. Using constant US dollars makes the US consumption basket the numeraire, which is a reasonable basis for our real model. As sovereign bonds are not typically indexed to inflation, our model does not incorporate the fact that surprise dollar inflation represents a windfall to debtors. However, given the low and stable US inflation over this period, this is not likely to be quantitatively important.
issued domestically (whether in dollars or local currency), and thus is not reflected in the average maturity of external debt. For simplicity, our model only has external debt of constant maturity, raising the question of how to accurately capture a world in which maturity varies over time and the amount due (and to whom) in any given quarter is not uniform. Given our focus on crises, we set the average maturity length to a value that is relatively short.

The endowment while in default is $Y^D(s) = (1 - \phi(s))Y(s)$, where $Y(s)$ is the endowment absent default. In particular, we set:

$$Y^D(s) = \begin{cases} Y(s) & \text{if default occurs this period} \\ (1 - d)e^{\bar{z} - z(s)}Y(s) & \text{otherwise} \end{cases}$$

where $d \in (0, 1)$ is the adjustment to income due to being in default status. A few things are of note in this specification. One is that the cost of default is linear in output, as in Aguiar and Gopinath (2006). In contrast, Arellano (2008) introduced a non-linear cost of default which made default disproportionately more costly in good endowment states and “forgiven” – at least in terms of output costs – in low endowment states. The Arellano specification amplifies the impact of endowment fluctuations in the decision to default while also making default a better insurance option in low-endowment states. This helps the model generate additional volatility of spreads and frequency of default, but does so by making endowment risk more important rather than less. The empirical facts outlined above and in complementary work like Tomz and Wright (2007) suggests that this pulls the model in the wrong direction relative to the data. The parameter $d$ is set below by matching moments.

A second feature of $\phi(s)$ is that no output is lost in the initial period of default. This is not a crucial assumption given the ability to adjust $d$, but does simplify the exposition of how growth is correlated with the switch to default status. The timing is as if default occurs at the end of a quarter and output is not affected until the next quarter.

Finally, for computational convenience, we fix the transitory income shock to be a constant $\bar{z}$ while in default. Thus $Y(s)$ is adjusted by $e^{\bar{z} - z(s)}$, where $z(s)$ is the transitory shock associated with state $s$. Given that $d$ is a free parameter, the level of $\bar{z}$ can be normalized to 0 without loss.

In addition to lost output, default also brings exclusion from financial markets. We set the re-entry probability after default to 0.125 quarterly; that is, the average duration of
default is two years. This is in the range documented by Gelos, Sahay, and Sandleris (2011) for the 1990s, but lower than Tomz and Wright (2013)’s median of 6.5 years using a much longer sample.

We assume that the wealth of the available pool of creditors (relative to the economy’s endowment) follows an AR(1):

\[ w_{t+1} = (1 - \rho_w)\bar{w} + \rho_w w_t + u_{t+1}, \]

where \( u \sim N(0, \sigma^2_w) \) and \( iid \) over time and orthogonal to the other shocks. In the model, shocks to \( w \) generate fluctuations in the risk premium on sovereign bonds conditional on the other state variables. Based on the data presented in Section 2 and the related work in the finance literature, we use fluctuations in the price-earnings ratio of the S&P 500 as a proxy of the empirical appetite for risk. Specifically, the autocorrelation of the P/E ratio between 1993Q4 and 2014Q4 is 0.91. We therefore set \( \rho_w = 0.91 \). The remaining parameters, \((\bar{w}, \sigma_w)\) are set below by matching moments from the model. We discretize the AR(1) process using 5 grid points spanning \( \pm 3\sigma_w/\sqrt{1 - \rho^2_w} \).

**Creditor Beliefs**

We calibrate creditor beliefs as follows. Recall that beliefs are characterized by a three-state Markov regime-switching process. One regime (“crisis”) is the realization of a rollover crisis that generates default. The remaining regimes (“tranquil” and “vulnerable”) differ in the probability of the crisis regime. We estimate a two-state regime-switching model to Mexico’s EMBI spread series from 1993Q4-2014Q4. The rationale for the two-regime estimation rather than the model’s three regimes is the absence of default in the sample period. We estimate a transition probability of moving from the low-spread to the high-spread regime of 0.026, and the reverse transition of 0.177. The time series of the EMBI spread and the associated probability of being in the vulnerable regimes are depicted in Figure 3. We can see that the Mexico crisis of 1994-95 and the spillover from the Russia crisis of 1998 are evident in spreads and the probability of being in the “vulnerable” regime. The 2008 spike in spreads is not interpreted as a shift in regimes.

In estimating the regime-switching model, we would like to separate movements in creditor beliefs from movements in risk premia. For this reason, we have also estimated a regime-switching model for Mexico which includes the S&P P/E ratio as an explanatory variable, with a coefficient that varies by regime. The respective transition probabilities for
this augmented model are nearly identical (0.026 and 0.176, respectively). If we subtract a linear time-trend from the spread series, the transition probabilities are 0.061 and 0.077, which implies slightly more time in the vulnerable regime.

In the calibrated model, we therefore set the transition probabilities at 0.028 and 0.12 [to be updated]. This leaves the probability of a rollover crisis in the vulnerable regime. We set this at 20 percent, and perform robustness checks regarding this parameter. The probability of transiting from the tranquil regime to a crisis is set to the product of transiting to vulnerable and the probability of a crisis conditional on vulnerable; that is, we allow the regime to transit from tranquil to crisis via vulnerable within the same quarter. This probability is nearly zero: \((0.028)(0.20) = 0.006\).

Figure 3: Sovereign Spread Regimes for Mexico

Simulated Matched Moments

We calibrate the remaining moments by simulating the model and matching targeted empirical moments. Specifically, the remaining parameters are the government’s discount factor, the proportion of output lost during default, and the mean and variance of creditor wealth-to-GDP \((\bar{w}, \sigma_w)\).

For Mexico, we have external debt to annual GDP (both in US dollars) for the period
2002Q1 through 2014Q3. The average over this period is 16.4 percent, which translates into a quarterly debt-to-income ratio of 65.6 percent. This measure of debt includes external debt by the government as well as banks. A longer time series exists for a narrower stock of debt issued by the federal government. This series suggests that debt levels were higher in the 1990s and have been falling in the 2000s and 2010s. Hence our measure of 65 percent may be an understatement. The average EMBI spread for Mexico over the entire period is 3.4 percent. These moments are natural targets as their simulated counterparts are particularly sensitive to the government’s discount factor and the default cost.

The spread is a combination of risk premium and default probability. To help separately identify these, we also target a frequency of default of 2 percent per annum, which is in line with the facts documented by Tomz and Wright (2013). An additional moment related to the role of risk premia in spreads is based on the regression of spreads on P/E ratios reported in Table 4. In particular, this regression has an $R^2$ of 0.29. The model counterpart is the $R^2$ of a regression of the spread on creditor wealth (which is our proxy for risk aversion).

In the simulated model, the mean debt-to-income ratio and the spread is conditional on not being in the default state. More specifically, we compute the mean conditional on being out of the default state for at least 25 quarters. The reason we condition on being in good credit standing for a significant period of time is that the government exits default status with zero debt. Zero debt after default is not a realistic feature of the model and hence we focus on the ergodic distribution conditional on having sufficient time to rebuild debt.

These four targets, the model counterparts, and the associated parameters are reported in Table 6.

4.1 Solution

The fact that $u$ and $v$ are homogenous functions and the budget set for the government’s problem is homogenous of degree one in $Y$ implies that the level of endowment $Y$ is not a relevant state variable for the equilibrium price schedule and associated policies, where recall that the policy functions for debt issuance and bond demand have been defined as ratios to current endowment $Y$. Therefore we may solve for an equilibrium price schedule and the associated government’s and lender’s problems on a truncated (“detrended”) state space that

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10 See Alvarez and Stokey (1998) for a formal treatment of dynamic programming with homogenous functions.
Table 5: Parameters I: Set Prior to Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endowments:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(1 - \rho_g)\bar{g}$</td>
<td>0.0034</td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.445</td>
<td>Mexico GDP Data</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.012</td>
<td>1980Q1-2015Q1</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td><strong>Preferences:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sovereign CRRA ($\sigma$)</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>Creditor CRRA ($\gamma$)</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>Financial Markets:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarterly Risk-free rate ($r^*$)</td>
<td>0.01</td>
<td>Standard</td>
</tr>
<tr>
<td>Reciprocal of Avg. Maturity ($\lambda$)</td>
<td>0.125</td>
<td>N/A</td>
</tr>
<tr>
<td>Default Re-entry Prob ($\xi$)</td>
<td>0.125</td>
<td>Gelos et all (2011)</td>
</tr>
<tr>
<td>Autocorrelation of Creditor Wealth ($\rho_w$)</td>
<td>0.91</td>
<td>P/E Ratio AR(1) Coeff</td>
</tr>
<tr>
<td><strong>Belief Regimes:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Pr(\rho' = r_V</td>
<td>\rho = r_T)$</td>
<td>0.028</td>
</tr>
<tr>
<td>$Pr(\rho' = r_T</td>
<td>\rho = r_V)$</td>
<td>0.120</td>
</tr>
<tr>
<td>$Pr(\rho' = r_C</td>
<td>\rho = r_V)$</td>
<td>0.200</td>
</tr>
<tr>
<td>$Pr(\rho' = r_C</td>
<td>\rho = r_T)$</td>
<td>0.006</td>
</tr>
</tbody>
</table>

omits $Y$. With these policies in hand, we can simulate the economy by drawing a sequence $\{g_t, z_t\}_{t=0}^\infty$ and then iterating on the detrended policy functions to obtain equilibrium paths for debt-to-income as well as prices and default outcomes. Finally, the path of endowment $Y_t$ can be constructed from $\{g_t, z_t\}_{t=0}^\infty$, and the level of any endogenous variable can be obtained by scaling up its ratio with income.
Table 6: Parameters II: Simulated Method of Moments

<table>
<thead>
<tr>
<th>Target Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt-to-Income (Quarterly)</td>
<td>65.6%</td>
<td>66.1%</td>
</tr>
<tr>
<td>Mean Spread (Annual)</td>
<td>3.4%</td>
<td>3.3%</td>
</tr>
<tr>
<td>Default Frequency (Annually)</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$R^2$ Reg of Spread on Risk Measure</td>
<td>0.29</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor ($\beta$)</td>
<td>0.835</td>
</tr>
<tr>
<td>Default Cost ($d$)</td>
<td>0.068</td>
</tr>
<tr>
<td>Mean Creditor Wealth Relative to $Y$ ($\bar{w}$)</td>
<td>2.53</td>
</tr>
<tr>
<td>Std Dev Creditor Wealth ($\sigma_w$)</td>
<td>2.64</td>
</tr>
</tbody>
</table>

5 Quantitative Results

5.1 Equilibrium Prices

In this section we characterize the equilibrium bond price schedules and policy functions for debt issuance. In Figure 4a we plot the price schedule $q$ as a function of $b'$ given that $(g, z, w)$ are set at their mean values and beliefs are “Tranquil” ($\rho = \rho_T$). Recall that we normalize the coupon payment so that the price of a risk free bond is one.

The dominating feature of the price schedule is the extreme nonlinearity once default becomes a likely scenario. That is, in this region, the probability of default next period is sensitive to the choice of how much debt to issue. This is a well known feature of these models, as discussed in Aguiar and Gopinath (2006) and Aguiar and Amador (2014a). The decision to issue debt in this region has an average cost $q$ but also a marginal cost related to $\partial q/\partial b'$, which quickly becomes large in the nonlinear range.

Recall that we target the conditional mean of $b$ at 0.69. This is close to the start of the extremely nonlinear portion of the price schedule. To get a better sense of the relevant state space, Figure 5 plots the histogram of $b$ in the simulated model, conditional on being in good credit standing for at least 25 quarters. The middle 98th percentiles spanning $[0.665, 0.709]$. 
This reflects the fact that impatience drives the sovereign to accumulate debt, but the non-linearity of the price schedule places a limit on leverage. The ergodic distribution therefore clusters around this region.

In Figure 4b plots three price schedules over this domain conditional on \( w \) being at its mean and \( \rho = \rho_T \), and with each schedule depicting an alternative \( g \) realization. In particular, we depict schedules associated with the mean \( g \) as well as plus and minus \( 3\sigma_g/(1 - \rho_g^2) \). Note that as \( g \) increases, the price schedule shifts up and flattens. This generates a pattern in which high growth leads to more borrowing, as the average and marginal cost of debt is relatively low. This feature of the model is reminiscent of Aguiar and Gopinath (2006), which presents a quantitative Eaton-Gersovitz model in which the source of risk is a stochastic growth process.

Figure 4c explores the response of equilibrium prices given different creditor wealth levels (conditional on \( g \) at its mean and \( \rho = \rho_T \)). Note that for high levels of \( b' \), a high level of creditor wealth raises \( q \) as creditors are more willing to tolerate risk. Interestingly, for lower levels of debt, a positive wealth shock lowers prices. This is due to a disciplining effect. A low level of current wealth, given persistence, deters the sovereign from accumulating a lot of wealth in the future. Therefore, the incentive to dilute today’s new bondholders is mitigated, raising today’s price. This effect depends on the fact that bonds have a maturity greater than one period and that the wealth state is persistent.

Figure 4d depicts the shift in price schedule between the tranquil and vulnerable regimes. The shift in creditor beliefs to Vulnerable generates a sharp drop in prices, reflecting the higher probability of a rollover crisis in the near term.
Figure 4: Equilibrium Price Schedules

(a) Equilibrium Price Schedule (Tranquil, Mean $g, w$)

(b) Alternative $g$

(c) Alternative $w$

(d) Alternative Beliefs
5.2 Policy Functions

In Figure 6 we plot the policy function for debt issuances as a function of inherited debt and for two belief regimes. We integrate over the shocks to $g$ and $w$ using the ergodic distribution. We plot the policy functions over the relevant range of debt from the ergodic distribution conditional on being out of the default state for at least 25 quarters.

Two striking facts emerge from the figure. First, debt policy functions are quite flat around the 45 degree line: The optimal policy features sharp leveraging and deleveraging that offsets the impact of good and bad growth shocks, respectively, and returns $b'$ to the neighborhood of the crossing point quite rapidly. The second fact is that the switch to the Vulnerable regime leads to rapid deleveraging (or default). The jump in spreads provides an incentive to de-lever, a feature reminiscent of Cole and Kehoe (2000). The ergodic distribution by belief regime is depicted in Figure 7. The shift down in the debt-issuance policy function in the Vulnerable regime is reflected in the leftward shift of the conditional distribution of debt.

6 Equilibrium Outcomes

[Still to be updated]
In this section we lay out the results of our benchmark model in order to both understand its mechanics and how it compares to the data we discussed. We will also be interested in comparing the results of our benchmark model to various other permutations which we discuss later. For now, focus on the first column where we report the basic statistics from our benchmark model. The first three statistics, which were targeted, match the long-run
data for Mexico and is in the ball park for other emerging economies. The correlation of the average excess return and the growth rate of output also seems in the neighborhood of the other emerging markets in that the correlation is quite weak, as it is in the data. The notable deviation relative to facts is that the volatility of spreads, which is very low in the model. We report the coefficient of variation and this should be roughly 1.00 to match the data. We will come back to this point later.

Table 7: Benchmark and Alternative Models of Beliefs

<table>
<thead>
<tr>
<th></th>
<th>Benchmark Model</th>
<th>Tranquil Only</th>
<th>Vulnerable Always</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B/Y$</td>
<td>0.69</td>
<td>TBA</td>
<td>TBA</td>
</tr>
<tr>
<td>Default Freq.</td>
<td>0.01</td>
<td>TBA</td>
<td>TBA</td>
</tr>
<tr>
<td>$E{r - r^*}$</td>
<td>0.02</td>
<td>TBA</td>
<td>TBA</td>
</tr>
<tr>
<td>$\sigma (r - r^*)$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>TBA</td>
<td>TBA</td>
</tr>
<tr>
<td>$corr(r - r^*, \Delta y)$</td>
<td>-0.16</td>
<td>TBA</td>
<td>TBA</td>
</tr>
</tbody>
</table>

The ergodic distribution of debt-to-income by belief regime is depicted in Figure 6. The tranquil regime is both more frequent and has a higher average debt level relative to the vulnerable regime. The crisis regime is the least frequent belief state. This figure reflects the impact of the current growth shock on the debt ratio. The value of $B_t/Y_{t-1}$ is distributed much more tightly, as indicated by the debt policy functions. Note that higher debt levels do occur in the vulnerable and crisis regimes, just as lower debt levels arise in the tranquil regime. These occur during transitions between the tranquil and either of the two risky regimes (vulnerable or crisis).

The associated distribution of interest rate spreads is depicted in Figure 8. Higher spreads are almost always associated with our risky regimes - vulnerable or crisis - and low spreads are almost always associated with the tranquil regime. However, a careful eye can detect some lower spread values for the risky regimes. These arise from high growth rate shocks that lower the debt-to-GDP ratio and hence spreads. Figure 9 displays the distribution of the change in spreads. We can see that large upward jumps in spreads tend to happen in the vulnerable regime while large drops in the tranquil regime. Sometimes, large upward jumps in the spread can happen in the tranquil regime if the growth realization turns out to be particularly bad.

This spread can be decomposed into a default premium and a risk premium. Specifically,
the risk premium is the standard difference between the expected implied yield on sovereign bonds and the risk free interest rate. The default premium is the promised yield that would equate the expected return on sovereign bonds (inclusive of default) to a risk-free bond; that is, the yield that would leave a risk-neutral lender indifferent. Panel (a) of Figure 10 depicts
the risk premium and Panel (b) depicts the default premium.

Figure 10: Decomposition of Spread

(a) Risk Premium

(b) Risk-Neutral Spread

Higher defaults spreads and risk premia are associated with our vulnerable regimes, while lower values occur under the tranquil regime. The correlation of the default spread and the risk premia is high, indicating that default risk is driving up both components of the spread.

Figure 11: Interest Rate Crises

(a) Distribution of Growth Rates: Crises and Non-Crises

(b) Frequency of Crisis Growth Rates: By Belief Regime

Figure 11 Panel (a) reports the distribution of growth rates during crisis and non-crisis periods. In the model, there is a clear tendency for crisis to happen during low growth periods but it is also the case the crises occur when the growth rate is above its mean. These crises during “good times” are crises that accompany a shift from Tranquil to Vulnerable regimes.

38
Table 8: Interest Rate Crises

<table>
<thead>
<tr>
<th>Regime</th>
<th>Share by $g$ Collapse</th>
<th>Share with $z$ Collapse</th>
<th>Share with Belief Change to $\rho_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tranquil</td>
<td>51.60</td>
<td>4.19</td>
<td>1.03</td>
</tr>
<tr>
<td>Vulnerable</td>
<td>48.40</td>
<td>2.25</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>27.94</td>
</tr>
</tbody>
</table>

Table 9: Defaults

<table>
<thead>
<tr>
<th>Regime</th>
<th>Share by $g$ Collapse</th>
<th>Share with $z$ Collapse</th>
<th>What if Tranquil (Counterfactual)</th>
<th>What if Always Tranquil (Counterfactual)</th>
<th>Share with Belief Change from $\rho_{t-1} = \rho_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tranquil</td>
<td>47.16</td>
<td>45.03</td>
<td>4.13</td>
<td>47.16</td>
<td>1.15</td>
</tr>
<tr>
<td>Vulnerable</td>
<td>9.79</td>
<td>7.60</td>
<td>0.44</td>
<td>3.19</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.54</td>
</tr>
<tr>
<td>Crisis</td>
<td>43.05</td>
<td>12.03</td>
<td>2.25</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21.38</td>
</tr>
</tbody>
</table>

Figure 11 Panel (b) depicts the frequency of crisis at different growth rates for each type of regime. For this purposes of this graph, a crisis is defined as jump up in spreads that are in 95th percentile of the distribution shown in Figure 9. Table 8 provides some crisis statistics. Crisis are somewhat more likely in a Vulnerable regime as compared to a Tranquil regime (the jump in spread in the Crisis regime is not reported because it is infinite as the sovereign defaults in the Crisis regime). To see how important growth shocks were to crisis, we report the fraction of crises in each regime caused by a 3 standard deviation drop in $g$ from the previous period or a a negative 1 standard deviation shock to $z$, episode we refer to as “collapses.” Growth collapses account for close to one-half of all crises in the Tranquil regime, although crises happen for less extreme shocks.\textsuperscript{11} In contrast, growth collapses account for a less than a fifth of the crises in the Vulnerable regime. Instead, crises tend to be triggered by the change in beliefs that accompany the shift from tranquillity to vulnerability.

Figure 12 and Table 9 reports some default related properties and statistics for our

\textsuperscript{11}A good chunk of crisis happen when the transitory shock $z$ is low. Since transitory shocks do not cause adverse shifts in the pricing function for debt, the sovereign has the motivation as well as the opportunity to smooth through the shock by borrowing. Such “consumption-smoothing” induced borrowing, if large, can cause the interest rate on debt to jump up.
benchmark economy. From the table one can see that each regime accounts for roughly one third of the overall defaults. One can see that virtually all of the defaults in the tranquil regime were accompanied by a growth collapse, while only 60-70 percent of defaults were in the non-tranquil regimes.

Table 9 also presents some statistics to gauge the importance of beliefs to our defaults. We report how many of them would have taken place if the regime was tranquil. All of the tranquil defaults would still occur by construction, however virtually none of the vulnerable or risky defaults would have occurred. To indicate that in some sense even the tranquil defaults are a product of beliefs we look at how many of them would have occurred if beliefs were always tranquil. The answer is none of them.

This result can be understood in the context of Figure 13 which plots default indifference curves for our three regimes and also for the case in which beliefs were always tranquil or always vulnerable. Because both the continuation payoffs of the sovereign and the price schedules offered by the lenders are effected by the different belief configurations, the default indifference curves as a function of the debt-to-output level at the end of the last period and the growth shocks end up being close to parallel shifts of each other. This results the fact that there is essentially a fixed realized debt-to-output regime which triggers default due to the low correlation in growth shocks.

Finally, to gauge how important shifts in beliefs were to our defaults, we report the share of defaults that were accompanied by a belief shift. Because a shift to the tranquil
In closing we stress two important findings. First, as shown in Figure 10 our model generates the sort of patterns we saw in the data regarding crisis and non-crisis periods. Crises periods tend to be periods of lower growth but crisis regularly also happen when fundamentals (as measured by the growth in output) seem good. Thus, we largely reproduce this key pattern in the data.

Second, as show in Table 7, our Markov structure on beliefs generates time variation in
default risk and that helps to get time variation in spreads. If we compare our benchmark results to a model, along the lines focused on by much of the literature which features the most generous possible price schedule (called the always tranquil) we see a much lower default frequency, average spread, and volatility of the spread. Moreover, the correlation of the excess return and the growth rate increases. These results arise because output shocks are relatively controllable risk, and the tendency of the price of default risk to increase sharply with the level of the default risk strongly encourages countries to adjust their debt-to-output ratio so as to keep these risks at constant low level. If we compare our benchmark model to a quantitative version of the original Cole-Kehoe model (called vulnerable always) which features constant risk of a crisis, we see that the debt-to-output ratio falls in response to this high level of risk, while the frequency of default and the average spread increases. However, the constancy of the risk of a rollover crisis leads to a fall in the relative volatility of the spread. Thus, the time variation in the nonfundamental risk of default coming from our Markov structure is helping the model match the data. But clearly more needs to be done in this dimension as the model is off by an order of magnitude.

7 Crises

[To be Updated]

In this section, we explore sovereign debt crises viewed through the context of the model. In the spirit of our empirical exercise, we first consider the economy’s behavior conditional on a sharp increase in spreads. We do this in the Tranquil Regime as well as the Vulnerable Regime. These event studies are unconditional on what underlying shock triggers the increase in spreads. To shed light on the role of specific shocks, we then consider the responses to a large decline in income growth as well as a shift in creditor beliefs.

7.1 Crises in the Tranquil Regime

In this section, we consider debt crises conditional on Tranquil Regime beliefs. Specifically, we condition on an increase in spreads greater than xx during periods when $\rho = r_T$. The threshold for a crisis is the 95th percentile of the unconditional ergodic distribution of the change in spreads conditional on repayment. We stress that the crisis analysis conditions on repayment; by construction, quarters in which the sovereign defaults are associated with
an infinite spread. Recall that the empirical analysis included two default episodes, so the model analysis represents a slight departure in this regard.

Figure 14 depicts the path of spreads (Panel a) and the implied short-long yield curve (Panel b) conditional on a significant increase in spreads between periods $t = -1$ and $t = 0$ and $\rho_0 = r_T$. Note that the period before the crisis is one of relatively low spreads. This reflects an accumulation in debt as a fraction of output leading up to the crisis, as a high debt-to-income ratio is necessary to sustain the subsequent high spread. Panel (b) indicates the increase in one-period spreads is greater than the long (benchmark maturity) spread. This reflects that short-run default risk exceeds the longer run risk due to mean reversion.

Figure 15 depicts the average growth rate of the economy conditional on a crisis at time $t = 0$. Note that growth experiences a slight boom before the crisis, and then a sharp bust. This reflects the boom-bust nature of crises absent shifts in creditor beliefs. The crisis itself is triggered by a large decline in economic growth. Given the ex ante boom, this large decline is relatively unexpected. This reflects that the sovereign would like to avoid states in which a crisis is likely, and thus periods with a large spike in spreads in the Tranquil Regime requires an unexpected deterioration in fundamentals.

[to be added: path of debt; default probability versus risk premia]
Figure 14: Tranquil-Regime Crises: Mean Spreads

(a) Mean Spread

(b) Short-Long Spread
7.2 Vulnerable-Regime Crises

We now consider a similar event study but conditional on Vulnerable-Regime beliefs. Figure 16 depicts the path of spreads and the implied yield curve conditional on an increase in spreads at time $t = 0$ and $\rho_0 = r_V$. Comparing Figure 16 to 14 we see that conditional on meeting the crisis threshold, the average increase in spreads is much greater in the Vulnerable Regime.

Moreover, the ex ante decline in spreads is much smaller in the Vulnerable-Regime crisis. This reflects a number of differences between crises in the two alternative belief regimes. Under Vulnerable beliefs, default is likely at lower debt levels due to the high probability of a rollover crisis. Thus high spreads do not need a large run up in debt beforehand. Moreover, a change in beliefs from Tranquil to Vulnerable will by itself induce an increase in spreads. Our calibration sets the probability of this transition at 2 percent. Therefore, belief shifts to
Vulnerable are always surprises. Table 8 indicates that more than half of the crisis episodes conditional on Vulnerable Regime are associated with a shift from Tranquil to Vulnerable beliefs.

Figure 17 plots the path of growth around the crisis event. Similar to spreads, we see a distinction in the average growth relative to the Tranquil Regime’s behavior depicted in Figure 15. Growth does not boom as much leading up to the Vulnerable-Regime crises and does not decline as much during the crisis. This is consistent with the above discussion on belief changes; the surprise in a Vulnerable-Regime crisis is due to shifts in creditor beliefs, not unexpected declines in income. It is also sheds light on Figure 11. That figure indicated that Vulnerable-Regime crises were associated with somewhat higher growth rates than the Tranquil-Regime crises; Figure 17 indicates that the Vulnerable crises are also associated with lower growth ex ante, and thus a smaller decline in growth during the periods surrounding a crisis.

[to be added: path of debt; default probability versus risk premia]
Figure 16: Vulnerable-Regime Crises: Mean Spreads

(a) Mean Spread

(b) Short-Long Spread
7.3 Growth Crises

Our discussion of crises that occur under Tranquil-Regime beliefs highlighted the importance of boom-bust dynamics for output. We now explore the economy’s behavior conditional on a large negative realization to output growth. Specifically, our event study is conditional on a growth realization three standard deviations below average in period $t = 0$. Figure 18 plots the path of income growth conditional on the realized innovation to the growth process exceeding $xx$ at $t = 0$. The low persistence of growth is reflected in the quick recovery of growth to its unconditional mean.

Figure 19 depicts the response of spreads conditional on Tranquil-Regime beliefs (Panel a) and Vulnerable-Regime beliefs (Panel b). Panels (c) and (d) depict the respective response to the Long-Short spread. The figure indicates that the impact of a negative growth innovation on spreads is amplified by Vulnerable-Regime beliefs. This indicates that the sensitivity of
default to fundamentals is enhanced by the high probability of a rollover crisis.

[to be added: path of debt; default probability versus risk premia]
7.4 Belief Crises

We now explore a shift in creditor beliefs. Specifically, we condition on a shift in beliefs from Tranquil at $t = -1$ to Vulnerable at $t = 0$. Figure 20 plots the response of spreads and the yield curve. As anticipated by the discussion of crises conditional on Vulnerable-Regime beliefs, a negative shift in beliefs generates a large increase in spreads.

[to be added: path of debt; default probability versus risk premia]
8 Conclusion and Directions for Future Research
[to be added]
References


