RELATIONAL CONTRACTS WITH SUBJECTIVE PEER EVALUATIONS*

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ABSTRACT. This article analyzes the optimal use of peer evaluations in the provision of incentives within a team, and its interplay with relational contracts. We consider an environment in which the firm pays a discretionary bonus based on a publicly observed team output but may further sharpen incentives by using privately reported peer evaluations. We characterize the optimal contract, and show that peer evaluations can help sustain relational contracts. Peer evaluations are used when the firm is less patient and the associated level of surplus destruction is small. Moreover, peer evaluation affects a worker's pay only when the public output is at its lowest level and the co-worker sends the worst report. Noticeably, a worker's report does not affect his own pay, as the provision of effort incentives cannot be decoupled from the incentive for truthful reporting of peer performance. To induce the workers both to put in effort and to report truthfully, the firm may find it optimal to neglect signals that are informative of the worker's effort.

1. INTRODUCTION

In modern labor markets, most workers perform jobs for which objective performance measures are hard to obtain (see, for example, Prendergast and Topel, 1993; Prendergast, 1999). When performance measures are subjective (and, thus, non-verifiable) but publicly observed, a common channel for incentive provision is a relational contract (see Bull, 1987; Levin, 2003; and Malcomson, 2010, for a survey of the literature on relational contracts). In a relational contract, a firm offers workers a discretionary bonus based on performance, and lives up to its promised payments as reneging may result in future retaliation by the workers.

However, even the subjective performance measures available to a firm may be imperfect. With the increasing prevalence of team-based organizations (see Che and Yoo, 2001, and the references therein,) coupled with the increasing complexity of tasks and decentralization of authority, information about performance of individual workers may be diffused in the organization. In such team settings, peers may have the most information about an individual's performance and contribution towards the overall team outcome (Fedor et al., 1999; May and Gueldenzoph, 2006). Thus, peer evaluations may provide a valuable source of information about workers' performances, and the firm can try to augment the incentives generated through relational contracts by offering additional performance pay that is based on the peer evaluations. Indeed, the management literature documents that "companies are turning to 360-degree multirater feedback and intragroup peer evaluation systems for the purpose of managing performance and determining compensation rewards" (May and Gueldenzoph,

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2006, p. 5). Edwards and Ewen (1996) reports that an estimated 90 percent of Fortune 1000 firms have implemented some form of multi-source assessment that includes peer evaluations.

But peer evaluations are inherently private and, hence, subjective. This feature may make the acquisition and use of this information costly for the firm: The firm must provide workers with the right incentives to report their evaluations truthfully and in turn, must itself have the incentive to report the true evaluation to the worker.

The central objective of this article is to analyze the optimal use of peer evaluation in the provision of incentives and its interplay with relational contracts. We seek to answer three questions: Why should peer evaluations be used in addition to standard performance measures of team output? When should peer evaluations be used? And, most importantly, how should they be used? We cast these questions in a minimal model in which one firm repeatedly interacts with two agents. In each period, the firm observes a public, but non-contractible, joint (or "team") output from the agents, and, in addition, the agents observes a private signal about each other's effort. Incentive is provided through two channels: a discretionary bonus payment based on the public signal on joint output and a "subjective pay" based on an agent's peer evaluation privately reported by his co-workers to the firm. We solve for the optimal contract in this setting.

The model can be thought of as a description of the compensation policy in a number of industries, especially in the financial services sectors. In mutual funds or investment banks, for example, a bonus often constitutes a significant proportion of the total compensation. Moreover, a manager's bonus usually depends on his division's performance and a subjective assessment of his teamwork and individual performance typically assessed through structured feedback collected from team members and other colleagues.

For example, Field (2010) presents a Harvard Business Review (HBR) case study of a mutual fund company in which the performance evaluation system for portfolio managers involves paying bonuses where 60 percent of the bonus is determined by the financial performance of the fund they direct and 40 percent determined by the quality of teamwork, which is assessed through structured feedback (gathered from team members and analyzed by top managers). Another HBR case study by Rose and Sesia (2010) on the investment bank Credit Suisse highlights a similar compensation structure. They report that

> "[a]s was the case at every Wall Street firm, the total compensation for professionals at Credit Suisse generally took two forms: salary and incentive compensation or bonuses. In normal years, the CEO would ask the compensation committee of [the company's] board to approve a portion of the firm's income that would be allocated towards incentive compensation ("the bonus pool"). Eric Varvel, [a regional head], explained that 'Individuals would be allocated their portion of their unit's compensation pool based on a combination of their specific economic contribution to the unit and division results, and a qualitative evaluation of their performance and broader contribution.'"[1]

Note that some aspects of the divisional results, such as sales volume or revenue generation in a given fiscal year, are verifiable, and can, in principle, affect a worker's compensation through explicit contracts. But the examples above indicate that the firms often refrain from using such explicit contracts. A particular reason for such a policy is that these verifiable measures are often misaligned with the firm's long-term goals and give incentives to the workers to manipulate such measures for their short-term gains. Consequently, firms often

[1] In addition to the financial service sector, similar compensation policy is also documented in other industries, such as consumer goods. Simon and Kindred (2012) reports a case study on Henkel, a manufacturer of personal care products, where managerial compensation includes a significant variable pay (bonus) component, that depends on overall organization performance team results and individual contribution.

rely on relational contracts based on alternative measures of the divisional performance that are better aligned with their objectives (see, e.g., Baker et al., 1994, and the references therein).

Our analysis yields three sets of results. First, we answer the "why" question: We show that peer evaluations are used because they can help sustain the relational contract. The logic is straightforward. In the absence of peer evaluations, the firm would use only a discretionary bonus payment to motivate the worker to exert effort. Since the output is not contractible, the firm can potentially renege on the bonus payment. The availability of peer evaluations reduces the firm's dependence on bonus payments to provide incentives. Now, the firm can use an alternate channel of performance pay and motivate the worker, while reducing the discretionary bonus payment. A relational contract with a smaller bonus payment is easier to sustain, as the firm is now less likely to renege.

Second, we characterize when peer evaluations are used. We show that the firm offers subjective pay only when the firm is impatient and the surplus destruction associated with this subjective pay is small. The intuition is simple. It is well known that if subjective measures of performance are to be used, surplus destruction on the equilibrium path is necessary (see, for example, Levin, 2003; MacLeod, 2003; and Fuchs, 2007). Recall that the peer evaluations are reported privately to the firm. So, the firm may have an incentive to understate a worker's evaluation in order to lower his wage payment. Thus, incentive pay based on such private peer reports calls for a "joint punishment" in the form of surplus destruction—whenever the firm punishes the worker by lowering his wage due to poor evaluation, the firm must also incur a cost in the form of lost surplus that it could have accrued otherwise. This implies that the firm would prefer to use relational contracts alone (and avoid surplus destruction) whenever possible. It turns out that when the firm is patient enough, relational contracts suffice to provide effort incentives. In contrast, when the firm is impatient, peer evaluations can help sustain the relational contract. However, even in this case, subjective pay helps only if the associated level of surplus destruction is small. If a large share of the surplus must be eroded, then relational contracts are unsustainable—the firm may actually prefer to renege on its bonus promise since it has little to lose in terms of future surplus.

Third, we answer the "how" of peer evaluations. In the central result of the paper, we explicitly characterize the optimal contract and show that peer evaluations affect a worker's pay only rarely: Peer evaluation may be used to lower a worker's pay, but this happens only in the unlikely event that both the public output is at its lowest level and his co-worker sends the worst possible report about him. This finding speaks to the debate in the management literature on the use of peer evaluation in organizations. Several scholars have observed that peer evaluation, albeit commonplace, is seldom used to determine pay; rather, it is used mainly for development and training purposes (see, Pieperl, 1999, and the references therein). The main reason cited is that peer evaluations are often plagued with various forms of rater biases, and, therefore, firms often find it unsuitable for use in determining compensation (Pieperl, 1999; May and Gueldenzoph, 2006). Our finding is consistent with the empirical evidence on the sparing use of peer evaluations in determining compensation, but we provide a different rationale that is derived from the firm's optimal contracting problem. This result is reminiscent of the findings in the subjective evaluation literature on individual-worker compensation (MacLeod, 2003; Fuchs, 2007) and indicates that wage compression takes place even in a more general environment with multiple agents.

The intuition for this result is related to the idea of minimizing surplus destruction, as highlighted in the context of the individual-worker compensation, but there is one key difference. In particular, one might expect that in a multi-agent setting, minimizing surplus destruction involves punishing the workers only when the realized signals are the most indicative of shirking (the least likely event in equilibrium)–i.e., when the public output is the lowest possible one, and both workers send the worst peer reports. However, compensation schemes with this feature turn out to be suboptimal for the reason of "double-deviation."

Specifically, unlike the individual-worker setting, workers in our model have two incentive constraints: to exert effort and to tell the truth about their peer. An important observation from our analysis is that these two constraints cannot be decoupled. There exist compensation schemes, including the one described above that minimize surplus destruction, that discourage each single deviation separately but fail to be incentive-compatible when both deviations are used simultaneously: After the worker shirks, he may also want to lie about his co-worker's performance. In other words, the fact that the worker's effort incentives cannot be decoupled with his truth-telling incentives may lead the firm to make a worker's pay in the optimal contract independent of what the worker reports about his peer. This is somewhat counterintuitive, since the firm may be disregarding some information in designing the optimal contract.

To better understand this particular feature of the optimal contract, we study a variation of the baseline model in which the workers' private signals about each other are correlated. In such a setting, notice that a worker's report about his peer contains information about the worker's own effort. Surprisingly, we find that even in this setting, in the optimal contract, a worker's pay is still independent of the report he submits about his co-worker. Our analysis, therefore, implies that it can be optimal to neglect some information about an agent's effort to better induce truth-telling—the benefit of using such information may be outweighed by the cost of eliciting it in the first place. This result stands in contrast to the celebrated "Informativeness Principle," à la Holmström (1979), and shows that the nature of optimal contracting with subjective private peer evaluations is fundamentally different from optimal contracting with publicly observable signals.

Finally, we show that if we allow the agent's effort to affect the degree of correlation between the agents' private signals, then an agent's pay may depend on his submitted peer report (in the optimal contract). To see the intuition, suppose that the signals are perfectly correlated when both agents work, but uncorrelated when at least one of them shirks. Then, the firm can detect shirking by simply checking whether there is a mismatch between the two agents' reports. And by rewarding the agents only when their reported evaluations match (rather than solely based on their respective peer evaluations), the firm may be able to offer incentives more efficiently.

Related literature: Starting with the influential article by Bull (1987), a vast literature in relational contracts has flourished over the last few decades. This literature primarily highlights how repeated interaction between parties may alleviate moral hazard problems even in the absence of court-enforceable incentive contracts (see Malcomson, 2010 for a survey of this literature). A recent strand of the literature (e.g., Che and Yoo, 2001; Kvaløy and Olsen, 2006; and Rayo, 2007) studies the role of relational incentive contracts in teams, but these models assume that the performance measure is necessarily public. Another strand of the literature has focused on private evaluations (Levin, 2003; MacLeod, 2003; Fuchs, 2007; Chan and Zheng, 2011; Maestri, 2012), but these papers consider the relationship between the firm and an individual worker, and largely abstract away from the role of public signals.

This paper bridges these two strands of the literature by analyzing a team environment with both private and public subjective performance measures—a setting that seems particularly relevant in practice. The key contribution of this article is to highlight how relational contracts based on publicly observed performance measures interact with subjective pay based on privately observed peer evaluations. In particular, we offer a characterization of the optimal contract that illustrates when and how the firm may be able to sharpen incentives by using subjective pay.

Our article closely relates to the existing literature on subjective evaluation mentioned above. Levin (2003) shows that the optimal contract involves just two pay levels, and prescribes termination following poor performance. In a related paper, MacLeod (2003) analyzes a static setting with a single risk-averse agent, and derives the optimal contract when there is a private subjective performance measure. He shows that the optimal contract results in more compressed pay relative to the case with verifiable (objective) performance measures, and it entails the use of bonus pay rather than the threat of dismissal. Fuchs (2007) considers a dynamic version of MacLeod's setting. He characterizes the optimal contract for a finite horizon, and shows (similar to our setting) that some burning of resources must happen in equilibrium. However, resources are burnt only after the worst possible realization sequence. It is important to note that while these articles neither consider team production nor do they allow for public signals, the key theme of this literature is also reflected in our findings—the firm uses the private signal sparingly, as such incentive payments must involve surplus destruction, and, consequently, the optimal contract leads to wage compression.

Our focus on the interaction between different types of performance measures also bears some resemblance to Baker, Gibbons and Murphy, 1994; (BGM hereafter). BGM studies the interaction between explicit contracts based on verifiable measures and relational contracts based on measures that are publicly observable but not verifiable (also see MacLeod and Malcomson, 1989). They show that explicit contracts can help the relationship because they provide another source of incentive. However, explicit contracts can also hurt the relationship because they improve the firm's fallback option, making it more tempting to renege on the relational contract. As a result, explicit and relational contracts can either be complements or substitutes. Unlike BGM in which performance measures are all publicly observable, in our model, each agent has private information about the other's performance. Therefore, the optimal relational contract must be designed to induce the truthful revelation of information. Consequently, unlike that in BGM, the optimal relational contract in our paper features surplus destruction that affects the future value of the relationship and, in turn, governs the sustenance of the relational contract.

The rest of the paper is structured as follows. In Section 2, we present the main model. Section 3 analyzes the firm's problem, and, characterizes the optimal contract. In Section 5, we consider an environment in which we allow the private and public subjective measures to be correlated, conditional on the agents' effort choices. Section 6 concludes. All proofs are provided in the Appendix.

2. Model

*Players***.** A principal, or a firm, F, employs two agents, A_1 and A_2, who work as a team on a given project. The firm and the agents are long-lived, and in each period $t \in \{1, 2, \ldots\}$, they play a stage-game as described below.

STAGE-GAME: At the start of every period, the firm F offers a contract to the agents. The stage-game is defined in terms of its three key components: *technology, contracts,* and *payoffs.*[2]

Technology. Each agent A_i chooses an effort level $e_i \in \{0, 1\}$ towards producing a team output, Y. Let the cost of effort be c when $e_i = 1$ and 0 otherwise. Effort is privately observed by the agent, giving rise to a moral hazard problem. There are N different output levels, $y_1 < y_2 < ... < y_N$, where

[2]For expositional clarity, we suppress the time index t for all variables unless mentioned otherwise.

$$\Pr(Y = y_j \mid \mathbf{e}) = \begin{cases} \alpha_j & if\ \mathbf{e} = \mathbf{1} \\ \beta_j & otherwise. \end{cases} \tag{1}$$

We assume that $\alpha_j \geq 0,\ \beta_j \geq 0,\ \sum_{j=1}^{N} \alpha_j = \sum_{j=1}^{N} \beta_j = 1$, and $\alpha_j/\beta_j > \alpha_k/\beta_k$ for any $j > k$. That is, the output Y is obtained through team production, and there is complementarity of effort in the production process.[3] Moreover, Y satisfies the monotone likelihood ratio property (MLRP). In practice, we can think of Y as an outcome of the project, and measures how successful the project was. We assume that Y is publicly observable but not verifiable and is realized only at the end of the period.

Before the team output is realized (and after the agents make their effort choices), each agent A_i also obtains a noisy signal $s_i \in S = \{1, 2, ..., n\}$ about his co-worker A_{-i}'s performance or contribution to the project, and each agent privately reports his signal to the firm. Let

$$\Pr(s_i = s \mid e_{-i}) = \begin{cases} p_s & if\ e_{-i} = 1 \\ q_s & otherwise. \end{cases} \tag{2}$$

We assume that $p_s \geq 0,\ q_s \geq 0,\ \sum_{s \in S} p_s = \sum_{s \in S} q_s = 1$, and $p_s/q_s > p_r/q_r$ for any $s > r$. As in the case of the team output, we maintain the MLRP condition. The signal s_i is a private signal of agent A_i: It is unverifiable, and also not observed by the other players (or any third party). For most of the paper, we assume that the set of signals $\{Y, s_1, s_2\}$ are pair-wise independent *conditional on* effort.

As the signals s_i are private, they are inherently subjective. Thus, one may interpret A_i's report on A_{-i} as his subjective evaluation of his co-worker's performance. Our assumption of private reporting is consistent with what is commonly observed in reality. In most peer evaluation systems in practice, the feedback is submitted privately, often even anonymously, in order to encourage candid reporting (Edwards and Ewen, 1996). In this context, also note that we do not allow for self-evaluation in our environment. While the agents observe and report a signal on their co-worker's performance, they do not observe any additional signal on their own performance. This modeling choice is also driven by what we observe in practice. Even though self-appraisals are commonplace in many organizations, much of the literature suggests that self-appraisal is used more as a developmental tool than as an accurate evaluation tool (see, for instance, Mabe and West, 1982; and Wexley and Klimoski, 1984).[4]

*Contracts***.** Since the performance measures are unverifiable, the firm cannot offer an explicit pay-for-performance contract. However, the firm can offer incentives to the agents through two channels.

First, the firm can offer a discretionary bonus that is based on the team output Y. Suppose that the firm offers a bonus of $b_i(y_j)$ to agent A_i if $Y = y_j$. This payment is not contractually enforced but is a part of a relational contract that can be sustained through the threat of future retaliation by the agents should the firm renege on it.

[3]The complementarity of the effort is not essential for our findings, but it considerably simplifies the notation. Also, the same results can be obtained if we have separate publicly observable signals for each agent.

[4]In the management literature, the usefulness of self-appraisals (except as a vehicle for personal development) is highly debated, mainly because of the lack of convergence between self-appraisals and supervisors' ratings (see Campbell and Lee, 1998). Such inconsistencies in feedback are often attributed to the so called "leniency bias" in self-evaluation (Xie et al., 2006; Nilsen and Campbell, 1993; Yammarino and Atwater, 1997).

Second, the firm can also offer a performance-based pay to A_i using the private reports that it has obtained from the agents; in particular, the firm can offer a payment to A_i, based on the private peer reports (s_i, s_{-i}) that it receives from the agents. We refer to compensation based on the peer evaluation as "subjective pay." Since the peer reports are made to the firm privately, both the agents and the firm may have incentives to misrepresent the signal—A_j may lie in his peer evaluation, potentially to "steal credit" from his co-worker, and the firm may misrepresent the peer evaluations in order to reduce the wage payment to the agents. Any contract that uses peer evaluations must, therefore, address the misreporting incentives of both the agents and the firm. To address the firm's misreporting incentives, notice that the firm may be tempted to claim that it has received a poor evaluation in order to reduce the subjective pay. As the reports are private, the firm must be indifferent between the various wage payments that it offers to the agents based on the different profile of reports.

One standard way to ensure truth-telling by the firm is to have the firm commit to spend a fixed wage pool w for the two agents, irrespective of the reports (see, e.g., MacLeod, 2003; Rajan and Reichelstein, 2006). When A_1's report about A_2 is r, A_2's report about A_1 is s (for any $r, s \in S$) and $Y = y_j$, let the payments to A_1 and A_2 be denoted by $w^1_{rs}(y_j)$and $w^2_{sr}(y_j)$, respectively (i.e., $w^i_{rs}(\cdot)$ denotes the wage to A_i when his report on A_{-i} is r and A_{-i}'s report on A_i is s). We assume that if $w^1_{rs}(y_j) + w^2_{sr}(y_j) < w$, then the firm commits to give the remaining sum (i.e., $w - w^1_{rs} - w^2_{sr}$) to a third party. Note that the firm's truth-telling problem is trivially solved by such a contract. The management and accounting literatures provide several examples documenting the use of such committed wage pools in practice.[5]

For the sake of tractability, we limit attention to symmetric contracts. We assume that $b_1(y_j) = b_2(y_j) = b(y_j)$ and $w^1_{rs}(y_j) = w^2_{rs}(y_j) = w_{rs}(y_j)$ for all y_j, r and s. A contract in the stage-game is completely characterized by the tuple $\phi = \{w, b(Y), w_{rs}(Y) : s,\ r = 1, ..., n\}$. Let Φ be the set of all such contracts.[6]

Note that the truth-telling constraint on the firm emerges because agents submit private reports to the firm. Our assumption of private reports is consistent with peer evaluation systems that are observed in organizations, where workers indeed submit their peer assessments to the firm privately. In practice, peer assessments are designed as private reports mainly to ensure anonymity, as it is believed that workers find it easier to provide truthful feedback

[5]For example, the literature documents the practice of using "managerial bonus pools"—where the firm commits to a bonus pool that is based on a common measure of performance, but the allocation among business units and managers is discretionary and decided later. For example, Baiman and Rajan (1995) discusses managerial bonus pools and shows that the use of a bonus pool can increase efficiency by enabling a firm to exploit non-contractible information to motivate agents. The paper provides some examples: At Waterhouse Investors Services, Inc., a US brokerage firm "[t]he plan provides for a bonus pool to be reserved in the amount of approximately 20 percent of the company's pre-tax income for all incentive awards[...]Individual executive awards are determined based on the committee's discretionary evaluation (Waterhouse Investors Services, Inc.; Proxy Statement filed with the SEC, January 4, 1995.)" In Quick and Reilly Group, Inc., another US brokerage firm, "[W]hile the standard for determining the amount of the bonus pool for each profit center is based entirely on financial performance criteria, the allocation of the bonus pool among the employees entitled to participate therein involves subjective considerations as well. Awards from the various bonus pools to executive officers are made based upon the Board's subjective evaluation of corporate performance, business unit performance (in the case of executive officers with business unit responsibility) and individual performance. (Quick and Reilly Group, Inc.; Proxy Statement filed with the SEC, May 20, 1994.)" In Crawford & Co., a US claim management firm, "[T]he amount distributable under the Plan consisted of 15 percent of the three year audited pre-tax earnings increase of the Company. Distribution of individual awards under the bonus pool was made at the discretion of the Chairman of the Board and President (Crawford & Co.; Proxy Statement filed with the SEC, March 31, 1992.)" Ederhof et al. (2010) also provides an example from the US health insurance firm, Aetna, and examine the structure of efficient bonus pools in the presence of subjective performance measures.

[6]Note that we do not explicitly define a fixed salary part of the compensation package. This is without loss of generality, as any fixed salary payment can be conceived as a part of the fixed wage pool w.

when they do so anonymously. One may argue that, in our model, with only two agents, private reporting does not preserve this spirit of anonymity, as each worker has exactly one peer who submits a report about him.

We retain the assumption of private monitoring even with two agents for two reasons. First, this assumption keeps our model closer to what we observe in practice. Generalizing the model to more than two agents is possible, even though the analysis would be less tractable. Second, the truth-telling constraint for the firm can be important in other settings, as well. Even with two agents and publicly observed peer evaluations, the truth-telling constraint on the firm would be essential if we assumed that the firm observes additional private signals on each agent's individual contribution to the team. The analysis of the optimal contract with publicly observed team output, publicly announced peer evaluations and private signals available to the firm would be qualitatively similar to our current setting with publicly observed team output and privately reported peer evaluations. In both settings, the optimal contract generates effort with minimal surplus destruction on equilibrium path leading to significant wage compression.

Finally, it is worth noting that the literature on mechanism design and repeated games has suggested alternative monitoring devices (see, e.g., Cremer and McLean, 1988; and Ben-Porath and Kahneman, 1996). A commonly used strategy in this literature is to cross-check the reports from interested parties and punish them if there is a mismatch. In a recent article, Rahman (2012) adopts a different approach of "monitoring the monitor." In his setting, a principal hires a monitor to supervise a worker, but the monitor privately observes the worker's deviation. To overcome the private monitoring problem, on occasions, the principal secretly asks the agent to shirk and rewards the monitor if he is able to detect these "prompted deviations."

While these strategies effectively overcome the issue of private monitoring, their applicability in our setting is questionable. In order to use cross-checking strategies to induce truth-telling by the firm, the agents would need to observe each other's peer reports or credibly communicate their reports to each other. However, this is against the spirit of private peer evaluations in the first place. Also, the monitoring strategies à la Rahman (2012) may be applicable to inducing effort and truth-telling by the agents, but to induce truth-telling by the firm using such strategies, one would require that the agents be able to prompt each other on what to report and that they also be able to make side payments between themselves. Such a setting is, again, not consistent with what we observe in practice.

*Payoffs***.** We assume that all players are risk-neutral. Thus, the expected payoff of the firm is

$$\pi := \mathbb{E}\left[Y|\mathbf{e}\right] - \left\{w + 2\mathbb{E}\left[b\left(Y\right)|\mathbf{e}\right]\right\},$$

and the expected payoff of agent A_i is

$$u_i := \mathbb{E}\left[b\left(Y\right) + w_{rs}\left(Y\right)|\mathbf{e}\right] - ce_i.$$

Let the outside options for all players be 0.

Timeline. The stage-game is summarized in the timeline below:

- **Period** $t.0$. Firm offers contract ϕ.
- **Period** $t.1$. Agents accept or rejects. If both agents accept, the game continues to the next stage.
- **Period** $t.2$. The agents exert effort e_i simultaneously.

- **Period** $t.3$. The agents privately obtains signal s_i and privately reports peer evaluation to the firm.
- **Period** $t.4$. Team output Y is realized.
- **End of period** t. Wages paid (bonus and subjective pay) and the game moves to the next period $t+1$.

Repeated game: The stage-game defined above is repeated in every period, and all players have a common discount factor $\delta \in (0,1)$. The public history of period t is given by the tuple $h_t = \{\phi, y_j, \hat{b}\}_t$ where $\hat{b}$ is the bonus paid in period t. Let the (public) history in period t be $h^t = \{\{\phi, y_j, \hat{b}\}_\tau \mid \tau = 1, ..., t-1\}$ and let H^t be the set of all (public) histories in period t.

Strategies and Equilibrium. The strategy of an agent A_i has three components: Given the history and the contract offered in the current period, he must choose (i) whether to accept or reject the contract; (ii) an effort level e_i; and (iii) a reporting strategy $m_i : S \to S$ that maps his privately observed signal about A_{-i}'s performance into the set of performance signals (by the Revelation Principle, it is without loss of generality to restrict the message space to the set of performance signals). The firm's strategy is to choose a contract $\phi \in \Phi$ given the history of the game and choose the actual payout of the discretionary bonus $\hat{b}$ (which may differ from the promised amount).

We restrict attention to *perfect public equilibrium* (*PPE*), which is a standard solution concept in models of relational contracts. Note that the restriction to PPE, however, is not without loss of generality because here, private strategies can help the players obtain better payoffs; for example, see Fuchs (2008) for an analysis of private strategies in a similar setting. However, allowing for private strategies would significantly complicate the analysis, and to keep the model tractable we restrict attention to PPE (as in Levin, 2003).

To characterize the optimal PPE, we assume that following a publicly observable deviation (i.e., if $\hat{b} \neq b(y_j)$ for some agent for some period t and some public signal y_j), the players take their outside options forever. This assumption is without loss of generality because the outside options give all players their minmax payoffs (Abreu, 1988). Notice that there is another static Nash equilibrium that gives all players 0: The firm pays out a wage of 0 under all contingencies and neither agents puts in effort. Since both options yield the same payoffs to all players and do not affect the structure of the optimal contract, either option can be chosen.[7]

3. The firm's problem

We begin our analysis by first delineating the firm's optimal contracting problem. We focus on the case in which it is optimal to induce effort, as the case of no effort is trivial. Following Levin (2003), we can restrict attention to the class of stationary relational contracts where the contract $\phi = \{w, b(Y), w_{rs}(Y)\}$ is invariant over time.[8] Therefore, the firm's problem is

[7]An alternative modeling choice is to assume that the parties switch to the optimal subjective contracts following a publicly observed deviation (See, for example, Baker, et. al, 1994). The main results of the model remain unaltered under this alternative assumption. However, we have not opted for this assumption because it is conceptually less appealing. Specifically, under the optimal subjective contracts, surplus destruction hurts only the agents and not the principal. As a result, the agents may be concerned that more surplus is destroyed than what is specified in the contract. Such a concern is especially relevant if the principal has deviated from equilibrium in the past. Therefore, following a public deviation, the agents may prefer their outside options to the optimal subjective contracts.

[8]The formal proof of this claim uses a line of argument very similar to that in Levin (2003) and is available from the authors.

to maximize its per-period payoff, π, subject to the following set of constraints related to the participation by the agents and incentive compatibility for both the agents and the firm.

First, consider the wage payment based on the peer evaluation (i.e., subjective pay). As mentioned earlier, any contract based on peer evaluation involves a commitment from the firm to pay out a fixed sum w in wages irrespective of the agents' reports. So, for any report profile, the total subjective pay must satisfy a budget constraint:

$$(B) \qquad w \geq w_{rs}(y_j) + w_{sr}(y_j) \quad \forall j = \{1, ..., N\}, \text{ and } r,\ s \in S.$$

Second, as we are limiting attention to direct mechanisms (by the Revelation Principle), the contract should induce the agents to report their signals truthfully. Thus, we must have the following truth-telling constraint on the equilibrium path:

$$\mathbb{E}_{\{y_j,r\}}\left[w_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}, s_i = s\right] \geq \mathbb{E}_{\{y_j,r\}}\left[w_{rs'}(y_j)\,|\mathbf{e}=\mathbf{1}, s_i = s\right] \quad \forall\ s \text{ and } s' \in S.$$

Note that because the truth-telling constraint must hold for all pairs of signals (s, s'), it must hold with equality, and we have:

$$(T) \qquad \mathbb{E}_{\{y_j,r\}}\left[w_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}, s_i = s\right] = \mathbb{E}_{\{y_j,r\}}\left[w_{rs'}(y_j)\,|\mathbf{e}=\mathbf{1}, s_i = s\right] \quad \forall\ s \text{ and } s' \in S.$$

Next, consider the bonus payment based on the (publicly observed) team output. As the bonus payment is sustained through the threat of future punishment, it must be the case that the firm's payoff from honoring its bonus promise exceeds its payoff from reneging. Also, since both agents trigger punishment if the firm "cheats" either of the two agents, if the firm decided to renege on its promise, it would renege with *both* agents. Recall that we have assumed that if the firm reneges, off the equilibrium path, the agents do not exert any effort and the firm and the agents earn their outside options of 0. Therefore, we must have the following dynamic enforcement constraint:[9]

$$(DE) \qquad \frac{\delta}{1-\delta}\pi \geq \max_j 2b(y_j).$$

Finally there are two constraints on the agents. First, the contract offered must ensure participation by each agent:

$$(IR) \qquad u_i = \mathbb{E}_{\{y_j,r,s\}}\left[b(y_j) + w_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}\right] - c \geq 0.$$

Second, the contract should make it optimal for the agent to exert effort (rather than shirk). It is worth noting that the incentive-compatibility constraint, (IC), significantly differs from that of the canonical moral hazard problem. In our setting, we must take into account the fact that, after shirking, the agent may find it optimal to not report his signal truthfully. In other words, a deviation in effort provision may be coupled with a deviation in the peer evaluation reporting strategy. The incentive constraint below reflects the fact that, conditional on shirking, the agent will always report the signal that maximizes his expected payoff. As we will see later, this potential "double deviation" plays a critical role in the characterization of the optimal contract. The incentive constraint can be written as:

[9] Recall that the contract is assumed to be symmetric. So, for any y_j the aggregate bonus promise is $2b(y_j)$.

$$\mathbb{E}_{\{y_j,r,s\}}\left[b\left(y_j\right)+w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1}\right]-c\geq\max_{s'}\mathbb{E}_{\{y_j,r\}}\left[b\left(y_j\right)+w_{rs'}\left(y_j\right)|e_i=0,e_{-i}=1\right].$$

We can also represent this constraint in two parts. Let $s^*\in S$ be the signal for which A_i's payoff is maximized conditional on him shirking and A_{-i} exerting effort. Now, the above constraint can be written as:

$$(IC)\qquad \mathbb{E}_{\{y_j,r,s\}}\left[b\left(y_j\right)+w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1}\right]-c\geq\mathbb{E}_{\{y_j,r\}}\left[b\left(y_j\right)+w_{rs^*}\left(y_j\right)|e_i=0,e_{-i}=1\right],$$

and

$$(IC')\qquad \mathbb{E}_{\{y_j,r\}}\left[w_{rs^*}\left(y_j\right)|e_i=0,,e_{-i}=1\right]\geq\mathbb{E}_{\{y_j,r\}}\left[w_{rs}\left(y_j\right)|e_i=0,e_{-i}=1\right]\quad\forall s\in S.$$

Denote $v=\mathbb{E}\left[Y|\mathbf{e}=\mathbf{1}\right]$. The firm's optimal contracting problem is as follows:

$$\mathcal{P}:\begin{cases}\max_{\phi\in\Phi}\pi=v-\left\{w+2\mathbb{E}\left[b\left(Y\right)|\mathbf{e}=\mathbf{1}\right]\right\}\\ s.t.\quad (B),\ (T),\ (DE),\ (IR),\ (IC),\ \text{and}\ (IC').\end{cases}$$

Note that at the optimum, the participation constraints of both agents must bind. Otherwise, the firm can reduce $w_{rs}\left(y_j\right)$ by a small positive amount for all r, s and y_j and increase its payoff without violating any constraints. As the (IR) constraints for both agents are binding, we can use them to eliminate $\mathbb{E}\left[b\left(Y\right)|\mathbf{e}\right]$ in the firm's objective function. Thus, the firm's objective function reduces to the total surplus generated by the relationship–i.e.,

$$\pi=\left\{v-c\right\}-\left\{w-2\mathbb{E}_{\{y_j,r,s\}}\left[w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1}\right]\right\}.$$

Denote $z:=w-2\mathbb{E}_{\{y_j,r,s\}}\left[w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1}\right]$–i.e., the surplus that is "destroyed" (in expectation) on equilibrium path when the firm relies on the peer evaluation. The firm's problem can be conceived as one of minimizing z subject to the set of constraints given in program $\mathcal{P}$; i.e., we can reformulate the problem as:

$$\mathcal{P}':\begin{cases}\min_{\phi\in\Phi}\quad z=w-2\mathbb{E}_{\{y_j,r,s\}}\left[w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1}\right]\\ s.t.\quad (B),\ (T),\ (DE),\ (IC),\ \text{and}\ (IC').\end{cases}$$

The optimal contract is the solution to the above program.

4. The optimal contract

4.1. **The stage-game equilibria.** In order to characterize the optimal contract in the repeated game, we first characterize the equilibrium in the stage-game in which effort is induced through subjective pay. We will see later that this characterization is informative about what happens in the repeated game. The optimal contract in the stage-game must solve the firm's program that is obtained by setting $b(y_j) = 0\ \forall j$ in the program $\mathcal{P}$. That is, the firm's optimization problem is as follows:

$$\hat{\mathcal{P}}: \begin{cases} \max_{\{w, w_{rs}(y_j)\}} \quad v - w \\ s.t. \quad w \geq w_{rs}(y_j) + w_{sr}(y_j) \quad \forall j \in \{1, ..., N\}, \text{ and } r,\ s \in S, & (B) \\ \mathbb{E}_{\{y_j, r\}}[w_{rs}(y_j) | \mathbf{e} = \mathbf{1}, s_i = s] = \mathbb{E}_{\{y_j, r\}}[w_{rs'}(y_j) | \mathbf{e} = \mathbf{1}, s_i = s] \quad \forall\ s \text{ and } s' \in S, & (T) \\ \mathbb{E}_{\{y_j, r, s\}}[w_{rs}(y_j) | \mathbf{e} = \mathbf{1}] - c \geq 0, & (IR) \\ \mathbb{E}_{\{y_j, r, s\}}[w_{rs}(y_j) | \mathbf{e} = \mathbf{1}] - c \geq \mathbb{E}_{\{y_j, r\}}[w_{rs'}(y_j) | e_i = 0, e_{-i} = 1] \quad \forall s' \in S. & (IC) \end{cases}$$

The following proposition characterizes the optimal contract (that induces effort) in the stage-game.

Proposition 1. ***(Optimal contract with effort in stage-game)*** *In the optimal contract that induces effort in the stage game, the firm commits to a wage pool*

$$\hat{w} = \frac{2q_1\beta_1}{\beta_1 q_1 - \alpha_1 p_1} c,$$

and offers subjective pay

$$\forall\ s \in S,\ \hat{w}_{rs}(y_j) = \begin{cases} \frac{1}{2}\hat{w} - \frac{c}{\beta_1 q_1 - \alpha_1 p_1} & if\ j = 1 \text{ and } r = 1 \\ \frac{1}{2}\hat{w} & otherwise. \end{cases}$$

It is important to note two key features of the optimal contract in the stage-game. First, the wage of an agent, A_i, depends only on his co-worker A_{-i}'s report—what A_i reports on his co-worker's performance does not affect his own wages. This, in particular, implies that it is incentive-compatible for each agent to report the signal about his co-worker's performance truthfully.[10]

Second, the agents receive the entire committed wage pool, except in the case where both the public and private signals are the worst possible ones–i.e., the team output is at its lowest and the agent gets the worst possible peer evaluation. To see the intuition behind this, recall that surplus destruction is necessary for inducing effort from the agents. As the agents' participation constraints always bind, at the optimal contract, the firm maximizes the total surplus generated in the employment relationship, which is equivalent to minimizing the amount of surplus destruction. This is achieved by punishing the agent only when both public and private signals indicate the worst performance. As this signal is unlikely to be realized when the agent is, indeed, exerting effort, such a contract minimizes surplus destruction in equilibrium (of the static game) while giving the agent sufficient incentive to exert effort.

[10]It may appear counterintuitive that the firm gives up the opportunity to sharpen incentives using relative and/or joint performance evaluation (see, for instance, Che and Yoo, 2001). We return to this issue later.

This finding is consistent with the observation in the management literature that firms often do not use peer evaluations to determine pay but, rather, use them as inputs for staff training and developmental initiatives (Budman and Rice, 1994; Pieperl, 1999). While this literature cites inherent biases in peer evaluations as the key reason behind this observation (May and Gueldenzoph, 2006), our finding highlights that in some contracting environments, such insensitivity of wages to peer evaluation may indeed be the optimal compensation policy.

Our result also echoes the findings in the literature on the individual-worker compensation with subjective evaluation. In these models, the worker is "punished" only when the firm obtains the *least* likely performance signal (conditional on effort)—in a static setting à la MacLeod (2003), wages are compressed over the private signal; and in a dynamic setting à la Fuchs (2007), wages are compressed over the histories of past performances. While our result indicates that the intuition developed in the context of individual worker compensation continues to hold in a team environment, there is a subtle yet critical difference between our findings and those discussed above: In Proposition 1, the signal for which the firm punishes the agents is *not* the least likely signal (conditional on effort).

In principle, even less surplus would be destroyed in equilibrium if peer evaluations mattered only in the *least* likely event that *both* agents receive the worst peer evaluations and the team output is at its lowest. However, such a contract turns out to be suboptimal in equilibrium. The reason is that, since each agent makes a private report about his co-worker, we need to take into account, the fact that, if an agent chooses to shirk, he may also find it optimal not to report his signal truthfully. In other words, a deviation in effort provision may be coupled with a deviation in the peer evaluation reporting strategy. This potential "double deviation" by agents implies that "punishing only when both agents fail"—indeed, the least likely event on the equilibrium path—is not optimal. A shirking agent can always avoid such a punishment by lying on his report, and, therefore, such a contract fails to generate adequate effort incentive. As we will see in the next section, these characteristics of the stage-game contract continue to hold on equilibrium path in the repeated game, as well.

Observe that the expected surplus destruction under the optimal contract in Proposition 1 is given by

$$\hat{z} := \hat{w} - 2\mathbb{E}_{\{y_j,r,s\}}\left[\hat{w}_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1}\right] = \frac{2\alpha_1 p_1}{\beta_1 q_1 - \alpha_1 p_1}c. \tag{3}$$

Thus, the firm's payoff under the optimal contract is $\hat{\pi} = v - c - \hat{z}$. It follows that $\hat{\pi} \geq 0$ if and only if

$$\frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v+c}{v-c}. \tag{4}$$

Specifically, if the above condition fails, then the unique stage-game equilibrium is the one with no effort exertion. Otherwise, there are multiple equilibria in which the firm induces effort using subjective pay, with the optimal one described in Proposition **??**. We state this formally in the proposition below.

Proposition 2. ***(Stage-game Nash equilibria)*** *Suppose that $\frac{\beta_1 q_1}{\alpha_1 p_1} < \frac{v+c}{v-c}$. Then there exists a unique Nash equilibrium in the stage-game in which agents do not exert any effort. Otherwise, there exist multiple Nash equilibria in the stage game:*

(i) In the worst equilibrium for the firm, the agents do not exert effort and all players get their outside option of 0.

(ii) In the best equilibrium for the firm, agents exert effort and the firm earns $v - \frac{\beta_1 q_1 + \alpha_1 p_1}{\beta_1 q_1 - \alpha_1 p_1} c$.

To see the intuition behind this result, consider the situation in which the worst public and private signals are realized (i.e., $r = 1, Y = y_1$). The larger is the likelihood ratio $\frac{\beta_1 q_1}{\alpha_1 p_1}$, the more likely it is that the signals are drawn from the distribution where the agent shirked rather than worked. Thus, if the ratio $\frac{\beta_1 q_1}{\alpha_1 p_1}$ is large, it is easier for the firm to induce effort—a relatively small spread between the reward (for good performance) and punishment (for poor performance) would induce the agent to exert effort. Consequently, only a small amount of surplus needs to be destroyed in equilibrium (to induce effort). In this case, the firm may find it profitable to induce effort through subjective pay. But if $\frac{\beta_1 q_1}{\alpha_1 p_1}$ is small, it implies that effort exertion has little effect on the relative likelihood of the worst signals being realized, and the firm must then make the "punishment" wage of the agent sufficiently low in order to induce effort. This implies that a considerable amount of surplus must be destroyed and, consequently, the firm's payoff may become negative—a scenario in which it is optimal for the firm not to induce effort and earn its outside option of 0.

4.2. **Characterizing the optimal contract.** We now use our understanding of the stage-game to solve the firm's optimal relational contract. Proposition 2 shows that subjective pay is used in a one-shot game if and only if condition (4) holds. It turns out that the same condition continues to determine whether subjective pay is used in the repeated setting.

Proposition 3 below characterizes the optimal relational contract. But before we present the formal statement of this proposition, it is instructive to summarize the three key implications of this proposition. First, when the firm is patient enough (reputation concerns are strong), then the optimal incentive scheme uses only the relational contract: The firm pays a fixed bonus for all public signals above a certain threshold, and subjective pay is a lump sum that is independent of the agent's performance (consequently, there is no surplus destruction on the equilibrium path). Second, when the firm is not sufficiently patient (reputation concerns are weak), then it may use subjective pay to sharpen effort incentives. But its decision to use subjective pay depends on whether condition (4) holds—if condition (4) is violated, (i.e., in the stage-game equilibrium, effort cannot be induced with subjective pay), then the firm never uses subjective pay to generate effort incentives. Finally, whenever subjective pay is used to provide incentives, the optimal contract has features very similar to those in the stage-game: An agent is "punished" through low subjective pay only if both the lowest public output and the lowest private signal are realized, and an agent's subjective pay does not depend on the report he submits about his co-worker.

Proposition 3. ***(Optimal contract)*** *Let* y_k *denote the lowest team output such that* $\alpha_k > \beta_k$. *There exists a threshold discount factor* $\delta^* \in (0,1)$ *such that the followings hold:*

(A) If $\frac{\beta_1 q_1}{\alpha_1 p_1} < \frac{v+c}{v-c}$*, then (i) subjective pay is not used:* $w = w^*$ *and* $w_{rs}(y_j) = w^*/2$ *for all* $j \in \{1, ..., N\}$*, and* $r, s \in S$*; (ii) bonus payment is feasible if and only if* $\delta \geq \delta^*$*, and in this case, the bonus payment associated with the optimal contract is of the form*

$$b^*(y_j) = \begin{cases} b^* & if\ y_j \geq y_k \\ 0 & otherwise \end{cases},$$

where $b^* = c^{-1} \sum_{j=k}^{N} (\alpha_j - \beta_j)$*; and (iii) agents exert effort if and only if* $\delta \geq \delta^*$.

(B) If $\frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v+c}{v-c}$, then (i) for $\delta \geq \delta^*$, the optimal contract is as described above and both agents exert effort; (ii) for $\delta < \delta^*$, the subjective pay takes the form: $w = w^{**}$ and

$$w_{rs}(y_j) = \begin{cases} \frac{1}{2}w^{**} - \Delta & if\ j = 1\ and\ r = 1 \\ \frac{1}{2}w^{**} & otherwise \end{cases},$$

where $\Delta > 0$. The bonus payment takes the form

$$b^*(y_j) = \begin{cases} b^{**}(\delta) & if\ y_j \geq y_k \\ 0 & otherwise \end{cases},$$

where $b^{**}(\delta)$ is increasing in δ. Also, both agents exert effort.

Proposition 3 indicates that it is optimal to combine incentives only when the firm's reputation concerns are weak (i.e., the firm is less patient) and the private signal is a sharp measure of the agents' performance. Moreover, the subjective pay is used (to punish the agents) only when the lowest team output is realized and at least one of the agents receives the worst report from his co-worker. That is, in equilibrium, subjective pay is "compressed" across *both public and private* performance measures. This is similar to the results on wage compression discussed earlier in the context of the static game. The above result shows that the feature of wage compression not only extends to a team-production environment, but it also continues to hold in a richer set of performance signals that allows for both public and private measures.

To see the intuition behind Proposition 3, notice that providing incentives through subjective pay involves surplus destruction, but no surplus needs to be destroyed while providing incentives through bonus payments. However, a bonus payment requires the promise to be credible. The credibility of bonus payments depends on the future benefits from the relationship (this is the essence of the dynamic enforcement constraint (DE)). If the firm is sufficiently patient (i.e., when δ is high), the future gains from the relationship between the agents and the firm is high, and this means that the range of credible bonus payments is considerably large. Specifically, the firm can use bonus alone to generate sufficiently strong incentives to achieve the first best.

However, when the firm is not very patient (i.e., when δ is low), then its ability to offer large bonus payments is limited. Consequently, the firm will not be able to induce effort from the worker if it relies only on the bonus payments. In this case, subjective pay *may be* used to help induce effort. Notice that in Proposition 3 (part (B)), $b^{**}(\delta)$ is increasing in δ. This implies that the less patient the firm is, the more it will rely on the subjective payment to induce effort.

But subjective pay affects both sides of the dynamic enforcement (DE) constraint. On the one hand, subjective pay reduces the bonus amount necessary to induce effort. This makes (DE) more easily satisfied. On the other hand, subjective pay leads to surplus destruction, making (DE) harder to satisfy. When condition (4) holds, the benefit from reducing the bonus payment outweighs the cost of surplus destruction, and, thus, subjective pay can be used to relax (DE). Otherwise, the use of subjective pay hurts rather than helps the (DE) constraint, and so, it is never used in equilibrium.

Finally, the optimal use of subjective pay requires minimizing surplus destruction. Specifically, surplus should be destroyed only when the signals are "most" indicative that the agent has shirked. This occurs when the team output is at its lowest level and the peer report is the worst possible one. Of course, less surplus can be destroyed when *both* agents report the worst signal about each other. But such a contract suffers from the same "double deviation" problem as discussed in Proposition 1; i.e., the truth-telling constraint cannot be decoupled from the incentive constraint on effort. Consequently, just as in Proposition 1, an agent's report on his peer does not affect his own pay.

5. Optimal contracting with correlated signals

A key feature of the optimal contract discussed above is that an agent's subjective pay does not depend on his report about his co-worker. While this feature trivially ensures truth-telling by the agents, it also implies that in the optimal contract, the firm disregards some information about the agents' effort. This observation is, perhaps, not surprising when the private signals are statistically independent (conditional on effort).

In this section, we study the optimal contract when the signals are correlated. In this case, one might expect–in the spirit of the celebrated "informativeness principle" of optimal contracting (à la Holmström, 1979)–that an agent's subjective pay will depend on his report, as this signal contains information about his action. Surprisingly, it turns out that even with correlated signals, an agent's pay remains independent of his report, unless the degree of correlation itself is a function of the agents' effort level.

5.1. **A signal structure with correlation.** We consider a variation of our main model by assuming that the private signals are correlated (conditional on effort). We maintain the assumption that the public signal is still uncorrelated with both the private signals. To fix ideas, one can think of the public signal as being generated by an independent source outside the team–such as a client satisfaction survey–and, therefore, likely to be statistically independent of the private signals (conditional on effort) that the agents obtain.

To keep the analysis tractable, we restrict attention to binary private signals $S = \{l, h\}$. Consider two polar cases: perfect (positive) correlation between signals and independent signals (as in our main model). In the case of perfect correlation, suppose that $\Pr(r = s = h \mid \mathbf{e}) = p(\mathbf{e})$, $\Pr(r = s = l \mid \mathbf{e}) = 1 - p(\mathbf{e})$, and $\Pr(r \neq s \mid \mathbf{e}) = 0$. In the case of independence, let $\Pr(r = h \mid \mathbf{e}) = \Pr(s = h \mid \mathbf{e}) = p(\mathbf{e})$. Moreover, suppose that $p(1,1) = p$ and $p(e_1, e_2) = q$ for any $(e_1, e_2) \neq (1,1)$ where $p > q$. Now, consider a general signal structure that is a convex combination of these two cases. That is, let $\theta(\mathbf{e}) \in [0,1]$ be the "weight" on the correlated component of the signal and suppose that the signals are distributed as follows:

$$\Pr(r, s \mid \mathbf{e}) = \begin{cases} \theta(\mathbf{e})\, p(\mathbf{e}) + (1-\theta(\mathbf{e}))\,(p(\mathbf{e}))^2 & if\ r = s = h \\ \theta(\mathbf{e})\,(1-p(\mathbf{e})) + (1-\theta(\mathbf{e}))\,(1-p(\mathbf{e}))^2 & if\ r = s = l \\ 2\,(1-\theta(\mathbf{e}))\,p(\mathbf{e})\,(1-p(\mathbf{e})) & if\ r \neq s \end{cases}.$$

Finally, assume that

$$\theta(\mathbf{e}) = \begin{cases} \theta & if\ \ \mathbf{e} = (1,1) \\ \rho & otherwise \end{cases}.$$

It is worthwhile to highlight that our specification allows for the degree of correlation to depend on the agents' efforts. In other words, we assume that the agents' collective effort level can determine the extent to which the agents are exposed to a common shock.

5.2. **The optimal contract.** The firm's optimal contracting problem under this altered signal structure closely mirrors our earlier analysis of the independent signal case. That is, the firm solves the program $\mathcal{P}$ where the probability distribution over the private signals $\{r, s\}$ is the one given above. As before, we first state in Proposition 4 the optimal contract in the stage-game. This result not only pins down the firm's payoff under the optimal subjective contract, but also highlights how the optimal subjective pay depends on the underlying correlation structure. Using this result, we then derive the optimal contract in the repeated game in Proposition 5.

Proposition 4. ***(Optimal stage-game contract with effort under correlated signals)*** *The optimal contract in the stage-game is characterized as follows. There exist $\hat{w}_c$ and $\bar{w}_c$, such that the following holds:*

(A) If $(1-\theta)\,p > (1-\rho)\,q$, then $w = \hat{w}_c$ and

$$w_{rs}(y_j) = \begin{cases} \frac{1}{2}\hat{w}_c - \frac{c}{\beta_1(1-q)-\alpha_1(1-p)} & \text{if } j=1 \text{ and } r=l \\ \frac{1}{2}\hat{w}_c & \text{otherwise} \end{cases}.$$

(B) If $(1-\theta)\,p \leq (1-\rho)\,q$, then, $w = \bar{w}_c$ and

$$w_{rs}(y_j) = \begin{cases} \frac{1}{2}\bar{w}_c - \Delta_l^* & \text{if } j=1 \text{ and } (r,s)=(l,h) \\ \frac{1}{2}\bar{w}_c - \Delta_h^* & \text{if } j=1 \text{ and } (r,s)=(h,l) \\ \frac{1}{2}\bar{w}_c & \text{otherwise} \end{cases}.$$

where Δ_l^* and $\Delta_h^* > 0$.

Part (A) of the above proposition indicates that as long as

$$(1-\theta)\,p > (1-\rho)\,q, \tag{5}$$

the optimal contract with subjective evaluation takes the same form as in Proposition 1.[11] In particular, when effort does not affect the correlation–i.e., $\theta = \rho$ – condition (5) is always satisfied. In this case the contract disregards A_i's report about A_{-i} while determining A_i's pay. This holds even if $\theta = \rho \neq 0$ – that is, when there is correlation between the signals. This finding appears counterintuitive, as A_i's report about A_{-i} does contain information about A_i's own performance when signals are correlated. So, at the optimal contract, the firm is choosing not to use additional information that it could have used to sharpen incentives. This finding is in contrast with the informativeness principle (à la Holmström, 1979) in which using an additional signal always (weakly) improves the efficiency of the contract.[12] This indicates that the nature of the optimal compensation scheme with private peer evaluations is fundamentally different from its counterpart with public signals.

Part (B) shows that when condition (5) is violated, an agent's pay is affected not only by his co-worker's report, but also by his own report about his co-worker. In particular, surplus is destroyed only when the reports of the agents do not match. The intuition behind this result is that the firm can induce effort from the agents by comparing their reports. Specifically, the agents' reports are more likely to be aligned when they both exert effort. Therefore, by punishing the agent for a mismatch of reports, the firm induces each agent to exert effort.

Notice that condition (5) determines whether an agent's pay depends on his own report about his co-worker. This condition has a natural interpretation. In our setup there are two possible ways to detect whether or not an agent has shirked. First, an agent is more likely to have shirked when his evaluation is low. The effectiveness of detecting shirking by using peer evaluations is captured by the likelihood ratio p/q. Second, an agent is more likely to have shirked when reports of the agents do not match. The effectiveness of detecting shirking by comparing the reports is captured by the likelihood ratio $(1-\rho)\,/\,(1-\theta)$. When the latter likelihood ratio is larger than the former, comparison of reports is a more effective in detecting shirking.

[11] Notice that since the signals are binary, $(1-q)$ corresponds to the notation q_1 in our main model (similarly $(1-p)$ corresponds to p_1).

[12] This result continues to hold even if the agents are risk-averse. The proof is available from the authors.

To characterize the optimal contract with correlated signals, it is instructive to calculate the associated level of surplus destruction. Routine calculation gives that:

$$\hat{z}_c = \begin{cases} \frac{2\alpha_1(1-p)c}{\beta_1(1-q)-\alpha_1(1-p)} & \textit{if } \ (1-\theta)\,p > (1-\rho)\,q \\ \frac{2\alpha_1(1-\theta)p(1-p)c}{\beta_1 q(1-q)-\alpha_1(1-p)p(1-\theta)} & \textit{otherwise} \end{cases}.$$

(We use the subscript c to indicate that the signals are correlated.) The following remarks are in order. First, if the private signals are not perfectly correlated (i.e., $\theta \neq 1$), there is always surplus destruction in equilibrium. Second, under perfect correlation (i.e., $\theta = 1$), the optimal contract achieves the first best. This finding is similar to the finding by MacLeod (2003), who shows that when the information of the principal and the agent are perfectly correlated, there are no agency costs associated with the use of subjective evaluations. Put differently, the optimal contract with (perfectly correlated) subjective evaluation is the same as the optimal principal-agent contract with verifiable information. Third, while it is well known that surplus destruction is smaller when the signals are more correlated, in our setting, the amount of surplus destruction depends not only on the correlation of the signals, but also on how effort affects the degree of correlation. Finally, the above finding also indicates the condition under which it is optimal to induce effort in the stage-game. Note that the stage-game payoff under the optimal subjective contract (with effort inducement) is $\hat{\pi}_c := v - c - \hat{z}_c$. The sign of $\hat{\pi}_c$ is determined by the following:

If $(1-\theta)\,p > (1-\rho)\,q$ then $\hat{\pi}_c \geq 0$ if and only if:

$$\frac{\beta_1\,(1-q)}{\alpha_1\,(1-p)} \geq \frac{v+c}{v-c}. \tag{6}$$

And if $(1-\theta)\,p \leq (1-\rho)\,q$, then $\hat{\pi}_c \geq 0$ if and only if:

$$\frac{\beta_1\,(1-q)}{\alpha_1\,(1-p)} \cdot \frac{q}{(1-\theta)\,p} \geq \frac{v+c}{v-c}. \tag{7}$$

Using the above observations, the following proposition characterizes the optimal contract with correlated signals.

Proposition 5. ***(Optimal contract with correlated signals)*** *If $(1-\theta)\,p > (1-\rho)\,q$, then the optimal contract is exactly as given in Proposition 3. Otherwise, there exists a threshold discount factor $\delta^* \in (0,1)$ such that the following are true:*

(A) If $\delta \geq \delta^*$, then both agents exert effort, and subjective pay is not used: $w = w^*$ and $w_{rs}\,(y_j) = w^*/2$ for all $j \in \{1, ..., N\}$, and $r,\ s \in S$, and a bonus payment takes the form:

$$b^*\,(y_j) = \begin{cases} b^* & \textit{if } y_j \geq y_k \\ 0 & \textit{otherwise} \end{cases},$$

where $b^* = c^{-1} \sum_{j=k}^{N} (\alpha_j - \beta_j)$ and y_k denotes the lowest team output such that $\alpha_k > \beta_k$.

(B) If $\delta < \delta^*$, there are two cases: (i) If $\frac{\beta_1(1-q)}{\alpha_1(1-p)} \cdot \frac{q}{(1-\theta)p} < \frac{v+c}{v-c}$, then agents do not exert effort. (ii) Otherwise, both agents exert effort. The subjective pay takes the form: $w = \bar{w}_c$ and

$$w_{rs}(y_j) = \begin{cases} \frac{1}{2}\bar{w}_c - \Delta_l^* & \text{if } j = 1 \text{ and } (r,s) = (l,h) \\ \frac{1}{2}\bar{w}_c - \Delta_h^* & \text{if } j = 1 \text{ and } (r,s) = (h,l) \\ \frac{1}{2}\bar{w}_c & \text{otherwise} \end{cases},$$

where Δ_l^*, $\Delta_h^* > 0$. The bonus payment takes the form:

$$b^*(y_j) = \begin{cases} b_c^{**}(\delta) & \text{if } y_j \geq y_k \\ 0 & \text{otherwise} \end{cases},$$

where $b_c^{**}(\delta)$ is increasing in δ.

The above proposition closely parallels Proposition 3, in that subjective pay is used only when the firm is less patient, and when the surplus destruction it necessitates is small. But in contrast to Proposition 3, an agent's pay can now depend on his own report about his co-worker. Moreover, the surplus destruction can depend on the correlation between the signals. Specifically, the larger is θ, the smaller is the surplus destruction.

6. Conclusion

This paper studies the optimal design of peer evaluation in environments in which the firm motivates its workers through relational contracts. When co-workers have superior knowledge about each other's performance, peer evaluations can elicit such information and use it to better motivate the workers. Since peer evaluations are inherently private and subjective, the agents and the firm must have the right incentives for truthful revelation of information. And this process necessitates surplus destruction on the equilibrium path. Our analysis shows that while peer evaluations can help better sustain relational contracts, the use of peer evaluations in incentive contracts is intrinsically different from the use of publicly available information.

Specifically, we show that peer evaluations will be used in the optimal relational contract only when the associated level of surplus destruction is small and the firm is relatively impatient. More importantly, we show that the worker's effort incentives cannot be decoupled from his truth-telling incentives. In particular, when the worker's peer evaluation affects his own pay, contracts that are robust to single deviations may fall victim to a double-deviation in which the worker shirks and then lies about his peer's performance. The interconnectedness of effort and truth-telling incentives implies that the firm may neglect useful information about the worker's effort to elicit truthful peer evaluations, contrasting the celebrated "Informativeness Principle" (à la Holmström) for the public information case.

We conclude with brief remarks on two issues related to our model. First, we have assumed that subjective peer evaluation is performed in each period. While frequent feedback about performance has its advantages, it is not always optimal. For example, Fuchs (2007) shows that when subjective evaluation is used, reducing the frequency of evaluation can mitigate the amount of surplus destruction. In addition, Fong and Li (2010) shows that less information can sometimes help better sustain the relational contract by reducing the principal's temptation to renege on the bonus. A natural next step is to study how different information-collection processes affect organizational performance. In addition to questions about the frequency of peer evaluations, one can also ask how peer evaluations interact with other information sources, such as self-evaluations. These issues are crucial in practice, and further research in this direction is needed.

Second, an important issue related to information collection is potential collusion among the workers. Collusion affects our analysis because the workers can gain by always sending good reports about each other and avoiding the punishment associated with subjective evaluations.[13] Agreeing to always sending good reports, however, may not be the optimal way to collude because it destroys the value by exacerbating the free-riding problem associated with the team production. Specifically, if a worker is always guaranteed a good peer evaluation, he then strictly prefers to shirk under our optimal contract. By shirking, the worker exerts a negative externality on his co-worker through lowering the expected bonus associated with the public outputs. An interesting question to ask is how workers can collude while maintaining effort. More importantly, one can ask how collusion affects the design of peer evaluations and, thus, how information is collected within organizations. A complete analysis of this issue, however, is beyond the scope of our article. To see the complexity of the issue, note that such a collusion cannot be publicly monitored (as both the reports and the effort are privately observed) and such an analysis also needs to explicitly model the optimal punishment mechanism that the co-workers may adopt following a breakdown of the collusion (punishment may involve withholding of effort or misreporting of evaluation or a combination of the two). Nevertheless, understanding how peer evaluations are affected by collusion among workers is important for both business practice and economic theory and it is an interesting topic for future research.

Appendix

This appendix contains the proofs omitted in the text.

Proof of Proposition 1. **Step 1**. Observe that the truth-telling constraint (T) implies that $\mathbb{E}_{\{y_j,r\}}\left[w_{rs'}\left(y_j\right)|\mathbf{e}=\mathbf{1},s_i=s\right]$ must be independent of s' on the equilibrium path. Now, the (IR) can be rewritten as

$$\mathbb{E}_{s|\{y_j,r\}}\left[\mathbb{E}_{\{y_j,r\}}\left[w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1},s_i=s\right]\right]\geq 0.$$

Thus, it must be the case that

$$\mathbb{E}_{\{y_j,r\}}\left[w_{rs}\left(y_j\right)|\mathbf{e}=\mathbf{1},s_i=s\right]=0 \quad \forall s\in S. \quad \left(IR'\right).$$

Otherwise, the firm could achieve a higher payoff by reducing its committed wage pool w and all subjective pay $w_{rs}\left(y_j\right)$ by a small amount $\varepsilon>0$ while still not violating any constraints. Also, using (IR'), one can rewrite (IC) as

$$0\geq\mathbb{E}_{\{y_j,r\}}\left[w_{rs}\left(y_j\right)|e_i=0,s_i=s\right] \quad \forall s\in S. \quad \left(IC'\right)$$

Thus, the firm's problem reduces to maximizing $-w$ subject to (B), (T), (IR') and (IC').
Step 2. Define the Lagrangian for the firm's problem as

$$\begin{aligned}\mathcal{L}= \quad & (v-w)+\textstyle\sum_{s\in S}\sum_{r\in S}\sum_{j=1}^{N}\eta_{sr}^{j}(w-w_{rs}\left(y_j\right)-w_{sr}\left(y_j\right))+\\ & \textstyle\sum_{s\in S}\lambda_s\sum_{j=1}^{N}\alpha_j(\sum_{r\in S}p_r w_{rs}\left(y_j\right)-c)+\sum_{s\in S}\mu_s\sum_{j=1}^{N}\left(-\beta_j\sum_{r\in S}q_r w_{rs}\left(y_j\right)\right).\end{aligned}$$

[13]The extent of the problem of collusion, however, depends on the nature of the team. Following the classification of teams suggested by Scott and Einstein (2001), collusion is more of an issues in the "work and service" teams, where a group of workers are assigned to a "routine manufacturing or service tasks." In contrast, "project teams" are formed for a specific task or project and usually reconfigured upon completion of the project. One would expect collusion to be less of an issue in project teams, as the members do not anticipate repeated interaction over a long period of time.

Now, the first-order conditions with respect to w and $w_{rs}(y_j)$ $\forall j,\ s$, and r are:

$$\sum_{r,s\in S}\sum_{j=1}^{N}\eta_{sr}^{j} = 1, \text{ and} \tag{8}$$

$$\lambda_s\alpha_j p_r - \mu_s\beta_j q_r = \eta_{sr}^{j}+\eta_{rs}^{j}. \tag{9}$$

Also, the complementary slackness conditions are: $\forall j,\ r$, and s,

$$\eta_{sr}^{j}(w-w_{rs}(y_j)-w_{sr}(y_j)) = \lambda_s\sum_{j=1}^{N}\sum_{r\in S}\alpha_j p_r w_{rs}(y_j)-c = -\mu_s\sum_{j=1}^{N}\sum_{r\in S}\beta_j q_r w_{rs}(y_j)=0.$$

Step 3. We claim that there exists a set of non-negative Lagrange multipliers such that the proposed compensation schedule along with the set of multipliers satisfy the first-order conditions and the complementary slackness conditions.

Note that $\sum_{j=1}^{N}\alpha_j\sum_{r\in S}p_r w_{rs}(y_j)-c=\sum_{j=1}^{N}\beta_j\sum_{r\in S}q_r w_{rs}(y_j)=0\ \forall s\in S$. Also note that $\hat{w}-\hat{w}_{rs}(y_j)-\hat{w}_{sr}(y_j)=0\ \forall j>1$ and $\hat{w}-\hat{w}_{rs}(y_1)-\hat{w}_{sr}(y_1)=0\ \ \forall r,s>1$. Thus, any set of η_{sr}^{j}, λ_s and μ_s values satisfies the complementary slackness conditions as long as

$$\eta_{1r}^{1}=\eta_{s1}^{1}=0\ \ \forall\ r,s\in S. \tag{10}$$

Now, consider the following values of the multipliers:

$$\mu_s=\nu\,(q_1p_s-q_sp_1)\ \textit{for some}\ \nu>0,\ \text{and}\ \lambda_s=\mu_s\beta_1q_1/\alpha_1p_1. \tag{11}$$

By MLRP, $(q_1p_s-q_sp_1)=q_1q_s\,(p_s/q_s-p_1/q_1)>0$. Hence, $\mu_s\geq 0$, and therefore, $\lambda_s\geq 0$ $\forall s\in S$. Also, given the proposed values of the multipliers in (10) and (11), it is routine to check that the first-order condition (9) is satisfied if s and/or $r=1$.

Hence, it remains to show that one can find values of η_{sr}^{j} for $s,\ r>1$, such that they satisfy the first-order conditions.

Step 4. For $s,\ r>1$, let

$$\eta_{sr}^{j}=\eta_{rs}^{j}=\frac{1}{2}\left(\lambda_s\alpha_jp_r-\mu_s\beta_jq_r\right)=\frac{\nu}{2}\frac{\beta_1\beta_jq_1q_r}{\alpha_1p_1}\left(q_1p_s-q_sp_1\right)\left(\frac{\alpha_jp_r}{\beta_jq_r}-\frac{\alpha_1p_1}{\beta_1q_1}\right),$$

and

$$\nu=\left[\sum_{j=1}^{N}\sum_{s,r\in S}\frac{\beta_1\beta_jq_1q_r}{2\alpha_1p_1}\left(q_1p_s-q_sp_1\right)\left(\frac{\alpha_jp_r}{\beta_jq_r}-\frac{\alpha_1p_1}{\beta_1q_1}\right)\right]^{-1}.$$

As before, by MLRP, $\eta_{sr}^{j}\geq 0$. Also, by construction, the first-order conditions (9) are now satisfied. Thus, the proposed compensation scheme, along with the Lagrangian multipliers, satisfies all first-order conditions and the complementary slackness condition. Hence, the proposed compensation scheme is optimal. ■

Proof of Proposition 3. **Step 1.** We argue that without loss of generality, we can restrict attention to the class of contracts where

$$w=w_{rs}(y_j)+w_{sr}(y_j)\text{ for all }j>1.$$

We show this by contradiction. Suppose that this is not true for some $j>1$, r, and $s=s^*\in S$. Consider a new subjective payment scheme where

$$w'_{rs^*}(y_j)=w_{rs^*}(y_j)+\varepsilon,$$
$$w'_{rs^*}(y_1)=w_{rs^*}(y_1)-\tfrac{\alpha_j}{\alpha_1}\varepsilon,$$

and $w'_{rs}(y_j) = w_{rs}(y_j)$ for all other j, r and s. Note that under the new payment scheme, (a) the value in the objective function does not change; (b) constraint (B) remains satisfied; and (c) (DE) is not affected. Now to check (T), note that if the agent reports s^*, we have, by construction,

$$\mathbb{E}_{\{y_j,r\}}\left[w_{rs^*}(y_j)\,|\mathbf{e}=\mathbf{1}\right] = \mathbb{E}_{\{y_j,r\}}\left[w'_{rs^*}(y_j)\,|\mathbf{e}=\mathbf{1}\right].$$

And if the agent reports $s' \neq s^*$, then

$$\mathbb{E}_{\{y_j,r\}}\left[w_{rs'}(y_j)\,|\mathbf{e}=\mathbf{1}\right] = \mathbb{E}_{\{y_j,r\}}\left[w'_{rs'}(y_j)\,|\mathbf{e}=\mathbf{1}\right],$$

since $w'_{rs'}(y_j) = w_{rs'}(y_j)$ for all j, r and $s' \neq s^*$. Finally, to check (IC) (together with (IC')), we need to make sure that

$$\mathbb{E}_{\{y_j,r,s\}}\left[b(y_j) + w'_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}\right] - c \geq \mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w'_{rs'}(y_j)\,|e_i = 0\right] \ \ \forall s'.$$

Note that for all s, by construction,

$$\mathbb{E}_{\{y_j,r,s\}}\left[b(y_j) + w'_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}\right] - c = \mathbb{E}_{\{y_j,r,s\}}\left[b(y_j) + w_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}\right] - c.$$

On the right-hand side of (IC), if $s' \neq s^*$,

$$\mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w'_{rs'}(y_j)\,|e_i = 0\right] = \mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w_{rs}(y_j)\,|e_i = 0\right]$$

as none of the values are changed. And if $s = s^*$, we have

$$\begin{aligned}\mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w'_{rs^*}(y_j)\,|e_i = 0\right] &= \mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w_{rs}(y_j)\,|e_i = 0\right] + \beta_j\varepsilon - \beta_1\tfrac{\alpha_j}{\alpha_1}\varepsilon \\ &< \mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w_{rs}(y_j)\,|e_i = 0\right].\end{aligned}$$

This shows that if under any contract, we have $w > w_{rs}(y_j) + w_{sr}(y_j)$ some $j > 1$, r, and s, there always exists another contract that makes the inequality bind but gives the same payoff to the firm.

Step 2. We claim that if the surplus destruction is positive, then the agent's (IC) constraint (which combines (IC) and (IC'))

$$\mathbb{E}_{\{y_j,r,s\}}\left[b(y_j) + w_{rs}(y_j)\,|\mathbf{e}=\mathbf{1}\right] - c \geq \max_{s'} \mathbb{E}_{\{y_j,r\}}\left[b(y_j) + w_{rs'}(y_j)\,|e_i = 0\right] \ \ \forall s'$$

must binds with equality. To see this, suppose that the above is slack. Now define

$$w'_{rs}(y_1) = (1-\varepsilon)w_{rs}(y_1) + \frac{\varepsilon}{2}w.$$

In this case,

$$\begin{aligned}\mathbb{E}_{y_j}\left[w'_{rs}(y_1)\,|\mathbf{e}=\mathbf{1}\right] &= \textstyle\sum_{r\in S} p_r\left[(1-\varepsilon)w_{rs}(y_1) + \frac{\varepsilon}{2}w\right] \\ &= (1-\varepsilon)\textstyle\sum_{r\in S} p_r w_{rs}(y_1) + \frac{\varepsilon}{2}w.\end{aligned}$$

This implies that the truth-telling constraint (T) remains satisfied. We can also check that (B) and (DE) are also satisfied. For small enough ε, the (IC) constraint remains satisfied. Note that this change reduces the surplus destruction by ε (in proportion). This implies that the (IC) constraint must bind.

Step 3. Define k to be the unique index such that $\alpha_k > \beta_k$ but $\alpha_{k-1} < \beta_{k-1}$. If the surplus destruction is positive, then there exists a b such that

$$b(y_j) = \begin{cases} b & \text{if } j \geq k \\ 0 & \text{otherwise} \end{cases}.$$

Note that for $j < k$, by decreasing $b(y_j)$, we (weakly) relax (DE) and (IC), and we do not affect other constraints. Similarly, for $j \geq k$, by increasing $b(y_j)$, we relax (IC) and we do not affect other constraints. In this case, (DE) remains satisfied as long as $b(y_j) = \max_j\{b(y_j)\}$. Also, note that the above value of $b(y_j)$ is unique. If there exists a $b(y_j) < b$ for some $j \geq k$, we can increase b_j by ε_1 and relax the (IC) constraint. But from Step 2 above, we know that if (IC) is relaxed, then the firm can set

$$w'_{rs}(y_1) = (1-\varepsilon_2)w_{rs}(y_1) + \frac{\varepsilon_2}{2}w,$$

for some small enough ε_2, and reduces the the level of surplus destruction.

Step 4. Next, consider the case in which effort can be induced using only the relational contract (i.e., no surplus is destroyed in equilibrium). From (IC) and the formulation of the optimal bonus pay as given in Step 3, we obtain that $b = b^* := 2c/\sum_{j=k}^{N}(\alpha_j - \beta_j)$ if effort must be induced by using bonus pay alone. In this case, a necessary and sufficient condition for sustaining such a relational contract is

$$\frac{\delta}{1-\delta}\left[v-c\right] \geq \frac{2c}{\sum_{j=k}^{N}(\alpha_j - \beta_j)}. \tag{12}$$

The above condition is satisfied when δ is higher than a threshold, say δ^*, at which the above inequality binds.

Step 5. If $\delta < \delta^*$, the optimal contract may use subjective pay, and surplus will be destroyed. In this case, suppose that the maximum bonus is given by b; then, we can rewrite the (DE) constraint in the firm's program $\mathcal{P}'$ as

$$\frac{\delta}{1-\delta}\pi \geq 2b. \quad (DE)$$

(Recall that the punishment payoff of the firm, $\underline{\pi}$, is 0). Now the program can be solved in two steps. First, for a fixed b, we choose $w_{rs}(y_1)$ to minimize the surplus destruction, and second, we choose the largest b for which the above (DE) holds. Note that in the first step, the problem is the same as the static one with the agent's cost of effort equal to

$$c(b) = c - \sum_{j=k}^{N}(\alpha_j - \beta_j)b. \tag{13}$$

Therefore, we can apply Propositions 1 and 2 to conclude the following: If

$$\frac{\beta_1 q_1}{\alpha_1 p_1} \geq \frac{v + c(b)}{v - c(b)}, \tag{14}$$

then the firm uses the following subjective contract in equilibrium: There exists some w so that

$$w_{rs}(y_1) = \begin{cases} \frac{1}{2}w - \frac{\alpha_1 p_1 c(b)}{\beta_1 q_1 - \alpha_1 p_1} & if\ r = 1 \\ \frac{1}{2}w & otherwise, \end{cases}$$

and the expected surplus destruction in this case is

$$z = \frac{2\alpha_1 p_1 c(b)}{\beta_1 q_1 - \alpha_1 p_1}.$$

Otherwise, no subjective pay is used. This solves for the first step. In the second step, for (DE) to be satisfied, we need

$$\frac{\delta}{1-\delta}\left[v - c - \frac{2\alpha_1 p_1 (c - \sum_{j=k}^{N}(\alpha_j - \beta_j)b)}{\beta_1 q_1 - \alpha_1 p_1}\right] \geq 2b. \tag{15}$$

That is, we need to find the largest b for which the above condition (15) holds. Note that at $b = 0$, the left-hand side is

$$\frac{\delta}{1-\delta}\left[v - c - \frac{2\alpha_1 p_1 c}{\beta_1 q_1 - \alpha_1 p_1}\right] = \frac{\delta}{1-\delta}\left[v - c - \hat{z}\right].$$

Here we have two cases.

Case 1: If $\beta_1 q_1/(\alpha_1 p_1) < (v+c)/(v-c)$ (i.e., effort cannot be induced in a Nash equilibrium of the stage-game), $v - c - \hat{z} < 0$. Also, at $b = c/(\sum_{j=k}^N(\alpha_j - \beta_j))$, condition (15) is violated (recall that we are considering the case where (12) fails). Thus, there is no value of b that satisfies (15). So, $b = 0$. As (14) is also violated in this case (note that $c(0) = c$), no subjective pay is used either.

Case 2: If $\beta_1 q_1/(\alpha_1 p_1) \geq (v+c)/(v-c)$ (i.e., effort can be induced in a Nash equilibrium of the stage-game), $v - c - \hat{z} < 0$. Also, as noted above, at $b = c/(\sum_{j=k}^N(\alpha_j - \beta_j))$, the condition (15) is violated. Since both sides of condition (15) are linear in b, this implies that the highest b that satisfies (15), $b^{**}(\delta)$ (say), is the one at which (15) holds with equality. That is,

$$b^{**}(\delta) = \frac{\delta\left[\beta_1 q_1 (v-c) - \alpha_1 p_1 (v+c)\right]}{2\left[\beta_1 q_1 (1-\delta) - \alpha_1 p_1 \left(1 - \delta\left(1 - \sum_{j=k}^N(\alpha_j - \beta_j)\right)\right)\right]}.$$

This observation completes the proof. ■

Proof of Proposition 4. As noted earlier, the firm solves $\mathcal{P}'$ where the distribution over the private signals $\{r, s\}$ are given as in section 5.1. While the set of constraints remains the same substantively, the algebraic expressions for all constraints, except the budget constraint (B) and the dynamic enforcement constraint (DE), would change as the signals are no longer independent. Thus, before we present the proof, it is useful to rewrite the relevant program for the firm.

Let P_{sr} and Q_{sr} be the probabilities of obtaining the signals (s, r) on and off equilibrium, respectively. So, $P_{hh} = \theta p + (1-\theta)p^2$, $P_{lh} = P_{hl} = (1-\theta)p(1-p)$, $P_{ll} = \theta(1-p) + (1-\theta)(1-p)^2$, and similarly $Q_{hh} = \rho q + (1-\rho)q^2$, $Q_{lh} = Q_{hl} = (1-\rho)q(1-q)$, and $Q_{ll} = \rho(1-q) + (1-\rho)(1-q)^2$.

The (IR) constraint for A_i is:

$$U_A := \textstyle\sum_{j=1}^N \alpha_j \left[b(y_j) + \sum_{r,s} P_{rs} w_{rs}(y_j)\right] - c \geq 0, \quad (IR_i)$$

and the truth-telling constraints are:

$$\textstyle\sum_{j=1}^N \alpha_j \left[P_{hh} w_{hh}(y_j) + P_{lh} w_{lh}(y_j)\right] \geq \sum_{j=1}^N \alpha_j \left[P_{hh} w_{hl}(y_j) + P_{lh} w_{ll}(y_j)\right], \quad (T_h)$$
$$\textstyle\sum_{j=1}^N \alpha_j \left[P_{hl} w_{hl}(y_j) + P_{ll} w_{ll}(y_j)\right] \geq \sum_{j=1}^N \alpha_j \left[P_{hl} w_{hh}(y_j) + P_{ll} w_{lh}(y_j)\right]. \quad (T_l)$$

It is, however, more useful to represent the agent's incentive compatibility a bit differently than the way we defined it in the independent-signal case. Recall that, off the equilibrium path, the agent may not only deviate from his effort level, but also engage in manipulating his report on his colleague's performance. Given this observation, there are four (IC) constraints that we need to account for, each corresponding to one of four reporting strategies: always report l, always report h, report truthfully, and reverse the signals while reporting. In what follows, we state the respective (IC) constraints for each of these four types of (off the equilibrium) reporting strategies:

$$U_A \geq \sum_{j=1}^{N} \beta_j \left[b(y_j) + q w_{hl}(y_j) + (1-q) w_{ll}(y_j)\right], \qquad (IC_l)$$
$$U_A \geq \sum_{j=1}^{N} \beta_j \left[b(y_j) + q w_{hh}(y_j) + (1-q) w_{lh}(y_j)\right], \qquad (IC_h)$$
$$U_A \geq \sum_{j=1}^{N} \beta_j \left[b(y_j) + \sum_{r,s\in\{h,l\}} Q_{rs} w_{rs}(y_j)\right], \qquad (IC_t)$$
$$U_A \geq \sum_{j=1}^{N} \beta_j \left[b(y_j) + Q_{hh} w_{hl}(y_j) + Q_{hl} w_{hh}(y_j) + Q_{lh} w_{ll}(y_j) + Q_{ll} w_{lh}(y_j)\right]. \qquad (IC_r)$$

So, the firm's problem is to maximize its payoff subject to the above set of constraints. That is, in the repeated game, the firm's problem is:

$$\mathcal{P}_c \begin{cases} \max_{\phi\in\Phi} \ \pi = v - \left[w + \sum_{j=1}^{N} \alpha_j b(y_j)\right] \\ \\ s.t. \ (B), \ (T_h), \ (T_l), \ (DE), \ (IR_i), \ (IC_l), \ (IC_h), \ (IC_t), \text{ and } (IC_r). \end{cases}$$

The static game counterpart of this problem can be derived simply by ignoring (DE) and setting $b(y_j) = 0$ for all y_j. That is, the firm's problem in the stage-game is

$$\mathcal{P}_c' \begin{cases} \max_{\{w, w_{rs}(y_j)\}} \ \ v - w \\ \\ s.t. \ (B), \ (T_h), \ (T_l), \ (IR_i), \ (IC_l), \ (IC_h), \ (IC_t), \text{ and } (IC_r). \end{cases}$$

We are now ready to present the proof, which is given in the following steps:

Step 1. Let $\Lambda = (\eta_{hh}^j, \eta_{ll}^j, \eta_{hl}^j, \mu, \lambda_1, \lambda_2, \gamma_1, \gamma_2, \gamma_3, \gamma_4)$ be the respective Lagrange multipliers to the above set of constraints (η_{rs}^j is the multiplier for (B) when the signals are (y_j, r, s); also, note that (B) remains the same for the private signals (r, s) and (s, r)). The first-order conditions with respect to w, $w_{hh}(y_j)$, $w_{hl}(y_j)$, $w_{lh}(y_j)$, and $w_{ll}(y_j)$ are:

(16)

$$\begin{array}{ll} (i) & \sum_{j=1}^{N}(\eta_{hh}^j + \eta_{ll}^j + \eta_{hl}^j) = 1, \\ (ii) & -2\eta_{hh}^j + \alpha_j[[\mu + \sum_i \gamma_i]P_{hh} + \lambda_1 P_{hh} - \lambda_2 P_{hl}] = \beta_j\left[\gamma_2 q + \gamma_3 Q_{hh} + \gamma_4 Q_{hl}\right], \\ (iii) & -\eta_{hl}^j + \alpha_j[[\mu + \sum_i \gamma_i]P_{hl} - \lambda_1 P_{hh} + \lambda_2 P_{hl}] = \beta_j\left[\gamma_1 q + \gamma_3 Q_{hl} + \gamma_4 Q_{hh}\right], \\ (iv) & -\eta_{hl}^j + \alpha_j[[\mu + \sum_i \gamma_i]P_{lh} + \lambda_1 P_{lh} - \lambda_2 P_{ll}] = \beta_j\left[\gamma_2(1-q) + \gamma_3 Q_{lh} + \gamma_4 Q_{ll}\right], \\ (v) & -2\eta_{ll}^j + \alpha_j[[\mu + \sum_i \gamma_i]P_{ll} - \lambda_1 P_{lh} + \lambda_2 P_{ll}] = \beta_j\left[\gamma_1(1-q) + \gamma_3 Q_{ll} + \gamma_4 Q_{lh}\right]. \end{array}$$

Note that under the given contract, (B) is slack only when $j = 1$, r and/or $s = l$. So, we must have $\eta_{ll}^1 = \eta_{hl}^1 = 0$.

Step 2. Next, we solve for μ. Summing equations $(16\ ii - v)$, one obtains

$$-2\sum_{j=1}^{N}\left(\eta_{hh}^j + \eta_{hl}^j + \eta_{ll}^j\right) + \sum_{j=1}^{N}\alpha_j\left(\mu + \sum \gamma_i\right) = \sum_{j=1}^{N}\beta_j \sum \gamma_i,$$

or

$$\mu = 2\sum_{j=1}^{N}\left(\eta_{hh}^j + \eta_{hl}^j + \eta_{ll}^j\right).$$

Using $(16\ i)$ one obtains $\mu = 2$.

Step 3. Now, we further conjecture that $\gamma_1 = \gamma_2 = \gamma_4 = 0$ and $\gamma_3 > 0$. Plugging the values $\eta_{ll}^1 = \eta_{hl}^1 = 0$, $\mu = 2$, and $\gamma_1 = \gamma_2 = \gamma_4 = 0$ into equations $(16\ ii - v)$, one obtains the following solution for λ_is, γ_3 and η_{hh}^1:

$$\lambda_1 = \frac{2\beta_1(1-p)(1-q)((p-q)-(\theta p - \rho q))}{\theta p((1-q)\beta_1 - (1-p)\alpha_1)}, \quad \lambda_2 = \frac{2\beta_1(1-q)((p-q)-(\theta p-\rho q))}{\theta((1-q)\beta_1-(1-p)\alpha_1)},$$

and

$$\gamma_3 = \frac{2(1-p)\alpha_1}{(1-q)\beta_1-(1-p)\alpha_1}, \quad \eta^1_{hh} = \frac{(p-q)\alpha_1\beta_1}{(1-q)\beta_1-(1-p)\alpha_1}.$$

As $\beta_1 > \alpha_1$ (by MLRP), all of the above values are non-negative if and only if $(1-\theta)\,p \geq (1-\rho)\,q$.

Step 4. It remains to show that we can find values for η^j_{hh}, η^j_{hl}, and η^j_{ll} for $j > 1$ such that equations in (16) are also satisfied. Note that λs, μ and γs are independent of j. So, we plug the above values of λs, μ and γs into the first-order conditions ($16\ ii - v$) for any arbitrary $j > 1$ and back out the values for η^js that solves this set of equations. The solutions are:

$$\begin{aligned}
\eta^j_{hh} = {} & \beta_1\beta_j \frac{p\left[(1-q)\frac{\alpha_j}{\beta_j}(1-q(1-\rho))+q\frac{\alpha_1}{\beta_1}(q+q(1-\rho))\right]-q\left[(1-q)\frac{\alpha_j}{\beta_j}(1-\rho)+\frac{\alpha_1}{\beta_1}(q+q(1-\rho))\right]}{\beta_1(1-q)-\alpha_1(1-p)}, \\
\eta^j_{hl} = {} & \beta_1\beta_j \frac{2(1-p)(1-q)q(1-\rho)\left(\alpha_j/\beta_j-\alpha_1/\beta_1\right)}{\beta_1(1-q)-\alpha_1(1-p)}, \\
\eta^j_{ll} = {} & \beta_1\beta_j \frac{(1-p)(1-q)(1-q(1-\rho))\left(\alpha_j/\beta_j-\alpha_1/\beta_1\right)}{\beta_1(1-q)-\alpha_1(1-p)}.
\end{aligned}$$

By MLRP, η^j_{hl} and η^j_{ll} are positive. To see that η^j_{hh} is also positive, note that the numerator can be written as:

$$p\,(1-q)\,\frac{\alpha_j}{\beta_j}\left[1-\left(\frac{1-p}{p}\right)q\,(1-\rho)\right]-(1-p)\,q\frac{\alpha_1}{\beta_1}\,(\rho+(1-\rho)\,q)\,.$$

Now, $p > q$, $(1-q) > (1-p)$, and $\alpha_j/\beta_j > \alpha_1/\beta_1$. Also, it can be verified that $1-(1-p)\,q\,(1-\rho)\,/p > (\rho+(1-\rho)\,q)$ (note that this inequality can be simplified as $p > q$, which is true by assumption). Thus, all multipliers are positive. To ensure that these multipliers also satisfy ($16\ i$), one can scale all multipliers appropriately without violating any of the first-order conditions and the complementary slackness conditions.

Step 5. Next, we argue that if $(1-\theta)\,p < (1-\rho)\,q$, then the above contract is not optimal. This step involves the following sub-steps:

*Step 5a***:** Note that the initial contract was of the following form:

$$w_{hh}(y_1) - w_{hl}(y_1) = 0,\ w_{ll}(y_1) - w_{lh}(y_1) = 0,$$
$$\text{and } w_{hh}(y_1) - w_{ll}(y_1) = \frac{c}{\beta_1(1-q)-\alpha_1(1-p)}.$$

Consider the following variation $\{w'_{rs}(y_j)\}$ of the above contract: for $j > 1$, $w'_{rs}(y_j) = w_{rs}(y_j)$ and for $j = 1$,

$$w'_{hh}(y_1) = w_{hh}(y_1)\,,\ w'_{hl}(y_1) = w'_{hh}(y_1) - \Delta_h,\ w'_{ll}(y_1) = w'_{hh}(y_1) - \frac{c}{\beta_1(1-q)-\alpha_1(1-p)} + \Delta_{hl},$$
$$\text{and } w'_{lh}(y_1) = w'_{ll}(y_1) - \Delta_l.$$

So, in the new contract, $w'_{hh}(y_1) = w_{hh}(y_1)$, but

$$w'_{hh}(y_1) - w'_{hl}(y_1) = \Delta_h,\ w'_{ll}(y_1) - w'_{lh}(y_1) = \Delta_l,$$
$$\text{and } w_{hh}(y_1) - w_{ll}(y_1) = \frac{c}{\beta_1(1-q)-\alpha_1(1-p)} - \Delta_{hl}.$$

Note that such a contract changes the agent's payoff by

$$\alpha_1(1-p)(\Delta_{hl} - (1-\theta)p(\Delta_h + \Delta_l)). \tag{17}$$

We claim (as shown in the next step) that one can find values of Δ_h, Δ_l, and Δ_{hl} such that the term above is positive and all of the constraints remain satisfied. Note that if this is the case, then the firm can scale down all wages (w_{sr}) and, hence, also the aggregate wage pool w, and earn higher payoff. And, therefore, the initial contract is not optimal.

Step 5b: Note that (T_h) and (T_l) imply that

$$\Delta_l \in \left[\frac{(1-\theta)\,p}{\theta+(1-\theta)\,(1-p)}\Delta_h, \frac{\theta+(1-\theta)\,p}{(1-\theta)\,(1-p)}\Delta_h\right],$$

so we have Δ_h and Δ_l both positive. For the objective function to be positive, we must have Δ_{hl} positive, as well. In order to check the (IC)s, consider (IC_l) first. Note that (IC_l) can be written as

$$\begin{array}{l}\left[\sum_{j>1}\alpha_j\left[\sum_{r,s}P_{rs}w'_{rs}(y_j)\right]-\sum_{j\neq i}\beta_j\left[\sum_{r,s}P_{rs}w'_{rs}(y_j)\right]\right]+\\ \alpha_1P_{hh}w'_{hh}(y_1)+(\alpha_1P_{hl}-\beta_1q)\,w'_{hl}(y_1)+\alpha_1P_{lh}w'_{lh}(y_1)+(\alpha_1P_{ll}-\beta_1(1-q))\,w'_{ll}(y_1)\geq c.\end{array}$$

Now, using (i) the relationships between $w'_{rs}(y_1)$ values as given above and (ii) the fact that $\Delta_{hl} > (1-\theta)p(\Delta_h+\Delta_l)$ if there is strict increment in the agent's payoff, the above inequality boils down to:

$$\begin{array}{l}\beta_1 q\Delta_h-\alpha_1(1-p)(1-\theta)p(\Delta_h+\Delta_l)\\ \geq(\beta_1(1-q)-\alpha_1(1-p))\Delta_{hl}\\ >(\beta_1(1-q)-\alpha_1(1-p))(1-\theta)p(\Delta_h+\Delta_l)\end{array}, \tag{18}$$

or

$$q\frac{\Delta_h}{\Delta_h+\Delta_l} > (1-q)(1-\theta)p. \quad (IC'_l)$$

Similarly, the other (IC)s can be written as:

$$\begin{array}{rr}(1-q)\frac{\Delta_l}{\Delta_h+\Delta_l}>(1-q)(1-\theta)p, & (IC'_h)\\ (1-q)(1-\rho)q>(1-q)(1-\theta)p, & (IC'_t)\\ (\rho+(1-\rho)q)q\frac{\Delta_h}{\Delta_h+\Delta_l}+(\rho+(1-\rho)(1-q))(1-q)\frac{\Delta_l}{\Delta_h+\Delta_l}>(1-q)(1-\theta)p. & (IC'_r)\end{array}$$

Note that (IC'_t) is satisfied when

$$(1-\theta)p < (1-\rho)q.$$

Now suppose that the inequality above is satisfied. Note that if (IC'_l) and (IC'_h) are satisfied, (IC'_r) is automatically satisfied because $\rho+(1-\rho)q+\rho+(1-\rho)(1-q) = 1+\rho > 1$. Now choose Δ_h and Δ_l so that $\Delta_h/(\Delta_h+\Delta_l) = (1-q)$ – i.e., $\Delta_l = q\Delta_h/(1-q)$. To see that this is a feasible choice, note that

$$\frac{q}{1-q} < \frac{p}{1-p} < \frac{\theta+(1-\theta)\,p}{(1-\theta)\,(1-p)}.$$

In addition,

$$\frac{q}{1-q}-\frac{(1-\theta)\,p}{\theta+(1-\theta)\,(1-p)} = \frac{q-(1-\theta)p}{(1-q)(\theta+(1-\theta)\,(1-p))} > 0,$$

since $q > (1-\theta)p/(1-\rho)$. Finally, with $\Delta_h/(\Delta_h+\Delta_l) = (1-q)$, we have

$$q\frac{\Delta_h}{\Delta_h+\Delta_l} = (1-q)\frac{\Delta_l}{\Delta_h+\Delta_l} = (1-q)q > (1-q)(1-\theta)p.$$

Hence, all (IC')s are satisfied. Therefore, if $(1-\theta)\,p < (1-\rho)\,q$, the contract $\{w_{rs}(y_j)\}$ is not optimal.

Step 6. Next, we derive the optimal contract when $(1-\theta)\,p < (1-\rho)\,q$. This part of the proof, again, involves two sub-steps. First, we argue that at the optimum, we can set $w = w_{rs}(y_j) + w_{sr}(y_j)$ for all $j > 1$. Next, we derived the optimal contract by solving for the optimal values of Δs as given in Step 5 above.

Step 6a: We argue that at the optimum, we can set $w = w_{rs}(y_j) + w_{sr}(y_j)$ for all $j > 1$. This part of the proof is very similar to Step 1 in the proof of Proposition 3. Suppose this is not true for some $j > 1$, r, and s^*. Consider a new compensation scheme with

$$\begin{aligned} w'_{rs^*}(y_j) &= w_{rs^*}(y_j) + \varepsilon, \\ w'_{rs^*}(y_1) &= w_{rs^*}(y_1) - \tfrac{\alpha_j}{\alpha_1}\varepsilon, \end{aligned}$$

and $w'_{rs}(y_j) = w_{rs}(y_j)$ for all other j, r and $s \neq s^*$. Note that under the new compensation scheme, (i) (B) remains satisfied and (ii) (IR) remains unchanged. Now, to check (T), note that if the agent reports s^*, we have

$$\mathbb{E}_{\{y_j,r\}}\left[w_{rs^*}(y_j)\,|\mathbf{e} = \mathbf{1}\right] = \mathbb{E}_{\{y_j,r\}}\left[w'_{rs^*}(y_j)\,|\mathbf{e} = \mathbf{1}\right]$$

(by construction). If the agent reports $s' \neq s^*$,

$$\mathbb{E}_{\{y_j,r\}}\left[w_{rs'}(y_j)\,|\mathbf{e} = \mathbf{1}\right] = \mathbb{E}_{\{y_j,r\}}\left[w'_{rs'}(y_j)\,|\mathbf{e} = \mathbf{1}\right]$$

since $w'_{rs'}(y_j) = w_{rs'}(y_j)$ for all r and y_j. Finally, to check that (IC)s are satisfied, note that the left-hand side of all (IC)s do not change. So, we only need to consider the right-hand sides. Let $s^* = l$. Now, the right-hand side of (IC_h) does not change. So, (IC_h) is trivially satisfied. For (IC_l), the right-hand side becomes

$$\sum_{j=1}^{N}\beta_j\left[q w_{hh}(y_j) + (1-q)\,w_{lh}(y_j)\right] + \varepsilon\left[\frac{\beta_j}{\alpha_j} - \frac{\beta_1}{\alpha_1}\right] < \sum_{j=1}^{N}\beta_j\left[q w_{hh}(y_j) + (1-q)\,w_{lh}(y_j)\right]$$

(as $\beta_j/\alpha_j - \alpha_1/\beta_1 < 0$ by MLRP). Hence, (IC_l) is satisfied. Similarly, the right-hand sides of (IC_t) and (IC_r) boil down to

$$\sum_{j=1}^{N}\beta_j\sum_{r,s\in\{h,l\}} Q_{rs}w_{rs}(y_j) + (1-q)\,\varepsilon\left[\frac{\beta_j}{\alpha_j} - \frac{\beta_1}{\alpha_1}\right],$$

and

$$\sum_{j=1}^{N}\beta_j\left[Q_{hh}w_{hl}(y_j) + Q_{hl}w_{hh}(y_j) + Q_{lh}w_{ll}(y_j) + Q_{ll}w_{lh}(y_j)\right] + q\varepsilon\left[\frac{\beta_j}{\alpha_j} - \frac{\beta_1}{\alpha_1}\right].$$

As as $\beta_j/\alpha_j - \alpha_1/\beta_1 < 0$, both of these (IC)s are also satisfied. This shows that if $w > w_{rs}(y_j) + w_{sr}(y_j)$ some $j > 1$, r, and s, there always exists another contract that makes the inequality bind but gives the same payoff to the firm.

Step 6b. Now we can derive the optimal contract by choosing the values for the optimal deviations $(\Delta_h^*, \Delta_l^*, \Delta_{hl}^*)$ in the contract $w'_{rs}(y_1)$ that maximizes the agent's gain

$$\alpha_1(1-p)(\Delta_{hl}^* - (1-\theta)p(\Delta_h^* + \Delta_l^*)).$$

Note that if at the optimum, $\Delta_{hl}^* < \frac{c}{\beta_1(1-q)-\alpha_1(1-p)}$, we can then scale up $(\Delta_h^*,\ \Delta_l^*,\ \Delta_{hl}^*)$ by a common factor and get a larger gain for the agent. This implies that $\Delta_{hl}^* = \frac{c}{\beta_1(1-q)-\alpha_1(1-p)}$. In addition, (17) implies that the optimal contract minimizes $\Delta_h + \Delta_l$. Now, note that (IC'_l) and (IC'_h) can be written as (see equation (18)):

$$\begin{aligned} \beta_1 q\Delta_h^* - \alpha_1(1-p)(1-\theta)p(\Delta_h^* + \Delta_l^*) &\geq (\beta_1(1-q) - \alpha_1(1-p))\Delta_{hl}^*, \\ \beta_1(1-q)\,\Delta_h^* - \alpha_1(1-p)(1-\theta)p(\Delta_h^* + \Delta_l^*) &\geq (\beta_1(1-q) - \alpha_1(1-p))\Delta_{hl}^*. \end{aligned}$$

Using $\Delta_{hl}^* = \frac{c}{\beta_1(1-q)-\alpha_1(1-p)}$, we can rewrite the above as

$$\begin{aligned} \beta_1 q\Delta_h^* &\geq c + \alpha_1(1-p)(1-\theta)p(\Delta_h^* + \Delta_l^*), \\ \beta_1(1-q)\Delta_l^* &\geq c + \alpha_1(1-p)(1-\theta)p(\Delta_h^* + \Delta_l^*). \end{aligned}$$

Multiplying the first expression with $1-q$ and the second one with q, we get that

$$(\beta_1 q(1-q) - \alpha_1(1-p)p(1-\theta))(\Delta_h^* + \Delta_l^*) \geq c.$$

(Note that since $q > p(1-\theta)/(1-\rho)$ and $\beta_1 > \alpha_1$, the left-hand side is clearly positive.) So, we need to find values for Δ_h^* and Δ_l^* such that the above holds with equality and the (T) and (IC') constraints are satisfied. This is obtained by setting

$$\begin{aligned} \Delta_h^* &= \tfrac{(1-q)c}{\beta_1 q(1-q)-\alpha_1(1-p)p(1-\theta)}, \\ \Delta_l^* &= \tfrac{qc}{\beta_1 q(1-q)-\alpha_1(1-p)p(1-\theta)}. \end{aligned}$$

Hence the proof. ■

Proof of Proposition 5. This proof closely follows the proof of Proposition 3. Hence, in what follows, we present a brief outline of the proof but highlight the part where the current proof differs from the proof of Proposition 3.

Step 1. First, note that steps 1 to 3 of Proposition 3 continue to hold in this setting.

Step 2. Now, suppose $(1-\theta)\,p > (1-\rho)\,q$. By Proposition 4, the optimal contract in the static game is the same as the one derived under independent private signals. Also, condition (6) is identical to condition (4) (note that p_1 defined in the context of independent-signal structure is equal to $1-p$ as defined in the case of correlated signals; similarly, $q_1 = 1-q$). Thus, the optimal contract derived in the independent-signal case continues to be the optimal in this setting.

Step 3. Finally, consider the case in which $(1-\theta)\,p \leq (1-\rho)\,q$. Using the same logic as presented in Step 4 of Proposition 3, we argue that if $\delta \geq \delta^*$, then subjective pay is never used and effort is induced only by relational contract (the optimal contract is the same as defined in Step 4 of Proposition 3).

If $\delta < \delta^*$, the proof closely follows Step 5 of Proposition 3. Following this step and using Proposition 4, we conclude the following: If

$$\frac{\beta_1\,(1-q)}{\alpha_1\,(1-p)}\frac{q}{(1-\theta)\,p} > \frac{v + c\,(b)}{v - c\,(b)}, \tag{19}$$

then the firm uses the following subjective contract in equilibrium: There exists some $w = w_c$ and

$$w_{rs}\,(y_j) = \begin{cases} \frac{1}{2}w_c - \Delta_l & if\ j=1\ and\ (r,s)=(l,h) \\ \frac{1}{2}w_c - \Delta_h & if\ j=1\ and\ (r,s)=(h,l) \\ \frac{1}{2}w_c & otherwise, \end{cases}.$$

and the expected surplus destruction in this case is

$$z_c\,(b) = \frac{2\alpha_1(1-\theta)p(1-p)c\,(b)}{\beta_1 q(1-q) - \alpha_1(1-p)p(1-\theta)}.$$

Otherwise, no subjective pay is used. This solves for the first step.

In the second step, for (DE) to be satisfied, we need

$$\frac{\delta}{1-\delta}\left[v - c - z_c\,(b)\right] \geq 2b. \tag{20}$$

That is, we need to find the largest b for which the above condition (20) holds. Note that at $b = 0$, the left-hand side is

$$\frac{\delta}{1-\delta}\left[v - c - z_c\,(0)\right] = \frac{\delta}{1-\delta}\left[v - c - \hat{z}\right].$$

Here we have two cases.

Case 1: If (7) is violated (i.e., effort cannot be induced in a Nash equilibrium of the stage game), $v - c - \hat{z} \leq 0$. Also, at $b = c/(\sum_{j=k}^{N}(\alpha_j - \beta_j))$, condition (20) is violated (recall that we are considering the case in which (12) fails). Thus, there is no value of b that satisfies (20). So, $b = 0$. Also, if (7) is violated, then (19) is also violated as $c(0) = c$. Thus, no subjective pay is used when $b = 0$.

Case 2: If (7) holds (i.e., effort can be induced in a Nash equilibrium of the stage-game), $v - c - \hat{z} > 0$. But, as noted above, at $b = c/(\sum_{j=k}^{N}(\alpha_j - \beta_j))$, the condition (20) is violated. Since both sides of condition (20) are linear in b, this implies that the highest b that satisfies (20), $b_c^{**}(\delta)$ (say), is the one at which (20) holds with equality. This observation completes the proof. ■

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