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The Public Market Equivalent and Private Equity Performance

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Morten Sorensen Ravi Jagannathan

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Abstract

We provide a formal and rigorous justification for the Kaplan and Schoar (2005) public market equivalent (“PME”) measure of historical performance of private equity (“PE”) funds. The PME provides a valid economic performance measure when the investor (“LP”) has log-utility preferences and the return on the LP’s total wealth equals the market return. Notably, the PME is valid regardless of the risk of PE investments, and it is robust to variations in the timing and systematic risks of the underlying cash flows along with potential GP manipulations. Formally, we show that the PME is a generalized method-of-moments estimator using the stochastic discount factor implied by Rubinstein (1976)’s version of the capital asset pricing model.

Keywords: Private equity, public market equivalent, PME, log utility CAPM, ex-post performance evaluation, generalized method of moments.

JEL Classification: G12, G23, G24, G31.

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One complication that arises when evaluating the performance of private equity ("PE") funds (such as buyout or venture capital funds) is that these funds generate streams of cash flows but don’t have regularly quoted financial returns. Such funds predominantly own privately-held companies, without quoted market valuations, and hence they have no regularly quoted month-to-month or quarter-to-quarter market returns. This raises the general problem of evaluating performance based on cash flows alone. A natural starting point is to calculate the internal rate of return ("IRR"). The IRR, however, has several well-known problems: It is an absolute performance measure that does not adjust for the market return or the risk of the investments. It implicitly assumes investors can reinvest cash flows at the same rate. The IRR may not exist, and it may not be unique. Moreover, investors in PE funds ("LPs") have expressed concerns that PE funds can manipulate IRRs by deliberately choosing the timing and size of their investments. Consequently, in their evaluation of the Kauffman Foundation’s venture capital investments, Mulcahy, Weeks, and Bradley (2012) recommend that LPs “Reject performance marketing narratives that anchor on internal rate of return (IRR), top quartile, vintage year, or gross returns,”¹ and instead “Adopt PME as a consistent standard for VC performance reporting.”

The PME (or “Public Market Equivalent”) is an alternative performance measure, introduced by Kaplan and Schoar (2005).² This measure provides a way to evaluate performance based on cash flows alone. The PME calculation goes as follows. Let \( X(t) \) denote the cash

¹An earlier evaluation of IRRs, by PEI (2002), can be summarized succinctly by their quote from a fund manager that “you can’t eat IRRs.”

²An earlier but slightly different measure that was also originally called the public market equivalent, but later became known as the ACG Index Comparison Method, was introduced by Long and Nickels (1996). Long (2008) discusses various versions of these measures and their relationships.
flow from the fund to the LP at time $t$. This total cash-flow stream is divided into its positive and negative parts, called distributions ($\text{dist}(t)$) and capital calls ($\text{call}(t)$). Distributions are the cash flows returned to the LP from the PE fund (net of fees) when the fund successfully sells a company. Capital calls are the LP’s investments into the fund, including the payment of ongoing management fees. The distributions and capital calls are then valued by discounting them using the realized market returns over the same time period, and the PME is the ratio of the two resulting values:

$$PME = \frac{\sum_t \text{dist}(t)}{\sum_t \text{call}(t)} .$$

(1)

In this expression, $R_M(t)$ is the realized market return from the inception of the fund (at $t = 0$) to the time of the distribution or capital call. A PME greater than one is interpreted as the value of the distributions exceeding the value of the capital calls, which in turn suggests that the LP has benefitted from the investment in the fund.

From a standard valuation perspective, the PME calculation looks odd. The standard capital asset pricing model (“standard CAPM”) prescribes that the present value should be calculated by discounting the expected cash flow using the discount rate:

$$R_{CAPM} = R_F + \beta (E[R_M] - R_F) .$$

(2)

Here, $R_F$ is the risk-free rate, beta measures the cash flow’s systematic risk, and $E[R_M] - R_F$ is the expected market risk premium. The present value (“PV”) is then given as:

$$PV = \sum_t \frac{E[X(t)]}{(1 + R_{CAPM})^t} .$$

(3)
This standard CAPM valuation is the well-known textbook approach to evaluating potential investments ex-ante, because the systematic risk (beta) can be estimated from the market returns of comparable publicly-traded companies. When evaluating performance ex-post, after the actual cash flows from the investment are known, the expected cash flows are replaced by their realized values and averaged across several time periods or projects to estimate the average performance. Investments that earn a higher return than the cost of capital, $R_{CAPM}$, have a positive PV and those that earn a lower return have a negative PV.

Compared to the standard CAPM valuation in equation (3), the PME calculation in equation (1) appears to be quite different. The standard CAPM has no role for the realized market return, which is used as the discount rate in the PME calculation. Moreover, the PME calculation seems to be missing the beta that adjusts for the systematic risk of the cash flows in the standard CAPM.

One informal justification of the PME is that it gives the value to an LP if capital calls had instead been invested in the market portfolio and distributions are reinvested in the market. Another justification is that if the beta equals one, then the standard CAPM discount rate equals the expected market return. By replacing the expected market return by the realized return, the PME calculation begins to look like the standard CAPM. This intuition led Kaplan and Schoar (2005) to argue that the PME is valid when the beta equals one. Obviously, the beta of a PE fund’s cash flows may not be one, particularly for buyout funds, which invest in highly levered transactions. In response, some have calculated

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Based on this intuition, Rouvinez (2003) develops an informal PME+ version of the measure to account for portfolio constraints arising from this trading and reinvesting.
“tailored PMEs” that adjust the index returns used as discount rates to better reflect the specific risks of PE investments.\(^4\)

In contrast, we provide a formal theoretical justification for the PME measure, showing that it is a surprisingly general and robust performance measure. The PME measure does not require a beta of one. Without any adjustments, it can be justified under three assumptions, neither of which is particularly controversial nor restrictive: (1) markets are frictionless and the “law-of-one-price” holds; (2) the LP has log utility (or more generally, Kreps-Porteus (1978) preferences with relative risk aversion of one); and (3) the return on the LP’s total wealth portfolio equals the return on the public market. Importantly, the PME is a valid performance measure regardless of the risk of PE investments, even if this risk differs for capital calls and distributions or changes over time.

1 Stochastic Discount Factors

Before we analyze the PME, it is useful to review the concept of stochastic discount factors (“SDFs”). This material is standard, and the details can be found in any graduate-level asset-pricing textbook. Cochrane (2001) and Skiadas (2009) provide excellent discussions of SDFs.

Starting at time \(t = 0\), we can value a risky future cash-flow stream as follows. At each point in time, \(t\), a state of the world is realized, denoted \(s_t\), with probability \(p(s_t)\) (as

\(^4\)For example, Ljungqvist and Richardson (2003) defines a “Profitability Index” that discounts outflows using the risk-free rate and inflows at the S&P500 return. Phalippou and Gottschalg (2009) calculate a tailored version using indices for companies matched on industry and size. Robinson and Sensoy (2012) construct beta-adjusted PMEs to account for the higher systematic risk.
assessed by the investor). The market return varies across states, and the return on the market earned from time 0 to time \( t \), in state \( s_t \), is denoted \( R_M(s_t) \). The stochastic discount factor (“SDF”), denoted \( m(s_t) \), then gives the investor’s initial valuation (at time 0) of a future cash flow of one dollar at time \( t \) in state \( s_t \) (scaled by the probability). Using this basic building block, an investor can value any risky cash-flow stream, and the value of the cash flow \( X(s_t) \) is:

\[
PV = \sum_{t} \left[ \sum_{s_t} p(s_t)m(s_t)X(s_t) \right] = \sum_{t} E[m(s_t)X(s_t)] .
\] (4)

In this expression, \( m(s_t) \) is the discount factor for discounting the cash flows that occur at time \( t \) when the state is \( s_t \). Since the state \( s_t \) occurs with probability \( p(s_t) \), the investor’s present value of the cash flow \( X(s_t) \), in state \( s_t \), equals \( p(s_t)m(s_t)X(s_t) \).

The SDF is the fundamental building block of valuation models, and a valuation model is uniquely defined by its SDF. For any set of prices or valuations that are internally consistent (formally, the “law-of-one-price” holds), there exists a SDF that generates these valuations.

For risk-averse investors, cash flows that are higher when the market is down are more valuable, because they insure the investor against market downturns. Since the SDF specifies the value of cash flows across states, the SDF should be larger in those states where the market return is poor. Otherwise, \( m(s_t) \) is unrestricted, and without further assumptions or simplifications the SDF is difficult to determine in practice.

Most valuation models have two representations: (a) the commonly used linear beta form, where the expected excess return on an asset is a linear function of the asset’s factor betas; and (b) an equivalent SDF form. For example, the SDF representation of the standard
CAPM is:

\[ m(s_t) = a - b \left[ 1 + R_M(s_t) \right], \quad (5) \]

where \( a \) and \( b \) are constants that reflect the risk-free rate, the market risk, and the market price of risk (see Dybvig and Ingersoll (1982) and Hansen and Richard (1987)). With this SDF, the risk of any investment can be summarized by its correlation with the market, as measured by its \( \beta \). This standard CAPM, and the beta measure of systematic risk, forms the foundation of most valuations done in practice.

### 1.1 Rubinstein CAPM

Rubinstein (1976) derived an alternative capital asset pricing model (“R-CAPM”). While the standard CAPM is inherently a one-period static model, the R-CAPM is set in a dynamic model environment. Suppose that markets are frictionless, and that the investor is infinitely lived and chooses lifetime consumption and investment plans, subject to budget constraints, to maximize the lifetime expected utility as represented by the time-separable logarithmic (log) utility function:

\[ U(c(s_t)) = \sum_t \beta^t E \left[ \ln (c(s_t)) \right]. \quad (6) \]

With log-utility preferences, the investor’s SDF is \( m(s_t) = [c(s_t)/c(0)]^{-1} \), where \( c(s_t)/c(0) \) is the investor’s consumption growth from time 0 to time \( t \) in state \( s_t \). With log-utility, the investor consumes a constant fraction of total wealth, and \( c(s_t)/c(0) = 1 + R_W(s_t) \), where \( R_W(s_t) \) is the return on the investor’s total wealth. Assuming that \( R_W = R_M \), it follows
that:

\[ m(s_t) = \frac{1}{1 + R_M(s_t)}. \]  

(7)

This (non-linear) SDF is different from the SDF that defines the standard CAPM, so it implies a different valuation model. Under the Rubinstein CAPM, equation (4) implies that a cash-flow stream is valued by discounting it with the realized market return. The PV of \( X(s_t) \) is:

\[ PV = \sum_t E \left[ \frac{X(s_t)}{1 + R_M(s_t)} \right]. \]  

(8)

In both the Rubinstein and the standard CAPM, investors the only priced risk is systematic risk, which arises from the covariance between the cash flows and the market return, but the two models adjust for systematic risk in different ways. Under the standard CAPM, the systematic risk is measured by the \( \beta \) and a higher beta results in a greater discount rate and a lower present value. Under the Rubinstein CAPM, the systematic risk is captured by discounting with realized market returns. A cash-flow stream that pays off when the market return is low is discounted less, and it has a higher present value, as it should have. Formally, the PV of a one-period cash flow under the Rubinstein model is \( E[X/(1 + R_M)] \) and it equals \( E[X] \times E[1/(1 + R_M)] + COV[X, 1/(1 + R_M)] \). The last term is the covariance term, and by using realized return, the valuation implicitly incorporates this covariance term into the valuation.

A few comments remain. First, the derivation of the Rubinstein model assumes \( R_W \equiv R_M \). One justification for this assumption is that when markets are complete and when all investors have the same beliefs and preferences with constant relative risk aversion of
one, then $R_W = R_M$ in equilibrium. More generally, without complete markets and with heterogeneous agents, the equality $R_M = R_W$ may fail, and an investor with log-utility should then discount the cash flows using $R_W$ instead of $R_M$.

Second, with incomplete markets and heterogeneous beliefs, the valuation is the LP’s valuation of the PE fund, at the margin, using the state prices that are implied by the public market. A substantial investment in the PE fund, which is not part of the public market, may affect the state prices and the valuation of the fund.

As a third comment, the assumption that the LP has time-separable log-utility preferences can be relaxed. As long as the LP’s preferences belong to the scale-invariant Kreps-Porteus (1978) class with relative risk aversion of one, the SDF is $m(s_t) = 1/(1 + R_M(s_t))$, and the analysis holds (see Skiadas 2009 and Kim 2013). Log utility is one example of such preferences, along with the more general preferences from Epstein and Zin (1989) and Albuquerque, Rui, Eichenbaum, and Rebelo (2013).

2 Numerical Example

A numerical example helps illustrate the differences between the two models. The market structure is illustrated by the binomial tree in Panel A of Table 1. In each period (or year), with a 50% probability, the market either appreciates by 20% or depreciates by 10%. In the standard CAPM the risk-free rate and market risk premium are free parameters (determining the $a$ and $b$ in the SDF), but in the Rubinstein CAPM they are determined by the market structure. The implied risk-free rate is $R_F = 2.86\%$ and the market risk
premium is $E[R_M] - R_F = 2.14\%$.\(^5\) We use this risk-free rate and market risk premium in the standard CAPM as well, to make the valuations comparable.

Panel B of Table 1 shows the cash flows from the PE investment. Initially, at time 0, an LP invests $1,000 into the PE fund. After the first period, where the PE fund matures, the fund returns nothing. After the second period, depending on the market’s performance over the initial two periods, the PE fund returns either $405, $585, or $845. After the third and final period, again depending on the market’s performance over the fund’s entire life, the PE fund returns either $365, $527, $761, or $1,099. We ignore fees, but one can interpret these cash flows as the net-of-fees cash flows between the LP and the fund.\(^6\)

After observing several such funds, valuing the cash flow using the Rubinstein CAPM is straightforward. Each observed cash flow is discounted by the realized market return and the average for each period is in Table 2. The resulting PV is $107.68. No knowledge of beta or any other parameters is required for this valuation.

In contrast, the standard CAPM is more complicated. First, determine the cash flow’s beta. Below, we show that the beta in this example is 1.29, but in practice this beta must be determined by some other statistical analysis, typically involving publicly-traded comparable companies, when available. Second, determine the average cash flows in each period, as reported in Table 2. This second step is analogous to the calculation for the Rubinstein CAPM, except the cash flows are not discounted. Third, the CAPM discount

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\(^5\)The risk-free rate is calculated using $\frac{1}{1 + R_F} = E[R_M]$, which holds for SDFs in general.

\(^6\)These cash flows are constructed by assuming that the PE fund’s underlying valuation increases by 30% when the market appreciates and decreases by 10% when the market depreciates; and half the PE fund’s value is return after the second period and the remainder is returned at the end. But this underlying structure is unimportant for valuing the resulting cash flow.
rate is calculated using the beta and the market risk premium. In this example, \( R_{CAPM} = R_F + \beta \times (E[R_M] - R_F) = 2.86\% + 1.29 \times 2.14\% = 5.64\% \). Finally, the expected cash flow is discounted using the standard CAPM discount rate, and the resulting PV is $106.99. Despite the simplicity of the Rubinstein CAPM, in this example, it gives almost exactly the same valuation as the standard CAPM.

The above calculation illustrates how the valuation is done in practice. A more theoretical approach is to value the cash flows using the SDFs. The two SDFs are shown in Table 3. The SDF for the Rubinstein model is easy. It is \( m = 1/(1 + R_M) \). The SDF for the standard CAPM is more difficult. Remember, that the standard CAPM specifies \( m = a - b(1 + R_M) \), where \( a \) and \( b \) are determined by the risk-free rate and the market risk premium. In each periods, we solve for \( a \) and \( b \), and we find \( a = 1.944 \) and \( b = 0.926 \) in the first period. In the second period, \( a = 1.871 \) and \( b = 0.840 \), and in the third period, \( a = 1.801 \) and \( b = 0.762 \). The resulting SDF is in Table 3. The two resulting SDFs are fairly similar, and the greatest differences arise at the extreme market returns, with lower probabilities, in the third period.

In each period, we multiply the cash flow with the corresponding SDF and average the resulting values. The resulting valuations are numerically identical to those reported in Table 2, and we arrive at the exact same PVs of $107.68 and $106.99. In fact, we determined the beta of 1.29 by first valuing it using the SDF for the standard CAPM, and then solving for the implied beta, so the two valuations using the standard CAPM are identical by construction.

\[ \text{The } a \text{ and } b \text{ are calculated as follows. In each period, the following the equations for all SDFs, } 1 = (1 + R_f)E[m] \text{ and } 1 = E[mR_M]. \text{ Substituting the linear expression for } m \text{ into these two equations results in two equations in two unknowns, } a \text{ and } b. \text{ These equations are then solved by inverting a two-by-two matrix. Details are available from the authors upon request.} \]
In practice, however, the beta must determined statistically, and when beta is uncertain the two calculations using the standard CAPM may differ.

3 Public Market Equivalent

After presenting the properties of the Rubinstein CAPM, we can related this method to the PME calculation for the cash flows from a PE fund. With a sufficient number of observed deals for each PE fund, the expectation in equation (8) can be estimated using a generalized method of moments (“GMM”) estimator that replaces the expected values with the realized cash flows.\(^8\) Specifically, let the fund’s deals be indexed \(i = 1, 2, ..., N\), and separate the cash flows from each deal into the capital call, \(call_i(t_i)\), and the subsequent distribution, \(dist_i(t_i)\).\(^9\)

Let \(DIST\) and \(CALL\) denote the numerator and denominator in the PME definition, so \(DIST\) is:

\[
DIST = \sum_t \frac{dist(t)}{1 + R_M(t)} = \sum_i \frac{dist_i(t_i)}{1 + R_M(t_i)},
\]

where \(dist(t)\) equals the sum of the distributions at time \(t\) across deals. For a sufficiently large \(N\) (and under suitable regularity conditions ensuring the law of large numbers), the average of the actual distributions converges to their expected value:

\[
\frac{1}{N} DIST \sim E \left[ \frac{dist(s_i)}{1 + R_M(s_i)} \right] = PV_{dist}. \tag{10}
\]

\(^8\)We refer the interested reader to Driessen et al. (2012) for another interesting GMM approach to PE valuation.

\(^9\)To simplify the notation, we assume that each deal has only a single capital call and distribution. The argument holds with an arbitrary number of cash flows, but the notation becomes more complicated. Specifically, let \(i(c)\), for \(c = 1, ..., C(i)\) index the individual cash flows associated with deal \(i\). Replacing \(\sum_i \frac{dist_i(t_i)}{1 + R_M(t_i)}\) with \(\sum_i \left( \sum_c \frac{dist_i(c)(t_i)}{1 + R_M(t_i)} \right)\) and noting that \(E \left[ \left( \sum_c \frac{dist_i(c)(t_i)}{1 + R_M(t_i)} \right) \right] = PV_{dist}\) generalizes the derivation to an arbitrary number of cash flows for each deal.
A similar argument holds for the capital calls, and after canceling the $1/N$ terms, the PME equals:

$$PME = \frac{DIST}{CALL} \sim \frac{PV_{dist}}{PV_{call}}.$$  \hspace{1cm} (11)

The asymptotic sampling distribution of the PME can be computed with suitable further assumptions.

Hence, under the assumptions outlined above, the Kaplan and Schoar (2005) PME measure is a consistent estimate of the value of the distributions divided by the value of the capital calls. A PME greater than one then implies that the investment in the PE fund has been valuable for the LP.

4 Conclusion

To clarify the strengths and limitations of the PME measure for evaluating PE fund performance, we provide a formal axiomatic justification for the use of this PME measure. Under our assumptions, the PME is a valid performance measure regardless of the underlying risks. It does not require different treatments of cash flows associated with capital calls and distributions, despite their different properties. The PME does not require the investor to follow any particular trading strategy, such as reinvesting distributions dollar for dollar into the market portfolio. It is robust to potential manipulation of the timing of the cash flows. As long as the market proxy is reasonable, the risk of the cash flows can vary over time and across projects. Indeed, an advantage of the PME and Rubinstein CAPM for evaluating ex-post performance is that it does not require any knowledge of the betas or other pa-
rameters. The Rubinstein CAPM may also provide a more general and robust approach to evaluating ex-post performance than the standard CAPM in other situations where these benefits are important, such as when evaluating hedge fund or mutual fund performance. It seems particularly suited for valuing the irregular cash flows of PE investments.

Our analysis clarifies the difference between two fundamentally different ways to assess risk-adjusted performance. In the standard CAPM, expected cash flows are discounted with expected market returns, and the risk adjustment is performed by scaling this expected market return with a beta, which reflects the covariance between the returns on the investment and the market. Under Rubinstein (1976)’s valuation model, the risk-adjusted performance is calculated by discounting realized cash flows with realized market returns. By using realized returns, this calculation implicitly captures the covariance between the investment and the market, because $E[X/(1+R_M)] = E[X] 	imes E[1/(1+R_M)] + COV[X, 1/(1+R_M)]$. Under logarithmic preferences, the last covariance term is exactly the value of the risk adjustment. These two valuation approaches are difficult to reconcile, though, and attempts to evaluate PE performance by discounting realized cash flows with beta times realized market returns (i.e., by assuming $PV = E[X/(1 + R_F + \beta(R_M - R_F))]$) appear to “double count” the risk adjustment. We are unaware of any assumptions or SDF that justifies this calculation, and in our numerical example “double counting” gives a “PV” of only $17.01$, which is well below the other valuations. A more general caution is that actual performance measures often employ various “intuitive” adjustments for risk, illiquidity, portfolio constraints, timing, etc. without formally deriving these adjustments from a valuation model. As this analysis shows,
intuition alone is a poor guide for risk adjustments.

Our point is not that the standard CAPM should be discarded entirely in favor of the Rubinstein model. Each model has advantages and disadvantages. For ex-ante valuation, a disadvantage of the Rubinstein model is that it is difficult to project future correlations between cash flows and the market. Instead, betas can be estimated from the market returns of comparable publicly-traded companies. In contrast, ex-post, when evaluating past performance, these correlations can be assessed using their sample analogues, and an advantage of the Rubinstein model is then that it works for all cash flows, regardless of whether they are risky or safe, volatile or predictable.\(^\text{10}\)

Finally, while our analysis provides a formal foundation for the PME measure, it also opens some avenues for improving this measure. The statistical precision of the PME measure may be improved by optimally weighing individual observations, using standard methods for GMM estimators. When the LP’s coefficient of relative risk aversion differs from one, the PME may need some adjustment to reflect these preferences. If the return on the LP’s total wealth differs from the market return (e.g., when the LP is not fully invested in the market), it may be more appropriate to use a discount rate derived from the return on the LP’s total wealth. Finally, for LPs with larger PE allocations, the incomplete markets and illiquidity

\(^\text{10}\)Another issue is the empirical performance of these models for general valuation. Both have some problems. The Rubinstein model has difficulty reconciling the average market return with the risk-free rate, and given actual market returns over 1926–2012, this model implies a risk-free rate of 7.38%, which exceeds the actual average risk-free rate of 3.28%. Conversely, well-known problems with the standard CAPM are that it implies negative state prices for high market returns, which is unreasonable, and that actual returns do not appear to increase linearly in beta, as predicted by this model. As a consequence, a portfolio involving fairly priced written call options will have a spurious positive alpha when valued using the standard CAPM, as shown by Jagannathan and Korajczyk (1986). In contrast, the Rubinstein CAPM will value payoffs that exhibit option like features. When the return on the market has a log normal distribution, the Rubinstein CAPM results in the Black-Scholes formula.
of PE investments may introduce other distortions in the valuation of these investments (see Sorensen, Wang, and Yang 2013).
References


Mulcahy, Diane, Bill Weeks, and Harold Bradley (2012) “We Have Met the Enemy ... And He is Us,” *working paper: Kauffman Foundation*.  


TABLE 1: Market returns and cash flows from PE fund. Each panel shows a binomial tree, starting at time zero and spanning three periods. At each period, each of the two branches has a 50% probability. Panel A shows the market return at each node of the three. Panel B shows the private equity cash flows at each node.

Panel A: Market Return ($1+R_M$)

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<td></td>
<td>173%</td>
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<td>120%</td>
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Panel B: Private Equity Cash Flow

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<td>$761</td>
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<td></td>
<td>-$1,000</td>
<td>$585</td>
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<td>$527</td>
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<td>$405</td>
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**TABLE 2: Present Values.** The table shows expected and present values ("PVs") of the private equity cash flows for each of the three periods along with the total value of the entire cash flow stream.

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<th>2</th>
<th>3</th>
<th>PV</th>
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<td>$0</td>
<td>$542.53</td>
<td>$565.14</td>
<td>$107.68</td>
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TABLE 3: Stochastic Discount Factors (SDFs). The two panels show binomial trees that correspond to the binomial trees for the market return and cash flows in Table 1. Panel A shows the SDF under the standard CAPM, calculated as $m = a - b R_M$, where $a$ and $b$ are constant given in the text. Panel B shows the SDF under the Rubinstein CAPM, calculated as $m = 1/(1+R_M)$.

**Panel A: SDF for standard CAPM**

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**Panel B: SDF for Rubinstein CAPM**

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