Markets, Models, and Crises

Donald MacKenzie
1. Performativity
2. Black-Scholes-Merton option pricing theory
3. Performativity, counter-performativity and the credit crisis
Part 1: Performativity

*Performative utterance* (J.L. Austin):

‘I apologize’

‘I name this ship the Queen Elizabeth’
“generic” performativity: an aspect of economics (a theory, model, concept, procedure, data-set etc.) is used by participants in economic processes, regulators, etc.

“effective” performativity: the practical use of an aspect of economics has an effect on economic processes

“Barnesian” performativity: practical use of an aspect of economics makes economic processes more like their depiction by the aspect of economics in question

“counterperformativity”: practical use of an aspect of economics makes economic processes less like their depiction by the aspect of economics in question

A possible classification of the performativity of economics. The depicted sizes of the subsets are arbitrary: I have not attempted to estimate the prevalence of the different forms of performativity.
Part 2: Black-Scholes-Merton Option Pricing Theory

**Option** A financial derivative that gives a right but not an obligation, e.g. right:

- to buy 100 shares of ABC corporation at an ‘exercise price’ of $50 a share (‘call option’)
- to sell 100 shares of XYZ corporation at an exercise price of $30 a share (‘put option’)

**HOW DO MARKET PROCESSES DETERMINE PRICE OF OPTIONS?**
Black, Scholes, Merton: 1970-73. Given certain assumptions (price of underlying asset follows ‘lognormal’ random walk, no transaction costs, etc.), an option can be hedged perfectly by ‘replicating portfolio’: continuously-adjusted position in underlying asset and borrowing/lending cash.

A position consisting of an option hedged with ‘replicating portfolio’ is thus riskless. Must earn exactly the riskless rate of interest. If not, opportunity for arbitrage: for making profit with no risk and no net outlay.

Lognormal random walk: changes in natural logarithms of price of asset normally distributed.
Black-Scholes equation (1969-70):

\[
\frac{\partial w}{\partial t} = rw - rx \frac{\partial w}{\partial x} - \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 w}{\partial x^2}
\]

\(w\) is option price, \(x\) stock price, \(\sigma\) volatility of stock, \(r\) riskless rate of interest, \(t\) time. Stock pays no dividends & option ‘European’ (can be exercised only at expiry). Characteristics of option give boundary condition (at expiry, \(w\) is a known function of \(x\)).

**Volatility:** the extent of the fluctuations of the price of an asset (annualized standard deviation of continuously-compounded returns on the asset).

**Riskless rate:** the rate of interest paid by a borrower who creditors are certain will not default.
Solution (1970) for European call option, which gives the right to buy stock at ‘exercise price’ \( c \) at time \( t^* \):

\[
w = xN\left[ \frac{\ln(x/c) + (r + 1/2 \sigma^2)(t^* - t)}{\sigma \sqrt{t^* - t}} \right]
\]

\[
-c[\exp\{r(t - t^*)\}]N\left[ \frac{\ln(x/c) + (r - 1/2 \sigma^2)(t^* - t)}{\sigma \sqrt{t^* - t}} \right]
\]

where \( N \) is the (cumulative) distribution function of a normal distribution, and \( \ln \) indicates natural logarithm.
Option trading on Chicago Board Options Exchange (established 1973) and, later, Chicago Mercantile Exchange:

Bodies and theorems
\[ w = x \mathcal{N}\left[ \frac{\ln(x/c) + (r + \frac{1}{2} \sigma^2)(t^*-t)}{\sigma \sqrt{t^*-t}} \right] \]

\[ -c[\exp\{r(t-t^*)\}] \mathcal{N}\left[ \frac{\ln(x/c) + (r - \frac{1}{2} \sigma^2)(t^*-t)}{\sigma \sqrt{t^*-t}} \right] \]
### Example of Black's Sheets

<table>
<thead>
<tr>
<th>Strike Prices</th>
<th>Stock Prices</th>
<th>Delta ( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>6.28, 6.66, 7.22</td>
<td>0.22</td>
</tr>
<tr>
<td>45</td>
<td>2.63, 3.69, 5.26</td>
<td>0.34</td>
</tr>
<tr>
<td>50</td>
<td>1.67, 3.13, 5.26</td>
<td>0.33</td>
</tr>
<tr>
<td>55</td>
<td>0.68, 0.46, 1.23</td>
<td>0.03</td>
</tr>
<tr>
<td>60</td>
<td>0.00, 0.00, 0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Expiry Dates, Riskless Interest Rates, Volatility, Dividends

<table>
<thead>
<tr>
<th>Expiry</th>
<th>Riskless Interest Rates</th>
<th>Volatility</th>
<th>Dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 76</td>
<td>5.65%</td>
<td>5.19%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Oct 76</td>
<td>6.23%</td>
<td>5.09%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Jan 77</td>
<td>6.63%</td>
<td>4.99%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

### United States Steel Corp

- **Strike Prices:**
  - June 09, 76
  - July 09, 76
  - Oct 09, 76
  - Jan 09, 76

- **Stock Prices:**
  - 6.28, 6.66, 7.22
  - 2.63, 3.69, 5.26
  - 1.67, 3.13, 5.26
  - 0.68, 0.46, 1.23
  - 0.00, 0.00, 0.00

**Delta:** \( \Delta \) = \( \frac{\Delta s}{\Delta o} \)

**Option Prices:**

- **Black-Scholes Options Prices:**
  - Number of shares that need purchased to hedge each call option sold.
Traders embrace Black-Scholes-Merton because of:

• authority of financial economics (?)
• public availability (qv PC vs Apple)
• cognitive simplicity (only one free parameter: volatility)
Black’s sheets and other implementations of the Black-Scholes-Merton model used:

1) (sometimes) to set option prices

2) to identify overpriced options to sell or (sometimes) underpriced options to buy

3) as a guide to hedging (‘delta’)

4) in ‘spreading’: simultaneous purchase of ‘underpriced’ option and sale of ‘overpriced’ option on same stock.
From 1973, option prices fell towards Black-Scholes levels, but this not a simple performative effect of use of model: ‘Chicago-style’ competitive market at least equally important.

However, option theory provided options trading with legitimacy:

‘Black-Scholes was really what enabled the exchange to thrive. ... [I]t gave a lot of legitimacy to the whole notion of hedging and efficient pricing, whereas we were faced, in the late 60s-early 70s with the issue of gambling. That issue fell away, and I think Black-Scholes made it fall away. It wasn’t speculation or gambling, it was efficient pricing. I think the SEC [Securities and Exchange Commission] very quickly thought of options as a useful mechanism in the securities markets and it’s probably - that’s my judgement - the effects of Black-Scholes. I never heard the word “gambling” again in relation to options’ [interview: Burton R. Rissman, former counsel, Chicago Board Options Exchange].
Black-Scholes-Merton and similar models also facilitating trading and risk management by making it easier to talk about options.

‘Implied volatility’: the volatility of the underlying asset consistent with price of option on the asset. Reduces plethora of contracts to metric that is (a) simple and (b) meaningful. E.g. implied volatility of S&P 500 options: 15% or less: ‘normal’ conditions. 30%: serious unease 40% or more: crisis.
If the Black-Scholes-Merton model is correct, the implied volatility of all options on the same stock with the same time to expiration should be the same, so the graph of implied volatility against strike price should be a flat line. Rubinstein used this as a test of the empirical validity of the model. “Spreaders” used it as a way of profiting from price discrepancies. They used the model to identity relatively cheap options to buy (such as point A on the graph) and, simultaneously, relatively expensive options to sell (point B). Such trading could be expected to have the effect of flattening the graph.
Fit even better when Rubinstein’s test applied to index options data from 1980s.

‘When judged by its ability to explain the empirical data, option pricing theory is the most successful theory not only in finance, but in all of economics’ (Steve Ross, 1987)
A nice smooth tale of performativity ...?

Portfolio insurance: use of Black-Scholes-like option pricing theory to synthesize a put (option to sell), and thus create a ‘floor’ to the value of a portfolio.

Grows rapidly in 1980s: by 1987, $60-90 billion under portfolio insurance.

Put synthesis demands stock or index-futures sales as stock price falls.

Index future: derivative whose value tracks the price of the stocks comprising an index such as S&P 500.
October 19-20, 1987: worst crisis of US financial markets since 1929. (Monday 19th: Dow falls 22.6%). Trading disruptions (NYSE and main stock-derivatives exchanges nearly forced to close) and fears of ramifying chain of bankruptcies and bank failures.

If portfolio insurance exacerbated 1987 crash—serious counterperformative effect. Crash grotesquely unlikely on lognormal random walk underpinning Black-Scholes: fall in price of S&P 500 2-month futures a -27σ event, prob. 10^{-160} (Jackwerth and Rubinstein)
‘the crowd detected a pattern of a guy who had to sell [as] the market went lower. So what do you do? You push lower ... and you see him getting even more nervous. ... It’s chemistry between participants. And here’s what happened. You understand, these guys are looking at each other for ten years ... They go to each other’s houses and they’re each other’s best friends and everything. Now one of them is a broker. He has an order to sell. They can read on his face if he’s nervous or not. They can read it. They’re animals. They detect things. So this is how it happened in the stock market crash. They kept selling. They see the guys sell more ...’ (Taleb interview).
Empirical history of option pricing:

1. *up to mid 1970s*: model fits reality only approximately

2. *mid 1970s to summer 1987*: reality adjusts to fit model; the Barnesian performativity of option theory

3. *Oct. 1987 to present*: reality deviates systematically from model; volatility skew. (Some evidence that skew ‘too big’: not just the result of empirical deviation of prices from lognormality.) The model did not *fully* create a world in its image, and it had ‘counterperformative’ effects.

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*Skew*: a pattern of option prices in which implied volatility is not independent of exercise price (as it should be on the Black-Scholes model).
Volatility skew of CBOE S&P index options, 9.00 a.m., July 1, 1987 (upper graph), and 10.00 a.m., January 2, 1990. From Rubinstein (1994, pp. 776-77) courtesy of Mark Rubinstein and Blackwell Publishing Ltd.
Part 3: Performativity, counterperformativity and the credit crisis (work-in-progress!)

The most obvious analogue of Black-Scholes-Merton is the Gaussian copula model used to value CDOs (collateralized debt obligations)
Slides that follow use the simplest Gaussian copula model (Vasicek’s single-period large homogeneous pool model) to show probability distribution of losses on large, highly granular portfolio of assets, each with default probability 0.02, recovery rate of zero, and identical pairwise asset-value correlations, as the level of that correlation varies.
Effective performativity of Gaussian copula. Eg: used by S&P (from 2001), Fitch (from 2003), and (to some extent) Moody’s to rate CDOs; ratings (esp. of ABS CDOs) crucial to crisis.

No clear evidence of Barnesian performativity: there’s always been a ‘correlation skew’ analogous to volatility skew. Hypotheses:
1. ‘arbitrage’ organizationally harder in credit derivatives than in options (might have to hold position for 5-10 years)
2. early crisis of strategies loosely equivalent to arbitraging correlation skew (2005 ‘correlation crisis’)

Counterperformativity of use of Gaussian copula by rating agencies to rate ABS CDOs. Assuming modest correlations and low default probabilities of underlying assets sets in train processes that dramatically undermine those assumptions:

1. ABSs change mortgage market
2. ABS CDOs change ABS market
<table>
<thead>
<tr>
<th></th>
<th>CDO Evaluator three-year default prob. assumptions, as of June 2006 (percent)</th>
<th>Realized incidence of default, as of July 2009 (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.008</td>
<td>0.10</td>
</tr>
<tr>
<td>AA+</td>
<td>0.014</td>
<td>1.68</td>
</tr>
<tr>
<td>AA</td>
<td>0.042</td>
<td>8.16</td>
</tr>
<tr>
<td>AA-</td>
<td>0.053</td>
<td>12.03</td>
</tr>
<tr>
<td>A+</td>
<td>0.061</td>
<td>20.96</td>
</tr>
<tr>
<td>A</td>
<td>0.088</td>
<td>29.21</td>
</tr>
<tr>
<td>A-</td>
<td>0.118</td>
<td>36.65</td>
</tr>
<tr>
<td>BBB+</td>
<td>0.340</td>
<td>48.73</td>
</tr>
<tr>
<td>BBB</td>
<td>0.488</td>
<td>56.10</td>
</tr>
<tr>
<td>BBB-</td>
<td>0.881</td>
<td>66.67</td>
</tr>
</tbody>
</table>

Counterperformativity of logistic regression or hazard rate models used to rate the underlying mortgage-backed securities: as use of these models become more central to mortgage lending, they became (much) less accurate.
Estimates from yearly regression of interest rates on FICO (borrowers’ credit scores) & LTV (loan-to-value ratio)

<table>
<thead>
<tr>
<th>Year</th>
<th>$FICO$</th>
<th>$LTV$</th>
<th>$R^2$ (in %)</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>-0.004</td>
<td>0.030</td>
<td>3</td>
<td>240,676</td>
</tr>
<tr>
<td>1998</td>
<td>-0.007</td>
<td>0.035</td>
<td>7</td>
<td>600,941</td>
</tr>
<tr>
<td>1999</td>
<td>-0.007</td>
<td>0.020</td>
<td>8</td>
<td>104,847</td>
</tr>
<tr>
<td>2000</td>
<td>-0.010</td>
<td>0.035</td>
<td>14</td>
<td>116,778</td>
</tr>
<tr>
<td>2001</td>
<td>-0.012</td>
<td>0.038</td>
<td>20</td>
<td>136,483</td>
</tr>
<tr>
<td>2002</td>
<td>-0.011</td>
<td>0.071</td>
<td>18</td>
<td>162,501</td>
</tr>
<tr>
<td>2003</td>
<td>-0.012</td>
<td>0.079</td>
<td>32</td>
<td>318,866</td>
</tr>
<tr>
<td>2004</td>
<td>-0.010</td>
<td>0.097</td>
<td>40</td>
<td>610,753</td>
</tr>
<tr>
<td>2005</td>
<td>-0.009</td>
<td>0.110</td>
<td>48</td>
<td>793,725</td>
</tr>
<tr>
<td>2006</td>
<td>-0.011</td>
<td>0.115</td>
<td>50</td>
<td>614,820</td>
</tr>
</tbody>
</table>

Source: adapted from table II of Uday Rajan, Amit Seru and Vikrant Vig, “The Failure of Models that Predict Failure: Distance, Incentives and Defaults” (October 2008), ssrn.com/abstract=1296982. Underlying dataset: 16.5 million US residential mortgages in private-label securitisations. Statistical significance of all $\beta$s 0.0005 or better.
Note the analogy between naïve versions of the ‘performativity of economics’ and linear views of technological innovation:

*The ‘science’ (economics) comes first*
*It is applied (its implications are deduced)*
*The resultant innovations diffuse unchanged.*

Current controversy (e.g. critique of performativity by Mirowski and Nik-Khah in *Performing Economics*, ed. MacKenzie, Muniesa, Siu) tends to read it this way.
We have known for some time that:

*Science is a resource, not a determinant*

*Application is not deduction*

*‘Users’ innovate (Fleck: ‘innofusion’)*

Same holds for the performativity of economics. Even the numbers on Black’s sheets aren’t merely solutions of the Black-Scholes equation: dividends; American puts; estimating volatility. Esp. after 1987, Black-Scholes becomes ‘practitioner Black-Scholes’, with volatility skew (supply & demand; industry structure; judgement).

‘American option’: can be exercised at any point up to its expiry.
Is Barnesian performativity just self-fulfilling prophecy?

Key role of arbitrage, both as process ‘enforcing’ the model, and as bound on models that can be self-fulfilling prophecies.

Qv credit derivatives and single-factor Gaussian copula. Closer to self-fulfilling prophecy case. Arbitrage options by constructing replicating portfolio. Doing same for 5- or 10-year CDO (Collateralized Debt Obligation) institutionally problematic: marking-to-market, annual bonuses etc.

‘Totem’ service: calibrate your CDO model against others’.