Abstract

How much influence can the media exert on the political process and how can regulation reduce such influence? We define media power as the ability of subsets of jointly owned media outlets to affect electoral outcomes by providing biased information to voters. We characterize the worst-case scenario media power index over a range of assumptions on the beliefs and attention patterns of voters. We identify circumstances where the power of a media organization can be expressed in terms of attention share. Within our model, the power index can be used to assess the maximal damage produced by media mergers.

1 Introduction

The media industry plays a crucial role in keeping government accountable. Most of the information that we as citizens have about our political leaders comes from the media. Powerful media owners can attempt to manipulate information for their own goals. William Randolph Hearst, the owner of the *Morning Journal* and the source of inspiration for Orson Welles’ *Citizen Kane*, inflamed the American public opinion against Spain through highly

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*This project developed from discussions with Mark Armstrong. I am greatly indebted to him. I am also thankful to Eli Noam and seminar participants at Columbia and Princeton for helpful suggestions.*
biased coverage of the Cuban Rebellion. Hearst’s propaganda is cited as a key cause of the Spanish-American War of 1898. At least since Hearst, there has been a debate on the risks associated with excessive concentration of media ownership. Perhaps, the most chilling quote on this topic is found in Adolf Hitler’s *Mein Kampf*: “By the skillful and sustained use of propaganda, one can make a people see even heaven as hell or an extremely wretched life as paradise.” It is no surprise that the issue of media power occupies a central position in public policy debate.

While the debate is often vehement, effective policy discussion is hampered by the lack of a widely accepted definition of what constitutes a dangerously high level of media concentration. Depending on the measure adopted, conclusions can be at polar opposites. For instance, a simple application of the Hirschmann-Herfindahl Index reveal that most US media industries have low levels of concentration: the HHI of radio, tv stations, and the daily press are respectively 545, 253, and 191 (Noam 2009) and the Department of Justice classifies industries with an HHI lower than 1500 as unconcentrated. Yet, one of the most influential scholarly texts in this area, tellingly entitled The Media Monopoly, uses market share measures to argue that the US media industry is dominated by five companies, whose "concentrated influence exercises political and cultural forces reminiscent of the royal decrees of monarchs rejected by the revolutionists of 1776" (Bagdikian 2004).

Currently, two sets of concentration measures have been discussed. The first one comprises standard indices used in competition policy, chiefly the Herfindahl Index. These measures are traditionally used to evaluate the effect of consumer welfare. These measures are appropriate if someone is interested in the effect of media concentration on *standard* consumer welfare. However, the media industry is different from other industries in that it has an indirect effect on welfare through information externalities imposed on the policy process. Concentration may be damaging in that it raises prices and reduces quantities, but the main worry is that owners may be able to manipulate democratic decision-making in a way that inflicts damage on citizens through an indirect channels. Antitrust provisions must be complemented by media-specific considerations (Polo 2005).

The problem is not the choice of a particular index – like the Herfindahl Index but the definition of the relevant market. If we use a standard notion based on demand, we will tend to define media markets in terms of platforms: radio, newspapers, tv, internet, etc. However, such definition is at the same time too narrow and too broad. It is too narrow, because what matters from
the point of view of democracy is whether citizens are informed, not whether they get their news from a mobile phone, a screen, a magazine, or while they drive their car. Moreover, many people get information from multiple platforms. It is also too broad, because not all media on a platform produce information. For instance, most radio stations or websites do not have news.

In response to that perceived limitation, in 2003 the US Federal Communications Commission attempted to introduce the Media Diversity Index. The index assigned a weight to every platform: broadcast TV (33.8%), newspapers (20.2%), weekly periodicals (8.6%), radio (24.9%), cable internet (2.3%), all other internet (10.2%). Within each platform, every outlet was given equal weight. The index was highly controversial and it was eventually struck down by the courts in *Prometheus Radio Project v. FCC* because of “irrational assumptions and inconsistencies.”

As the court’s decision underscores, the main difficulty with the Media Diversity Index and similar measures is that they cannot be related to standard welfare notions. A valid measure should be

The present paper attempts to bridge this gap by introducing a media power index based on an explicit political economy model. The underlying idea is that the damage that biased media can inflict on (Bayesian) voters corresponds to their ability of inducing them to make electoral decisions that they would not make if they had unbiased information (Besley and Prat 2006).

There are two possible ways to proceed. If we had sufficient information on the motives of media owners and on the way voters process news, we could use it to construct the primitives of our models and we could provide an exact measure of media power. However, despite the considerable progress made by the empirical literature on the political economy of mass media (summarized below), some key factors are intrinsically difficult to observe, like the intentions of the media owner, the ability of voters to detect an attempt to subvert democracy by biasing the media, the voters’ willingness to switch to unbiased media in such a case, and the voters’ ability to process multiple news items. So, this paper adopts a different approach. It considers a large set of possible parameters and identifies the combination that leads to a maximum possible damage to the electorate for a given media ownership structure.

The core contribution of the paper is the analysis of the worst-case scenario for a generic media landscape, defined by an ownership structure and media consumption matrix. Voters are heterogeneous in their media con-
sumption decisions. They are Bayesian and they use the information they receive from the media sources they follow to decide who to vote for. Voters have beliefs on the probability that media are captured. They also have a potentially bounded capacity to absorb information: they only observe or remember a certain number of news items from the various sources they follow. The relative quality of political candidates is stochastic and the media receive a large number of signals correlated with candidate quality. We entertain the possibility that one of the media owners has a political motive: he prefers one of the two candidates (and has no profit motive). While, for a given set of parameters, the equilibrium of this game is very difficult to characterize, we prove that the worst-case scenario can be expressed as the solution of a simple polynomial equation.

As one would expect, the worst case we identify corresponds to a naive electorate who cannot undo media bias. However, the role of news processing is more subtle. The worst case is not necessarily the one where voters have minimal bandwidth. In the two extreme cases – minimal and maximal bandwidth – the index can be expressed in a very simple form. With maximal bandwidth (and a bound on how many sources ever voter uses), it depends on the share of voters who consume one of the biased media.

With minimal bandwidth, we obtain a particularly simple power index. The power of a media owner depends on the share of attention that his media command: namely, if $n_i$ is the number of sources that voter $i \in I$ consumes and $b_i$ is the number of those sources that belong to the evil owner, then the owner’s power is

$$\sum_{i \in I} \frac{b_i}{n_i}.$$ 

The index is computed on the basis of a media consumption matrix, where each citizen is associated to the subset of media sources that he or she utilizes. It can therefore be implemented with standard data on media consumption patterns. Technological platforms are treated in a symmetric way and matter only inasmuch as they are associated to media consumption patterns. The index provides a unified conceptual framework to assess within-platform and across-platform ownership. Their relative danger depends only on consumption patterns.

The worst-case scenario index introduced here can be viewed as an alarm bell to identify situations that are potentially dangerous for citizens. Examples illustrate how the index can be used to assess the relative risk of
different media groups as well as the potential damage generated by different media mergers. Of course, the limit of our approach is that it may be too pessimistic. Media owners may be less evil – or just more profit-driven – and voters may be less naive. Just like other competition indices, the media power index should be seen as a raw indicator of where regulatory attention might be directed.

There is now a large body of empirical research on media bias and the influence of media on the democratic system. The fact that media scrutiny influences both policy chosen by elected officials and electoral outcomes is amply documented (See Prat and Stromberg 2012 for a survey). The presence of news slant has been documented through partisan references (Grosseclose-Milyo 2005), airtime (Durante-Knight 2006), space devoted to partisan issues (Puglisi 2006), and textual analysis (Gentzkow and Shapiro 2010). The link between media bias and electoral outcome is also present in the data. Della Vigna and Kaplan (2007) find a significant effect of Fox News entry on US voting patterns. Enikolopov, Petrova and Zhuravskaya (2011) find an even stronger effect of the entry of NTV into selected Russian regions. However, there is also evidence that US newspaper readers show some sophistication in the way they handle media bias (Durante and Knight, Chiang and Knight). Regarding the “evil” role of the media, we have forensic evidence of a masterplan to subvert democracy through media capture in Peru (Macmillan-Zoido). Also, Durante Knight (2006) document sudden and significant changes in public television coverage in Italy when Silvio Berlusconi came to power. However, Gentzkow and Shapiro (2010) find that owner identity has no significant effect on newspaper slant in the US.

On the theory side, media bias can be modeled as coming from two sources. Even though the media have no vested interests, consumers may demand biased coverage (e.g Mullainathan and Shleifer (2005), Gentzkow and Shapiro (2006)). However, bias can also be supply-driven (Baron (2006), Besley and Prat (2006), Balan, De-Graba, and Wickelgren (2009), Duggan and Martinelli (?), Anderson and McLaren 2012, Petrova 2012). Our model focuses on the second source of bias. Besley and Prat (2006) assumes that the goal of news manipulation is to influence the electoral process and determine conditions – chiefly higher media concentration – under which the goal is more likely to be reached. Anderson and McLaren (?) compares a media duopoly to a media monopoly, in the presence of politically motivated media owners, and analyzes the effect of a merger.

The present paper is loosely related to a very recent literature on agency
problems where the principal is unwilling or unable to specify all the primitives of the environment she operates in (e.g. Chassang 2011, Madarasz and Prat 2010). In such circumstance, theory can identify bounds to the performance of possible arrangements. The present paper extends this approach to regulatory issues.

The next section describes the model and characterizes the benchmark case where all media outlets are unbiased. Section 3 introduces an evil owner and characterizes the worst-case scenario when the voter bandwidth is known. Section 4 studies an even worse worst-case scenario when bandwidth is unknown. Section 5 concludes by mentioning policy implications.

2 Unbiased Media

We begin by stating and analyzing the model when the media are by assumption unbiased.

There are two candidates, say \( A \) and \( B \). The relative quality of candidate \( B \) over candidate \( A \) is a random variable \( \sigma \), uniformly distributed on \([0, 1]\). There is a mass of voters, who for now have homogenous preferences. In expectation, the two candidates are equally attractive, but given \( \sigma \) voters prefer candidate \( B \) if and only if \( \sigma \geq \frac{1}{2} \). Specifically, voters’ payoff is \( \frac{1}{2} \) if they elect \( A \) and \( \sigma \) if they elect \( B \).

However, voters do not observe the relative quality \( \sigma \) directly. They rely on the media for information. There is a set of media outlets, who do not observe \( \sigma \) directly either but they receive binary signals drawn from a binomial distribution with mean \( \sigma \). Let \( \mathcal{M} \) denote the finite set of media outlets, with typical individual outlet denoted \( 1 \leq m \leq |\mathcal{M}| \). Let \( x_m = (x_{m1}, \ldots, x_{mN}) \) denote a vector of \( N \) binary signals – news items – observed by outlet \( m \), with \( \Pr (x_{mi} = 1|\sigma) = 1 \). News items are, conditional on \( \sigma \), independent within and across media outlets.

In general voters may consume more than one outlet. Let \( M \subset \mathcal{M} \) denote some subset of outlets. Then voters are partitioned into segments, indexed by the subset \( M \) of outlets they consume, and for each \( M \subset \mathcal{M} \) let \( q_M \) be the fraction of voters who consume (exactly) the subset \( M \). Clearly

\[
\sum_{M \subset \mathcal{M}} q_M = 1 .
\]

(For simplicity, suppose that all voters see at least one outlet, so that \( q_{\emptyset} = 0 \).
This makes no difference at provided that we assume those consumers who see no outlet and hence see no messages merely vote randomly, with probability half for either candidate.)

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| Reach   | 30%   | 40% | 20% | 30% | 30% | 50% | 40% |
| Attention | 25%   | 14.1% | 15% | 8.3% | 9.1% | 15% | 16.6% |

Table 1: Example of a media landscape
(two television stations, three newspapers, two websites)

Unbiased media simply report all the $N$ signals they receive. Thus outlet $m$ reports $N$ binary numbers. A voter in group $M$ is exposed to $|M| N$ signals.

However, voters have potentially limited bandwidth. Voters in segment $M$ observe or remember a limited number $K_M \in \{1, ..., N\}$ of news items, randomly selected among the set of $|M| N$ items available. The assumption that all voters within segment $M$ have the same bandwidth is without loss of generality as segments with heterogeneous bandwidth can be subdivided into homogenous ones. Bandwidth plays no role in the unbiased case, but will be crucial once media can be biased.

A voter $i$ who observes $K$ binary news items will compute their average $s_i$, and that will clearly be the best unbiased estimator of $\sigma$, given $i$'s information. Voter $i$ prefers $B$ if $E[\sigma] \geq \frac{1}{2}$. Under sincere voting, he casts his ballot for $B$ if and only if $s_i \geq \frac{1}{2}$.

Let $s_m$ be the average of the $N$ signals received by outlet $m$. The aggregate election outcome can then be characterized as follows:
Proposition 1  

Candidate $B$ is elected if and only if  

$$
\sum_{M \subseteq M} q_M \sum_{m \in M} \frac{s_m}{|M|} \geq \frac{1}{2}.
$$

As $N \to \infty$, $B$ is elected if and only if $\sigma \geq \frac{1}{2}$.

An aggregate “error” occurs when $A$ is elected when $\sigma > \frac{1}{2}$ or $B$ is elected when $\sigma < \frac{1}{2}$. When all media report unbiased signals, voters cast their ballot according to the average signal they observe. An aggregate error occurs only when, by chance, the media observe skewed signal realizations. As the number of signals each outlet observes, denoted by $N$, increases, the probability of getting grossly skewed signal realization vectors decreases. In the limit, there is no aggregate error and the right candidate is always elected. As this paper is interested in error generated by deliberate bias, for the reminder of the paper we assume that $N$ is very large and we focus on the limiting case.

While the identity of the winning candidate in Proposition 1 is unaffected by assumptions on voter bandwidth, it is interesting to see how bandwidth affects the vote share. Figure 1 depicts the vote share of Candidate $A$ as a function of candidate quality differential $\sigma$ for four possible level of bandwidth, from the smallest: $K_M = 1$ to the limit as $K_M \to \infty$. As one expects, the vote share (of $A$) is decreasing in the quality (of $B$). However, bandwidth determines the slope. The probability that a voter chooses the wrong candidate, say $A$ when $\sigma > \frac{1}{2}$, corresponds to the chance that he observes/recalls a higher number of signals favorable to $A$ than to $B$. That decreases with $K_M$ and in the limit it goes to zero. This explains why vote share function
becomes increasingly S-shaped as bandwidth increases.

\[ A\text{'s vote share in segment as a function of quality } \sigma \]

\[ \begin{align*} \text{B-quality} & \quad A\text{-vote} \\ 0.0 & \quad 1.0 \\ 0.2 & \quad 0.9 \\ 0.4 & \quad 0.8 \\ 0.6 & \quad 0.6 \\ 0.8 & \quad 0.4 \\ 1.0 & \quad 0.2 \end{align*} \]

3 **Power Under Biased Media with Known Bandwidth**

Let us now entertain the possibility that an agent acquires control of a subset \( G \) of the set of active media \( \mathbb{M} \). In the worst-scenario spirit, this agent – henceforth known as the “evil media owner” – has one goal only: he wishes to see candidate \( A \) elected, independent of the relative quality of the two candidates. This excludes that the media owner might moderate his political bias because of commercial profit or journalistic integrity, but still allows for the possibility that he reports less biased news in order to bolster his ability to persuade voters. The goal of this section is to identify a set of conditions under which the evil media owner is successful in his attempt to get his candidate elected.

We introduce two standard measures of the market penetration of media group \( G \). The reach of subset \( G \), i.e., the fraction of voters who see outlet \( m \), is then

\[ r_G = \sum_{M \text{ s.t. } \exists m \in M, m \in G} q_M. \]
The attention share of media group $G$ in segment $M$ be defined as

$$g_M = \frac{|M \cap G|}{|M|}.$$

and let its overall attention share be

$$a_G = \sum_M q_M g_M.$$

In our pessimistic view of the world, the evil owner faces no constraint in the vector of news items that he can report. In particular, he can fail to report items that are favorable to $B$. This is consistent with a situation where the media can disclose news items selectively and voters only know the number of items they read, not the total number of items that are reported, not an unreasonable assumption given the limit on their bandwidth. Namely the evil owner chooses the frequency $s \in [0, 1]$ of signals that are favorable to $B$ that he reports. In one extreme case $s = 1$ and all signals are reported. In the other extreme $s = 0$ and only signals that are favorable to $A$ are reported.

How do voters react to the possible presence of an evil media owner? Let $\beta \in (0, 1)$ be the prior probability that voters assign to the presence of an evil media owner. This is a subjective parameter that captures the voters’ views on the possibility that $G$ is under the effective control of a unitary owner and that such owner is in biased in favor of candidate $A$.

Recall that a voter with bandwidth $K_M$ in group $M$ receives/remembers a $K_M$-sized vector of signal realizations randomly drawn from the media outlets in group $M$. As before, the number of signals that come from a particular outlet is random and the selection of signals within an outlet is random too. Now, however, the voter faces a more complex Bayesian updating process.

To analyze this, we begin by writing the probability that a voter in group $M$ observes a particular realization of the $K_M$-sized signal vector $y^i$ he receives from media outlets in $M$. The vector includes news items randomly drawn from outlets in $M$. Let $y^i_k$ denote the $k$th realization of the vector and let $m(k)$ denote the media outlet it is drawn from. Suppose the voter believes that the evil owner would use reporting strategy $\hat{s}$. Then, the probability of realization $y^i = Y$ would be given by:

$$\Pr(y^i = Y | \sigma, \hat{s}) = \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} \left( (1 - \beta) \sigma^{N_1(G)} (1 - \sigma)^{N_0(G)} + \beta (\hat{s} \sigma)^{N_1(G)} (1 - \hat{s} \sigma)^{N_0(G)} \right).$$
where \( N_y(M/G) \) is the number of signals with value \( y \) coming from unbiased outlets, while \( N_y(G) \) is the same variable for potentially biased outlets.

The voter computes the posterior on candidate quality as follows:

\[
E[\sigma|Y, \hat{s}] = \frac{\int_0^1 \Pr(y^i = Y|\sigma, \hat{s}) \sigma d\sigma}{\int_0^1 \Pr(y^i = Y|\sigma, \hat{s}) d\sigma}
\]

and votes for \( A \) if and only if \( E[\sigma|Y, \hat{s}] \leq \frac{1}{2} \).

We now compute a lower bound to posterior \( E[\sigma|Y, \hat{s}] \).

**Lemma 2** For any vector of signals \( Y \), let \( N_1(M/G) \) be the number of positive signals from unbiased media, let \( N_0(M/G) \) be the number of negative signals from unbiased media, and let \( K_G \) be the number of signals from biased media. The voter posterior \( E[\sigma|Y, \hat{s}] \) is bounded below by

\[
\frac{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G) + K_G} \sigma d\sigma}{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G) + K_G} d\sigma}.
\]

**Proof.** First, it is easy to see, that, for any value of \( \hat{s} \in [0, 1] \), \( E[\sigma|Y, \hat{s}] \) is nonincreasing in the number of signals that are favorable to \( A \). Hence, assume that all signals from biased media are zero’s: \( N_1(G) = 0 \) and \( N_0(G) = K_G \). Now the posterior is

\[
E[\sigma|Y, \hat{s}] = \frac{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} (1 - \beta) (1 - \sigma)^{N(G)} + \beta (1 - \hat{s}\sigma)^{N(G)} \sigma d\sigma}{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} (1 - \beta) (1 - \sigma)^{N(G)} + \beta (1 - \hat{s}\sigma)^{N(G)} \sigma d\sigma}
\]

Second, let us show that \( E[\sigma|Y, 1] \leq E[\sigma|Y, \hat{s}] \) for all \( \hat{s} \in [0, 1] \). Note that \( E[\sigma|Y, 1] \leq E[\sigma|Y, \hat{s}] \) if and only if

\[
\frac{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} (1 - \sigma)^{K_G} \sigma d\sigma}{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} (1 - \sigma)^{K_G} d\sigma} \leq \frac{A}{B}.
\]

Note that

\[
\text{sign} \left( \frac{d}{d\hat{s}} \frac{A}{B} \right) = \text{sign} \left( A \int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G) - K_G} \sigma d\sigma - B \int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G) - K_G} \sigma d\sigma \right)
\]

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because
\[
\frac{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} (1 - \hat{s} \sigma)^{K_G - 1} \sigma d\sigma}{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G)} (1 - \hat{s} \sigma)^{K_G - 1} d\sigma}
\]
corresponds to \(E \left[ \sigma | \bar{Y}, \hat{s} \right] \) where \(\bar{Y} \) is \(Y\) less a biased signal that was favorable to \(A\) and therefore \(E \left[ \sigma | \bar{Y}, \hat{s} \right] \geq E \left[ \sigma | Y, \hat{s} \right] = A / B\). This proves that \(\frac{d}{ds} \frac{A}{B} \leq 0\).

Thus, \(\frac{A}{B}\) is minimized when \(\hat{s} = 1\). This shows that the minimal value of \(E \left[ \sigma | Y, \hat{s} \right]\) is achieved when \(\hat{s} = 1\). Thus, a lower bound to \(E \left[ \sigma | Y, \hat{s} \right]\) is
\[
\frac{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G) + K_G} \sigma d\sigma}{\int_0^1 \sigma^{N_1(M/G)} (1 - \sigma)^{N_0(M/G) + K_G} d\sigma},
\]
which corresponds to a situation where all the signal realizations coming from the biased media are zero and the public believes that biased media report truthfully.

This simply shows that, given \(N_0(M/G)\) and \(N_1(M/G)\), the value of \(E \left[ \sigma | Y, \sigma \right]\) can never be lower than the value achieved when all the biased outlets’ news items are favorable to \(A\). \(\blacksquare\)

Now, let us translate the lower bound on the posterior into an upper bound on the vote share that candidate \(A\) can receive. The lower bound in (1) is greater or equal to \(\frac{1}{2}\) if and only if
\[
N_1(M/G) \geq N_0(M/G) + K_G
\]
In other words, the voter selects candidate \(B\) if and only if the number of signals in favor of Candidate \(A\) is weakly larger than the number of signals in favor of \(B\), including signals from both unbiased and potentially biased outlets. The “weakly” part comes from the fact that \(\beta > 0\). If the two candidates are supported by exactly the same number of signals, the voter would be exactly indifferent if \(\beta = 0\). But for any strictly positive \(\beta\), he must prefer \(B\).

The probability that the voter selects \(B\) is thus equal to:
\[
\Pr \left( N_1(M/G) \geq N_0(M/G) + K_G \right) = \Pr \left( \frac{N_1(M/G)}{N_1(M/G) + N_0(M/G) + K_G} \geq \frac{1}{2} \right)
\]
The probability that an individual signal takes value 1 is $(1 - g_M) \sigma + g_M \cdot 0$. The probability that a particular voter selects $A$ is given by the cumulative distribution of a binomial with parameter $(1 - g_M) \sigma$, with $K_M$ possible realizations, evaluated at the highest integral that is strictly smaller than $K_M/2T$. For $K_M = 1$ it is 0, for $K_M = 2$ it is 0, for $K_M = 3$ it is 1, etc. We denote it as the `ceiling’ of $K_M/2$ (the smallest integral that is at least as large as $K_M/2$) minus one:

$$p_A(g_M, K_M, \sigma) = \sum_{k=0}^{[K_M/2]-1} \binom{K_M}{k} ((1 - g_M) \sigma)^k (1 - (1 - g_M) \sigma)^{K_M-k}$$

By the law of large numbers, $p_A(g_M, K_M, \sigma)$ is the share of $A$ votes in segment $M$.

While $p_A(g_M, K_M, \sigma)$ is an upper bound to the vote share that $A$ can achieve, we do not know whether the upper bound can indeed be achieved. However, to see this just assume that the evil owner uses a strategy of reporting only zeros. When $\beta \to 0$, it is easy to verify that the vote share in $M$ does indeed tend to $p_A(g_M, K_M, \sigma)$ for any $K_M$.

**Proposition 3** The upper bound to $A$'s vote share in a segment where $G$ controls a share $g_M$ of outlets and voters have bandwidth $K_M$ is

$$p_A(g_M, K_M, \sigma) = \sum_{k=0}^{[K_M/2]-1} \binom{K_M}{k} ((1 - g_M) \sigma)^k (1 - (1 - g_M) \sigma)^{K_M-k}$$

Let us figure 1, which depicted $A$'s vote share in a segment with only unbiased media. With our current notation, we would express that as $p_A(0, K_M, \sigma)$. Let us compare it with a segment where, say, 1/4 of the outlets are biased:
Figure 2 now depicts $p_A\left(\frac{1}{4}, K_M, \sigma\right)$ for various values of $K_M$. 

\[ A\text{'s vote share in segment as a function of quality } \sigma \]

In Figure 2, $A$’s vote share is still a decreasing function of $\sigma$ and it is more s-shaped as bandwidth increases. However, now the curves have all shifted to the right. They intersect the $1/2$ horizontal line at $\sigma > \frac{1}{2}$.

The two extreme cases are particularly easy to characterize:

**Corollary 4**

(i) When bandwidth is minimal, $A$’s vote share is a linear function of $\sigma$:

\[ p_A(g_M, 1, \sigma) = (1 - g_M)(1 - \sigma) + g_M; \]

(ii) When bandwidth is maximal, the vote share is a step function

\[
\lim_{K_M \to \infty} p_A(g_M, K_M, \sigma) = \begin{cases} 
1 & \text{if } \sigma < \frac{1}{2(1-g_M)} \\
1/2 & \text{if } \sigma = \frac{1}{2(1-g_M)} \\
0 & \text{if } \sigma > \frac{1}{2(1-g_M)}
\end{cases}
\]

Now that we have characterized the vote share in each segment, let us move on to the overall vote share, and hence to characterizing the power of media group $G$.

Given any vector of segment bandwidth $K = (K_M)_{M \in M}$, the power of group $G$ is the highest value of $\bar{\sigma}(K)$ such that the $A$-vote share is at least $1/2$, namely the solution to

\[
\sum_{M \in M} q_M p_A(g_M, K_M, \bar{\sigma}(K)) = \frac{1}{2}
\]
Proposition 5 For a given bandwidth vector $K$, the power of group $G$, $\bar{\sigma} (K)$, is the minimum between one and the smallest solution greater than $1/2$ of the following polynomial equation:

$$\sum_{M \subseteq M} \left( \sum_{k=0}^{[K_M/2]-1} \binom{K_M}{k} ((1 - g_M) \bar{\sigma} (K))^k (1 - (1 - g_M) \bar{\sigma} (K))^{K_M-k} = \frac{1}{2} \right)$$

As one would expect, the index is monotonic in $g_M$. An increase in the attention share of media group $G$ in any segment causes an increase in $\bar{\sigma} (K)$. The increase is strict if $\bar{\sigma} (K) < 1$.

Instead, the effect of $K_M$ is non-monotonic. To see this, reconsider the two extreme cases of minimal and maximal bandwidth.

For the minimal case, suppose $K_M = 1$ in all segments. $A$’s overall vote share boils down to

$$1 - (1 - a_G) \sigma$$

where $a_G$ is the attention share of media group $G$ defined above. The power index is simply

$$\bar{\sigma} (1) = \min \left( 1, \frac{1}{2 (1 - a_G)} \right)$$

For the maximal case, instead we have

$$\lim_{K_M \to \infty, \text{all } M} \bar{\sigma} (K) = \min \left( 1, \frac{1}{2 (1 - \text{median} (g_M))} \right),$$

where $\text{median} (g_M)$ is defined as $G$’s attention share for the median voter.\footnote{Rank all voters in order of increase $g_M$ and pick the one corresponding to mass $1/2$. If this falls at the boundary between two segments, choose the segment with the lower $g_M$, a consequence of this being the limit of a worst case.}

If all voters follow at most two outlets, $g_M$ can only take three values: 0, 1/2, and 1. This means that the power index takes only two values. If the reach of $G$ is at least 50%, then power is absolute ($\bar{\sigma} (\infty) = 1$). If it is 50% or less, the group has no power ($\bar{\sigma} (\infty) = 1/2$).

To summarize the extreme cases:

**Corollary 6** (i) If bandwidth is minimal, media power is determined by attention share.

(ii) If bandwidth is maximal and no voter follows more than two outlets, media power is determined by reach.
To illustrate the use of our index in the extreme cases, we return to the example and compute the power of individual media outlets.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Share</th>
<th>Tv1</th>
<th>Tv2</th>
<th>Np1</th>
<th>Np2</th>
<th>Np3</th>
<th>Web1</th>
<th>Web2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power $K = 1$</td>
<td></td>
<td>0.666</td>
<td>0.582</td>
<td>0.588</td>
<td>0.545</td>
<td>0.550</td>
<td>0.588</td>
<td>0.576</td>
</tr>
<tr>
<td>Power $K \to \infty$</td>
<td></td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>

One of the lessons of this analysis is that bandwidth is a key variable to determine media power. To compute the power index in Proposition 5, one must know the vector of segment bandwidth $K$. However, this knowledge may not be available in practice.

4 Power with Unknown Bandwidth

We turn to the worst-case scenario under the assumption that the vector of bandwidth $K$ is unobservable. This is equivalent to choosing the vector $K$ to maximize $\sigma(K)$. As $K_M$ is segment-specific, focus on a particular segment $M$ and note that the upper envelope of $A$’s vote share over $K_M$ is simply

$$\max (p_A (g_M, 1, \sigma), p_A (g_M, \infty, \sigma))$$

Using the same definition of power as before, we obtain

**Proposition 7** If bandwidth is unknown, the power of group $G$ is bounded above by the highest value of $\sigma$ that solves

$$\sum_{M \subset M} q_M \max (p_A (g_M, 1, \sigma), p_A (g_M, \infty, \sigma)) = \frac{1}{2}$$

where

$$p_A (M, 1, \sigma) = 1 - (1 - g_M) \sigma$$

and

$$p_A (M, \infty, \sigma) \equiv \begin{cases} 0 & \text{if } (1 - g_M) \sigma \geq \frac{1}{2} \\ 1 & \text{if } (1 - g_M) \sigma < \frac{1}{2} \end{cases}$$

For example, we can compute the power of Website 1 as follows. In segments 1 through 5, where the outlet is not followed, the maximal power is 1 when $\sigma < 0.5$ and $1 - \sigma$ when $\sigma \geq 0.5$ (see figure below for a depiction of the maximal power in the different segments). In segments 6 and 10, the
maximal power is 1 when $\sigma < 3/4$ and $1 - \sigma$ when $\sigma \geq 3/4$, and in segments 7, 8, and 9, it is $\sigma < 2/3$ and $1 - \sigma$ when $\sigma \geq 1/3$. The maximal power is the piecewise linear function depicted in figure. The power index is 0.75, and it corresponds to a situation where $K_{1-5} = K_{7,9} = 1$ and $K_{6,10} = \infty$. Website 1’s power is maximal when voters have maximal bandwidth in the three segments where it is most popular and minimal bandwidth elsewhere.

By definition, the power index when bandwidth is unknown cannot be smaller than in the case where it known. In the example above, it is significantly greater than a situation where all voters have minimal bandwidth ($\bar{\sigma}(1) = 0.588$) and a situation where all voters have maximal bandwidth ($\bar{\sigma}(\infty) = 0.5$).

5 Policy Implications and Conclusion

The family of media power indices that we have developed can provide an indication of the amount of damage that a media group can impose on the democratic process.

One way of using the index is as an alarm bell. The regulator could set a certain index level as threshold for additional scrutiny. Media groups above the threshold may for instance be subject to additional checks to guarantee that their owners are independent of political forces and that their reporters enjoy effective journalistic autonomy.

Another way of using the index is as a first, rough measure of the potential risk of a media merger. For instance, in our example, suppose that Tv1 is for sale and the owners of Newspaper1 and Website1 have expressed an interest in acquiring it. Which of the two buyers poses a larger risk? Note that, on their own, both acquirers have the same power under minimal bandwidth and maximal bandwidth.

Newspaper1 and Website1 have the same attention share –15% – so under $K = 1$ the effect of a merger must be the same (Proposition ?). However, if bandwidth is maximal, or unknown, Website 1 becomes more dangerous, as table 7 below illustrates. Intuitively, this is because, when joint with Tv1 it has a sizeable presence in eight segments. At $\bar{\sigma} = \frac{10}{11} = 0.909$, it will still fully control segments 1, 2, and 3 (30% of votes), and it will get an additional 20% of the overall votes from segments 6 through 10. At the same level of $\bar{\sigma}$, a group composed of Tv1 and Newspaper1 cannot get a majority for candidate A.
<table>
<thead>
<tr>
<th>Segment</th>
<th>Tv1</th>
<th>Np1</th>
<th>Web1</th>
<th>Tv1+Np1</th>
<th>Tv1+Web1</th>
</tr>
</thead>
<tbody>
<tr>
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<td>100%</td>
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<td>50%</td>
<td>100%</td>
<td>50%</td>
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<td></td>
<td>100%</td>
<td></td>
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</tr>
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<td></td>
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<td>7</td>
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<td>33%</td>
<td></td>
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<td>9</td>
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<td>33%</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>25%</td>
<td></td>
<td>25%</td>
</tr>
</tbody>
</table>

Power Index $K = 1$

| Power Index $K = 1$ | 0.666 | 0.588 | 0.588 | 0.833 | 0.833 |
| Power Index $K \to \infty$ | 0.500 | 0.500 | 0.500 | 0.500 | 0.749 |
| Worst-case Power Index | 0.833 | 0.909 |

References


