Insider Trading and Earnings Management

Thomas Alan Issaevitch

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Abstract

Insider Trading, Earnings Management, and Informational Efficiency

Thomas Alan Issaevitch

I model the effect of insider trading on earnings management using first a two-period model and then an infinite-horizon overlapping generations model. The two-period reporting setting reveals that insider trading has a qualitative effect on investor uncertainty. The infinite-horizon setting demonstrates the robustness of the two-period model to both strategic insider trading as well as multiple accounts.

Using the two-period model, I study the consequences on investor information of varying regulatory stringency (the stringency of audits and enforcement of the ban on insider trading) and accounting precision. When regulatory stringency is high, earnings management and investor uncertainty show a decrease in accounting precision (the inherent accuracy of the accounting system). When regulatory stringency is low, earnings management and investor uncertainty show an increase in accounting precision. Income smoothing is largest at low regulatory stringency while ‘anti-smoothing’ is largest at intermediate regulatory stringency.

Using the infinite-horizon model, I again show that earnings management decreases investor informedness for regulatory stringency below a threshold value. In the infinite horizon model, the firm never liquidates and so true fundamental value is never revealed. Instead, I assume that an independent noisy (but unbiased) analyst report is issued after the manager’s insider trading and before his liquidation. I then show that the threshold regulatory stringency value is maximal for intermediate values of noise in the analyst report.
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Chapter 1

Introduction

This thesis considers the effect of insider trading on earnings management. While prior research has explored other motives for earnings management, insider trading is conspicuously absent. This is true despite the fact that insider trading has been suggested to increase investor information by providing a credible signalling mechanism. For example, if a manager believes that the current price undervalues the firm, he will buy shares and so credibly signal the undervaluedness.

Early models of insider trading did indeed support the idea that insider trading improves informational efficiency. But, this literature did not consider the possibility that insider trading may motivate the manager to distort accounting reports (by engaging in “earnings management”). Turning to the previous example, once the manager perceives the firm is undervalued, he is motivated to use accounting reports to increase this undervaluedness prior to his buying shares.

\footnote{In Leland (1992) insider trading improves information but lowers market liquidity. However Laffont and Maskin (1990) and Bhattacharya and Spiegel (1991) point out that managers act strategically if their trades impact prices (an effect usually modeled with a micro-structure model of Kyle (1985)) and show that pooling of (non-informative) equilibria can result. Ausubel (1990) also finds less optimistic results because investors anticipate suffering a loser’s curse with more informed traders. On the other hand, Fishman and Hagerty (1992) find that insider trading can lead to less informational efficiency because investors have less incentive to acquire information. This literature does not consider the effect of insider trading on earnings management because there are no reports to distort. On the other hand, Bushman and Indjejikian (1995) find results similar to Leland (1992) but in a voluntary disclosure setting.}
If the distortions cannot be unraveled by investors then insider trading may actually decrease total investor information. In particular, investors may have more uncertainty with insider trading even after they observe both the accounting and insider trading reports than when insider trading is impossible.

This paper studies the variation of investors' information (as measured by the precision of investor estimates of the firm's liquidation value and period earnings) when earnings management and insider trading are possible as a function of two variables. The first, accounting precision, describes the amount of noise the accounting system adds to "economic income" to produce "accounting income". Second, regulatory stringency describes the combined disutility incurred by the manager for insider trading and earnings management. To perform my analysis, I first use a two-period reporting model and then an infinite-period, overlapping-generations model.

The two-period model has two actors: a manager and a representative investor. In the first period, the manager privately observes two zero-mean random signals: economic income (with persistence to period two) and accounting error (which fully reverses in second period income). The manager next reports income: the sum of economic income, accounting error, and earnings management. This report is used by investors to rationally set a price for the firm's equity. Finally, the manager privately learns the firm's future (second period) terminal value and can then either buy or sell shares. Any of his remaining shares are liquidated at the terminal value per share. Opposing the manager's insider trading and earnings management are personal costs from audits and enforcement of the ban on insider trading. I use the term 'regulatory stringency' to denote the combined stringency of audits and enforcement of the ban on insider trading. The manager
thus chooses his earnings management and insider trading to maximize gains from insider trading less personal regulatory costs.

The model is similar to that of Fischer and Verrecchia (2000) with a key difference. While in Fischer and Verrecchia (2000) investor uncertainty as to the manager's reporting incentives arises exogenously, here uncertainty arises endogenously through insider trading. This is important because, from the viewpoint of the manager, insider trading and earnings management are complements: higher trade size increases the marginal benefit of earnings management and more earnings management increases the marginal benefit of insider trading.

My analysis leads to the following main result concerning whether higher accounting precision (i.e., the inverse of the amount of error generated by the accounting process) will improve accounting informativeness (defined as the increase in the precision of investor estimates of firm value due to the accounting report) when insider trading is possible. When regulatory stringency is low, then increasing accounting precision (defined as the precision of the noise introduced by the accounting system in estimating firm value) decreases the informativeness of the accounting report towards zero, while when regulatory stringency is high, increasing accounting precision increases the informativeness of the accounting report. Combined with my result that audit stringency and enforcement of insider trading restrictions are complements, this discontinuity suggests that more (presumably more accurate) fair value accounting may reduce informativeness either when it is harder to audit or when insider trading enforcement is sufficiently lax.

The infinite-period model is used to check the robustness of the conclusions of the two-period model. Secondarily, the infinite-period model provides a setting that is more readily testable empirically. I first discuss new issues found in
the multi-period model. I then discuss the specifics of the model and how it differs (superficially) from the simple 'repeated game' formulation of the two-period model.

The most important aspect of the infinite period model is that investor uncertainty is endogenized. In the two period model, uncertainty is described by the uncertainty in economic income, $m$, (which, without loss of generality, was normalized to one) relative to the accuracy of the accounting system (which had error precision $s$). This uncertainty in the accounting measurement of economic income is exogenous. In the infinite-period model, the stationary error covariance describing investor uncertainty of the analogue of $m$ is endogenous.

Also important is the fact that, in contrast to the two-period model, the manager's terminal payout in the infinite period model is endogenous. This change has two effects. The first relates to the mechanism underpinning the loss of investor information due to insider-trading-induced induced earnings management. The second relates to the fact that the endogenous liquidation value now depends on the manager's insider trading. This is because investors observe the manager's trade (and so update their beliefs) before setting this final price.

In the two-period model, loss of information due to insider trading comes from the fact that, in equilibrium, the manager always smooths economic income. By smoothing income, I mean that the magnitude of economic income relative to the reported value, is reduced, i.e., the "signal to noise ratio" is reduced. In turn, this smoothing follows from the fact that the market, knowing that accounting income contains both persistent and fully reversing components, applies an average persistence factor to earnings and so undervalues the persistent component. However, in the infinite-period model, the firm does not liquidate when the man-
ager sells his remaining shares. Consequently, the price received is endogenous and presumably also undervalues economic income. This motivates the consideration of a multi-period model. A multi-period model of earnings management was first considered in Dye (1988), albeit with no insider trading and a different focus (informational efficiency in my work vs. economic efficiency and the tension between current and future shareholders in his).

Finally, the infinite-period model requires modelling choices both to ease description as well as to simplify the analysis and conclusions. Once again, I assume a pure reporting model with no discretionary investment. I also assume that the manager liquidates any unsold shares before the next manager takes over, but after a new independent signal (e.g., analyst report) of the firm's economic assets is made. Thus, the generations in this model are non-overlapping. This non-overlapping assumption greatly simplifies the analysis (since the current manager need not conjecture as to the next earnings management manager's policies). However, it is a significant assumption whose effect I do not consider here.\footnote{Although preliminary work finds no qualitative change.}

The two-period model of Chapter 2 focuses on income and the only role of accounting is to create an "accounting error." In contrast, in a multi-period model, it is assets that best represent the state of the firm. This is because the effect of investments generally persist across periods and this persistence is captured by assets. Of course, nothing prevented an asset-based view from being used in the two-period model, but I chose an income view as more intuitive.

A proper treatment of the "economic" and "accounting" assets requires some care, and this is taken up in detail in Chapter 3. For now, I mention the following three aspects of the economics and accounting modeled here. First, accounting
error still has meaning as an error introduced in a given period, i.e., it is an accrual. However, accounting error no longer has the simple interpretation as the difference between accounting assets and economic assets since this difference would involve the accumulation of all past errors. Second, the accounting report now consists of three components: a report of accounting assets, a report of operations (e.g., revenues and expenses), a report of revaluations. All three reports provide investors with information concerning the firm's economic assets. Third, economic assets are defined as providing a sufficient statistic for predicting next periods' economic assets, including the changes in assets arising from revenues and expenses.

I now summarize the results I find. First, there are four ways in which insider trading affects overall investor informedness (as measured by the investor state-estimation error covariance matrix):

1. The manager's trade volume is reported which, all things equal, reduces investor error

2. The manager under-reports the magnitude by which accounting assets deviate from their respective means, but this has no informational impact

3. The manager anti-smooths the period accounting error which, all things equal, increases investor uncertainty, and

4. The manager can either under-report or over-report the magnitude by which economic assets deviate from their respective means, which, respectively and all things equal, reduces or increases investor error.

The first factor is exogenous since the report is mandatory and so always informative when investors are rational. The remaining effects are properties that I
find in the model’s equilibrium. Understanding the effect of insider trading on investor informativeness is thus answered by determining how the combination of these four forces varies with accounting accuracy and regulatory stringency.

To this end, I define an information environment to be poor when the precision of investor estimates just prior to the insider trading report is smaller than the precision of the signal contained in the insider trading report. I first show that when the information environment is poor, the comparison of accounting systems described above is possible. Next, I show that two quantities (algebraic functions of the model’s exogenous parameters) are sufficient to describe the precision of the no-insider trading firm accounting system as compared with the original insider trading firm accounting system (with the precision characterized as the determinant of the inverse accounting noise covariance matrix).

The first parameter characterizing the change in total investor informedness due to insider trading corresponds to the information lost in the accounting report due to earnings management. The second is a form of “value relevance”, but value relevance from the manager’s perspective, not that of the investors. The manager’s value relevance measures the gain of earnings management through insider trading profits relative to the manager’s personal audit disutility.

I show that if the precision of the insider trading firm’s accounting system exceeds a threshold, the accounting system precision of a firm without insider trading needed to achieve the same investor informedness would be lower than that of the insider trading firm’s accounting system. Since a more precise accounting system is needed in the insider trading firm to achieve the same level of investor uncertainty in the non-insider trading firm, insider trading has reduced overall investor informedness. Finally, the threshold is increasing in the value
relevnce parameter but has an interior minimum with respect to the accounting informativeness reduction parameter.

The intuition for these results is as follows. Hold fixed the earnings management -induced reduction in investor informedness from accounting. Then increasing the manager's value relevance (the gain of earnings management through insider trading profits relative to the manager's personal disutility) leads to increased profit opportunity from insider trading. This increases the manager's expected trade volume which increases the signal-to-noise ratio of the insider trading report.

Next, consider varying the earnings management -induced reduction in investor informedness due to accounting. Two opposing effects on information are observed. First, there is the direct effect that, by definition, the informativeness of the accounting report falls. However, a decrease in accounting informativeness (with value relevance held constant) can arise solely due to increased earnings management. In turn, this increased earnings management must be motivated by the potential of higher insider trading profits which again leads to higher trading volume, since only insider trading causes earnings management that affects information. Higher manager trading volume implies the signal-to-noise ratio, and thus the informativeness, of the insider trading report increases. For small accounting informativeness reductions, the decrease in accounting informativeness has a larger magnitude than the increase from higher insider trading volume (the opposite occurs for small accounting informativeness reductions). Consequently, there is an interior minimum accounting informativeness reduction for the threshold accounting precision dividing the insider trading better (for information) and insider trading worse regions.
1.0.1 Background and Literature Review

My work described in this thesis contributes to the earnings management and insider trading literatures. Most importantly, prior work does not simultaneously consider both earnings management and insider trading, as I do here.

I show that an increase in accounting precision can reduce accounting informativeness. Prior work has also found negative consequences from increasing accounting precision with different settings and metrics. Baiman and Verrecchia (1996) study the effect of insider trading rather than earnings management on investment efficiency (their model assumes no earnings management) and find that investment is reduced as accounting precision increases. Kanodia, Singh, and Spero (2005) (with no earnings management or insider trading) find that an intermediate level of accounting precision is best when investors are uncertain as to the productivity of an investment and when the manager's investment level is unverifiable. Ewert and Wagenhofer (2005) (extending Fischer and Verrecchia (2000)) have do not model insider trading but include real earnings management with accounting earnings management and find that earnings management can increase as accounting precision increases. However, they define the expected level of earnings management to include both the component that can be unraveled and the part that cannot. Consequently, their finding that earnings management may increase as regulatory stringency increases does not imply that accounting informativeness falls and, in fact, without insider trading they find that informativeness increases monotonically in accounting precision. In contrast, I focus solely on the component of earnings management that cannot be unraveled, as this is the only component that affects accounting informativeness.
My work finds that low regulatory stringency together with high accounting precision leads the manager to use earnings management to reduce the importance of economic income in his report to near zero. This is similar to (but not the same as) fully or partially pooling reporting equilibria found elsewhere.\textsuperscript{3} Even without accounting error, partially pooling equilibria are hard to find in my model because the manager profits from insider trading whether the equity price is too high or too low. This makes off-equilibrium, but rational, 'punishments' difficult to find.

I also demonstrate a threshold regulatory stringency (which separates equilibria in which accounting informativeness increases with accounting precision from those in which informativeness decreases with accounting precision) as well as both smoothing and anti-smoothing. A threshold of a different nature is found in the earnings management model of Sankar and Sunramanyam (2001). While they have no insider trading or accounting error, they allow the degree of reversal of earnings management to vary between zero and 100%. For low degree of reversal, the manager smooths income, while for high degree of reversal the manager anti-smooths without bound (they have no audit disutility). Kirschenheiter and Melumad (2002) also have both 'big-bath' (anti-smoothing) and smoothing. In their setting, the manager receives only a one dimensional signal, the sum of permanent and transitory income. Smoothing and anti-smoothing arise for different magnitudes of the combined signal due to nonlinearity (arising from the precision of transitory income not being common knowledge).

\footnote{Pooling may arise either from exogenous compensation nonlinearity (with respect to reported signals) or endogenously (e.g., if the manager has equity-based compensation and the pooling reporting equilibrium leads to nonlinear pricing). Examples of exogenous nonlinearity include Healy (1985) (via the assumed contract) and Liang (2004) (via a non-Normal distribution of values). Examples of endogenous nonlinearity include Guttman, Kadan and Kandel (2004) wherein nonlinearity arises from equity compensation and a pooling price equilibrium. Finally, Kirschenheiter and Melumad (2002) have both sources of nonlinearity.}
Finally, I mention that the link between insider trading and earnings management has been established empirically. Vargus and Beneish (2001) and Rogers and Stocken (2005) find evidence of earnings management before insider selling. Beneish and Vargus (2004) find evidence that insiders manage earnings upwards after selling when earnings dip (to avoid litigation). Richardson, Teoh, and Wysocki (2004) find evidence that managers ‘walkdown’ earnings estimates to beatable forecasts most when they sell shares after the subsequent earnings “surprise.” Bartov and Mohanram (2004) find that large stock option exercises are associated with past inflated earnings and subsequent abnormally low returns. Seyhun (1992) shows that stricter insider trading regulations has prevented neither insider trading nor insider trading profits when insider trading takes place after major news announcements.
Chapter 2

Two Period Model

This chapter introduces the basic model used to understand the effect of insider trading on earnings management. The question is whether higher accounting precision will improve accounting informativeness when insider trading is possible. My analysis leads to the following main result: when regulatory stringency is low, then increasing accounting precision decreases the informativeness of the accounting report towards zero. In contrast, when regulatory stringency is high, an increase in accounting precision increases the informativeness of the accounting report. This suggests that more (and presumably more accurate) fair value accounting may reduce informativeness either when it is harder to audit or when insider trading enforcement is sufficiently lax.

Insider trading allows the manager to profit from his information advantage whether "news" is good or bad. Importantly, how he does so differs for economic income\(^1\) and accounting error (which fully reverses in period two) due to the persistence of economic income. If the accounting error is positive (recall that both accounting error and economic income have zero mean so that zero divides good

\(^1\)Strictly speaking, what I call economic income isn't income. Instead, it is a signal of the future liquidation value of the firm. However, the liquidation value of the firm is proportional to economic income, and I simply refer to it as economic income for brevity.
and bad news), the firm is over-priced and so the manager sells more shares than usual. To increase profits from trading, the manager uses earnings management to increase any mispricing by increasing (anti-smoothing) the accounting error. The opposite considerations apply when accounting error is negative, but they lead to the same conclusion that accounting error is anti-smoothed (now, made more negative). Here, anti-smoothing has the same meaning as in Lambert (2001); i.e., it means increasing the magnitude of accounting error in the report while preserving the sign. Smoothing, in contrast, would imply reducing the effect in the report.

The motive to anti-smooth economic income is weaker than the motive to smooth. If the manager sells shares due to positive economic income, he will have fewer shares left to take advantage of the higher terminal value that follows from the persistence of economic income. In equilibrium, the response of the equity price to the manager's report (which I refer to as the earnings response coefficient, or ERC) is always less than one. Consequently, the immediate gain from insider trading (which increases the ERC) is less than the loss from forgone terminal value. It follows, then, that when economic income is positive, the manager will want to buy, and so will smooth by reducing the positive economic income in the report to get a lower price. When economic income is negative, the manager will want to sell, and so will smooth by increasing the economic income in the report to get a higher price. Note that this smoothing is purely an insider trading effect and not related to discounting or risk aversion.

It is the smoothing of economic income that is responsible for the equilibrium properties described above. The level to which the manager smooths economic income is determined by equating the marginal benefit (to the manager) of smooth-
ing economic income arising through insider trading to his marginal cost from regulatory enforcement. For large regulatory stringency, the gain from smoothing economic income falls as ERC increases because of a direct effect: higher ERC reduces the difference between the first period price and second period terminal value. Consequently, accounting informativeness increases in the ERC when regulatory stringency is high. In contrast, when regulatory stringency is low the gain from smoothing economic income increases as the ERC increases due to an indirect effect: insider trading and earnings management are complements in that higher insider trading increases the gains from earnings management. Since a higher ERC increases the gains from insider trading, it also increases the gains from earnings management. Consequently, when regulatory stringency is low, smoothing of economic income increases, and informativeness thus decreases in the ERC. Finally, since the ERC always increases with increasing accounting precision, I find that informativeness increases with accounting precision when regulatory stringency is high, but decreases when regulatory stringency is low.

These results are demonstrated when insider trading occurs after the accounting report. I also consider the effect of having insider trading precede the accounting report. This is an important difference because of the presumption that trading before the report allows the manager to exploit non-public material information. Consequently, as documented by Seyhun (1992), tighter regulatory stringency on insider trading has indeed significantly reduced insider trading before reports, but insider trading after reports has increased, as have insider profits. However, the intuition underlying relative tolerance of insider trading after accounting reports fails to consider the effects that future insider trading will have on the quality of reports. I show that the two orders have different impacts on
total investor information and that neither is unambiguously superior. conclusion. I don’t know how well this is emphasized in the Conclusion section.]

The remainder of this chapter is organized as follows. Section 1 introduces the basic two-period model of the firm. Section 2 describes the optimal manager policies. Section 3 discusses the solution for the price equilibrium. Section 4 describes the behavior of the price equilibrium. Section 5 concludes. All proofs are in Appendix A. Robustness is considered in Appendix B.

2.1 Model

There is a firm and two players: a group of identical investors and a manager. All parameters other than the manager's private information signals (economic income and accounting error) are common knowledge. The first period has two stages: reporting and insider trading. Briefly, in stage 1a, the manager receives two private signals: economic income $m_1 \sim N(0, 1)$ (which has persistence to the next period) and accounting error $\eta \sim N(0, 1/s)$ (which reverses in the next period). In stage 1b, he chooses his earnings management and in stage 1c he releases his report. In stage 2a, the manager privately observes the firm's liquidation value. In stage 2b, he engages in insider trading, and in stage 2c investors receive a truthful report of the manager's insider trading volume. The firm liquidates in period two. I now discuss these stages in more detail.

In period one, the intrinsic value of the firm is best predicted by economic income, $m_1$. Thus, in this basic model, $m_1$ combines two roles: it is a proxy not only for the first period economic income but also a sufficient statistic for the next period’s economic income. This combining of roles is purely for simplicity and not necessary for my results. The manager computes economic income using
both accounting and non-accounting information as well as superior knowledge of how this information predicts future value. Except to the manager, economic income is both unobservable and unverifiable.

In contrast, the "accounting system" (which includes both transactional records as well as the knowledge and experience of the auditors) produces an imperfect estimate of economic income, which I call accounting income. Accounting income is what the outside auditors believe the manager should report in stage 1c based on their information and knowledge of the firm and accounting. Taking an accounting-centric approach, I define the accounting error so that accounting income equals \( m_1 + \eta \).

Because auditors have limited power and recognize that their information and knowledge is inferior to that of the manager, the auditors allow the manager to add a deviation \( \delta \) to accounting income (at a cost described in the next paragraph) to yield the manager's "reported income,"

\[
I_1 = m_1 + \eta + \delta.
\]  

Since I assume that the manager observes \( m_1 \) exactly, accounting precision of \( s \to \infty \) could be achieved by requiring the manager to truthfully report \( m_1 \). This is analogous to the case of "managerial discretion" of Dye and Verrecchia (1995) because the valuation "algorithm" is for the manager to report his private information. Of course, the manager has no incentive to tell the truth (at least within my model), and such a prescription would be difficult to audit. Nonetheless, since the report (under this rule) should be \( m_1 \), all difference between \( m_1 \) and the actual report \( m_1 + \delta \) is due to the intentional deviation \( \delta \), and \( \eta = 0 \). Countermanding the accounting earnings management \( \delta \) is an auditor disutility.
(personal to the manager) $\nu \delta^2 / 2$, where $\nu$ models the audit stringency. Like $s$, $\nu$ is exogenous.

The manager receives compensation in each period as a function of reported income and is granted an initial equity stake $\epsilon$. Without loss of generality, I normalize the number of shares outstanding to be one so that $\epsilon \in [0,1)$. Wages are linear in reported income:

\begin{equation}
\omega_t = \theta_t + \phi_t I_t, \quad t = 1, 2,
\end{equation}

where $I_2$ is the second period reported income (equal also to accounting income since the firm liquidates) defined as

\begin{equation}
I_2 = m_1 + m_2 - I_1
\end{equation}

\begin{equation}
= m_2 - \eta - \delta.
\end{equation}

The contract parameters $\theta_t$ and $\phi_t$ are exogenous, and my conclusions are independent of their values, provided, as I assume henceforth, that $\phi_2 < 1$. $m_2$ is the second period economic income. Second period income relates to first period economic income by

\begin{equation}
m_2 = \rho m_1 + z,
\end{equation}

where $z \sim N(0, \sigma^2)$ is a shock to second-period economic income (discussed further in the next paragraph) and $\rho$ is the persistence of economic income. I assume that $-1 < \rho < 1$. Note that the second period accounting income $I_2$ reverses the accounting error $\eta$ as well as any first period earnings management $\delta$.

The shock $z$ represents any additional information impacting the terminal value learned by the manager at the time of his insider trading. For simplicity,
this is the only information impacting the terminal value, $TV$, which is thus given by

\[(2.6) \quad TV = m_1 + m_2 - \omega_1 - \omega_2.\]

After issuing the report $I_1$ in stage 1c, in stage 2a the manager observes $z$ privately and then, in stage 2b, trades an amount $e$ of his shares. In Section 5, I consider the effect of reversing the order of insider trading and the accounting report release. Thus, the economic shock $z$ allows one to distinguish these two cases.

The manager’s trade nets him cash $eP(I_1)$ with $P(I_1)$ the equity price after the report $I_1$. Given the trade volume $e$, the manager suffers the disutility $\chi e^2/2$. The exogenous parameter $\chi$ models the stringency of the enforcement of the ban on insider trading on the manager (insider trading stringency, for short). This includes such things as lawsuits, jail, pressure from the board of directors, etc. There are no short-sale restrictions and $\chi$ is exogenous. Together, the audit stringency parameter $\nu$ and the insider trading stringency parameter $\chi$ determine the overall regulatory stringency. I will soon introduce a single regulatory stringency parameter (a function of $\nu$ and $\chi$) to capture the combined effect of the individual stringencies $\nu$ (for the audit) and $\chi$ (for insider trading).

For simplicity, I assume that the manager’s trade volume is small enough that it has no informational effect at the time of the trade (there will be an informational effect when the trade volume is reported in stage 2c). In the extensions appendix, I report the effect of including strategic insider trading effects that arise when the manager’s trade volume is non-negligible and so imparts information to investors. I consider two cases: market microstructure effects (e.g., Kyle (1985));
a rational expectations equilibrium in which investors also observe private signals predicting the firm’s terminal value (e.g., Grossman, and Stiglitz (1980) or Grundy and McNichols (1989)). For both models, there is no qualitative change from the base model with negligible insider trading volume.

The manager is assumed risk-neutral and obtains utility from his terminal wealth, $W_2$. Thus, his utility is

\begin{equation}
    U = -\frac{1}{2}(\nu\delta^2 + \chi e^2) + W_2
\end{equation}

\begin{equation}
    W_2 = \omega_1 + eP(I_1) + \omega_2 + (\epsilon - \epsilon)TV
\end{equation}

I have repeated the calculations (not reported) with exponential utility and utility from intermediate consumption. There is no qualitative change in my results and so risk-neutrality is assumed to simplify the exposition.

**Definition 1** If insider trading is feasible, a perfect Bayesian equilibrium is one in which the manager’s policies $u^* = (e^*, \delta^*)$ are optimal given conjectures about the investors’ price function, the investors’ price function is Bayesian given their conjectures of the manager’s policies, and all conjectures are correct. Thus, if $\hat{P}(I_1)$ represents the manager’s conjecture concerning the post report price and $\hat{u}$ the investors’ conjecture, the equilibrium satisfies

\begin{align*}
    \delta^*(\hat{P}, m_1, \eta) &= \arg\max_{\delta} E_{\epsilon}(U(u, m_1, \eta, z)|\hat{P}, m_1, \eta, e^*) \\
    e^*(\hat{P}, m_1, \eta, z) &= \arg\max_{e} (U(u, m_1, \eta, z)|\hat{P}, m_1, \eta, z) \\
    P(I_1|\hat{u}) &= E_{m_1, \eta, \delta}(TV|I_1, \hat{u}) \\
    \hat{P}(\bullet) &= P(\bullet|\hat{u}) \quad \hat{u} = u^*
\end{align*}

Note that the manager’s optimizations are performed sequentially, first over $\delta$ with economic income $m_1$ and accounting error $\eta$ observed and then over $e$ with
also observed (this is suggested by the presence or absence of expectations). I only consider “pure-strategy” equilibria, i.e., equilibria in which the manager’s policies are chosen from pure strategies.

Because the manager’s private signal is higher dimensional than his message (here, two vs. one dimension) there is always pooling in the report. In other words, the entire one parameter family of accounting errors parameterized by \( m_1 \) as \( \eta = \mathcal{I}_1 - m_1 \) result in the same accounting income \( \mathcal{I}_1 \) and so cannot be distinguished, even if there is no earnings management. Nonetheless, because the price \( P(\mathcal{I}_1) \) depends only on the one-dimensional report \( \mathcal{I}_1 \), the price equilibrium can either be separating or pooling. I only consider linear separating equilibria (i.e., positive slope), and to ensure linearity it is sufficient (but not necessary) that investors’ conjecture that the manager’s policies are linear and pure-strategy\(^2\). Once specializing to linear equilibria, I write the manager’s earnings management and insider trading polices and investors’ price function as, respectively,

\[
\begin{align*}
\delta^* &= \delta_0 + \delta_\eta \eta + \delta_m m_1 \\
\epsilon^* &= \epsilon_0 + \epsilon_\eta \eta + \epsilon_m m_1 + \epsilon_z z \\
P(\mathcal{I}_1) &= p_0 + p_1 \mathcal{I}_1
\end{align*}
\]

That the manager anti-smoothes accounting error and smoothes economic income will be reflected by the facts (proved shortly) that \( \delta_\eta > 0 \) and \( \delta_m < 0 \). If \( \delta_m = -1 \), then the effect of economic income \( m_1 \) in the report vanishes (since then \( (1 + \delta_m)m_1 = 0 \)). I call this total smoothing.

The next section derives the manager’s optimal policies as a response to an assumed price function, \( P(\mathcal{I}_1) \). While these policies are functions of the as yet

\(^2\)See the extensions appendix for less restrictive assumptions leading to a linear equilibrium.
undetermined $P(I_1)$, nonetheless some conclusions can be reached even at this first step.

2.2 The Manager’s Policies

I begin with the benchmark case without insider trading and then turn to the case with insider trading. Of particular interest is the effect of earnings management on investor information. I characterize investor information by the precision (i.e., inverse of the variance) of the investors’ distribution of economic income $m_1$. The informativeness of a report is the increase in precision over that which was held before the report. I use the notation

\begin{equation}
Prec_{\text{NoIT}}(m_1|I_1) = (\text{Var}(m_1|I_1, e = 0))^{-1}
\end{equation}

(2.12)

to denote the precision without insider trading. Similarly, $Prec_{\text{IT}}(m_1|I_1)$ denotes the precision with insider trading after only the accounting report has been released (i.e., at stage $1c$) and $Prec_{\text{IT}}(m_1|I_1, e)$ denotes the precision with insider trading after both the accounting and insider trading reports have been released (i.e., at stage $2c$). To compute the “informativeness of accounting,” subtract the precision held by investors before the accounting report from that held after (note that prior to any reports, investors have precisions $Prec_0(m_1) = 1$ for economic income and $Prec_0(z) = 1/h^2$ for $z$). I also study informativeness with respect to the firm’s terminal value $TV$ but, unless explicitly mentioned, informativeness means informativeness with respect to the first period economic income, $m_1$. They differ because $TV$ depends on the stage $2a$ shock $z$ while $m_1$ does not.
2.2.1 Policies If Insider Trading is Impossible

If insider trading is impossible,

Lemma 1 If insider trading is impossible, the manager's earnings management policy is independent of his private information. Consequently, using equation (2.12), investor information is given by the precision

\[ \text{Prec}_{\text{NoIT}}(m_1|I_1) = 1 + s \]

The above result follows from the optimal earnings management policy

\[ \delta^* = \frac{1 - \epsilon}{\nu} (\phi_1 - \phi_2) \]

which is obtained by equating the marginal costs and benefits of accounting earnings management in the manager's quadratic certainty equivalent. If \( \phi_1 = \phi_2 \), accounting earnings management is identically zero. Of course, even if \( \phi_1 \neq \phi_2 \), since the accounting earnings management is a constant independent of \( m_1 \) or \( \eta \), it can be unraveled. Because earnings management can be unraveled, it does not affect investor inference, and the precision of the investors' \( m_1 \) estimate is unchanged from that of perfect auditability (which is \( 1 + s \) from Bayesian updating with a report of precision \( s \)). This result is consistent with Guttman, Kadan and Kandel (2004) who find earnings management is unraveled even with equity compensation if the price equilibrium is separating.

2.2.2 Policies If Insider Trading is Feasible

I now consider the manager's policies with insider trading. The manager maximizes his utility given in (2.7) over his two actions (insider trading \( e \) and earnings management \( \delta \)). These optimizations are performed sequentially, first over \( \delta \) with
economic income \( m_1 \) and accounting income \( \eta \) observed and then over \( e \) with \( z \) also observed.

Rational, risk-neutral pricing implies that the investor determined price is the investors' estimate of the terminal value based on the accounting report \( I_1 \). This can be decomposed into a part that is completely determined by the report and a component that still depends on the unknown first-period economic income, \( m_1 \). It is only this later component that, in equilibrium (i.e., when investor conjectures are correct), can contain earnings management that cannot be unraveled and so lead to manager profits from earnings management through insider trading. Thus, it is useful to isolate this portion of the price response to study the manager's policies and overall equilibrium.

To this end, begin with the \( P(I_1) = E_{m_1, \eta, z}(TV|I_1) \), and note that

\[
E_{m_1, \eta, z}(TV|I_1) = E_{m_1, \eta, z}(m_1 + m_2 - (\omega_1 + \omega_2)|I_1) = E_{m_1, \eta, z}((1 + \rho)m_1 + z - (\theta_1 + \phi_1 I_1 + \theta_2 + \phi_2((1 + \rho)m_1 + z - I_1)|I_1) = (\phi_2 - \phi_1)I_1 - (\theta_1 + \theta_2) + cE_{m_1, \eta, z}(m_1|I_1)
\]

(2.15)

where I have used \( E_{m_1, \eta, z}(z|I_1) = 0 \) and defined

(2.16)

\[
c = (1 + \rho)(1 - \phi_2)
\]

The component \( (\phi_2 - \phi_1)I_1 - (\theta_1 + \theta_2) \) is completely known once \( I_1 \) is reported while \( cE_{m_1, \eta, z}(m_1|I_1) \) is the price component that still requires investor conjectures and estimation.

The "terminal value coefficient," \( c \), has an important role to play in the subsequent analysis. For each dollar of economic income, the manager will receive \( c \) dollars per share that he still holds at liquidation. In contrast, for each dollar
of accounting error, he receives nothing at liquidation. As emphasized earlier, the key aspect of earnings management with insider trading is the differential treatment of economic income \( m_1 \) and accounting error \( \eta \), and we will see that \( c \) measures the extent of this difference.

I now complete the decomposition or price into a certain and uncertain component. Since the equilibrium is linear, the estimate \( cE_{m_1, \eta, z}(m_1|I_1) \) will be linear in \( I_1 \). Thus, define \( \bar{p}_1 \) and \( \bar{p}_0 \) by

\[
(2.17) \quad \bar{p}_0 + \bar{p}_1 I_1 = cE_{m_1, \eta, z}(m_1|I_1)
\]

From the definition of the linear price function, \( P(I_1) = p_0 + p_1 I_1 \) and equations (2.15) and (2.17)

\[
(2.18) \quad p_0 = \bar{p}_0 - (\theta_1 + \theta_2)
\]

\[
(2.19) \quad p_1 = \bar{p}_1 - \phi_1 + \phi_2
\]

The cum-wage ERC \( \bar{p}_1 \equiv p_1 + \phi_1 - \phi_2 \), in the following referred to simply as the ERC, is the component of the price response directly related to estimation of \( m_1 \). \( \bar{p}_1 \) plays an important role in the following analysis. Note that \( \bar{p}_1 \) is also the marginal benefit to earnings management arising through insider trading.

With these definitions, the manager's terminal wealth (defined in equation (2.8)) is

\[
W_2 = (1 - \epsilon)(\omega_1 + \omega_2) + \epsilon((1 + \rho)m_1 + z) + \epsilon(\bar{p}_0 + \bar{p}_1 I_1 - cm_1 - (1 - \phi_2)z)
\]

Note that \( \eta \) appears in \( W_2 \) only as a component of the accounting report \( I_1 \) while \( W_2 \) has an additional dependence on \( m_1 \) given by \(-cm_1\). This shows that

---

3Strictly speaking, I should use \( \hat{p}_0 \) etc. to denote the fact that the manager's response policies depend on a conjectured price. To avoid cumbersome notation (and, anticipating the fact that, in equilibrium, conjectures are correct), I drop the \( \hat{\cdot} \).
indeed $c$ captures the distinction between $m_1$ and $\eta$ with regard to the manager's wealth and hence with regard to his subsequent earnings management and insider trading policies.

I now proceed with the manager’s optimizations. In stage 2a, the manager observes $z$ and in stage 2b optimizes his insider trading volume $e$. The first order condition for his optimization over $e$ is

\begin{equation}
\chi e = p_0 + p_1 \mathcal{I}_1 - cm_1 - (1 - \phi_2)z
\end{equation}

which is solved for $e$ to yield

\begin{equation}
e^* = \frac{p_0 + p_1 \mathcal{I}_1 - cm_1 - (1 - \phi_2)z}{\chi}.
\end{equation}

Now consider the manager’s stage-1b optimization over $\delta$. $z$ has not yet been observed, but the manager knows his stage 2b policy $e^*$. We substitute the policy given in equation (2.21) and use $z \sim N(0, h^2)$ to obtain the manager’s expected program at stage 1b:

\begin{equation}
W_2^* = \max_{\delta} \mathbb{E}_x(W_2|m_1, \eta, \delta)
\end{equation}

\begin{equation}
= W_2^0 + \max_{\delta} \left( (1 - \epsilon)(\phi_1 - \phi_2)\delta + \frac{1}{2\chi}(p_0 + p_1 \mathcal{I}_1 - cm_1)^2 \right)
\end{equation}

where $W_2^0 = (1 - \epsilon)(\theta_1 + \theta_2 + (\phi_1 - (1 - \rho)\phi_2)m_1 + (\phi_1 - \phi_2)\eta) + \epsilon(1 + \rho)m_1$ is the part of the manager’s wealth independent of his earnings management $\delta$. Using $\mathcal{I}_1 = m_1 + \eta + \delta$, the manager’s first order condition for the optimization over $\delta$ gives

\begin{equation}
\delta^* = \frac{1}{\chi \nu - \nu^2} \left( \chi(1 - \epsilon)(\phi_1 - \phi_2) + \nu(p_0 + p_1 \eta + (\phi_1 - c)m_1) \right)
\end{equation}
Finally, the second order condition is that \( \chi \nu > \tilde{p}_1^2 \). As will be seen, this always holds in the linear equilibrium.

The result of the manager’s optimizations are policies whose key components are summarized by the following lemma.

**Lemma 2** Assume insider trading is feasible and the manager’s conjecture \( P(I_1) \) is linear. Then the manager’s policies are linear in his private information and are described by

\[
\delta_m = \frac{\bar{p}_1 (\bar{p}_1 - c)}{\chi \nu - \bar{p}_1^2}
\]

\[
\delta_\eta = \frac{\bar{p}_1^2}{\chi \nu - \bar{p}_1^2}
\]

\[
e_m = \frac{\nu}{\bar{p}_1} \delta_m, \quad e_\eta = \frac{\nu}{\bar{p}_1} \delta_\eta
\]

\[
e_z = -\frac{(1 - \phi_2)}{\chi}
\]

Because the component \( \delta_0 \) can be unraveled, only the components \( \delta_\eta \) and \( \delta_m \) are needed to understand informativeness and so only these values are reported and studied. Assuming, for the moment, that \( \bar{p}_1 > 0 \) (which will be shown to be true in equilibrium) this lemma has two key results: \( \delta_\eta > 0 \) and \( \delta_\eta > \delta_m \) (because \( \rho > -1 \) and \( \phi_2 < 1 \) imply \( c > 0 \)). Thus accounting error \( \eta \) is always anti-smoothed and always anti-smoothed more that economic income \( m_1 \), and the difference is increasing in the terminal value coefficient \( c \).

The fact that \( \delta_\eta > \delta_m \) has important consequences for investor inference. In particular, it implies that insider trading always reduces the “post-earnings management signal to noise ratio” (signal to noise ratio for short) from that which obtains without insider trading. The signal to noise ratio is an important concept in understanding investor inference. It is defined by
**Definition 2** The signal to noise ratio is

\[ R = \frac{\text{Var}((1 + \delta_m)m_1)}{\text{Var}((1 + \delta_\eta)\eta)} \]

Note that, using the policies given by equation (2.25) and (2.26),

\[ R = \sqrt{s} \frac{1 + \delta_m}{1 + \delta_\eta} \]

By Bayes rule, the precision of \( m_1 \) after the accounting report is \( \text{Prec}_{IT}(m_1|I_1) = 1 + R^2 \) (without insider trading \( \delta_m = \delta_\eta = 0 \) and so \( R = \sqrt{s} \) and \( \text{Prec}_{NoIT}(m_1|I_1) = 1 + s \), as found earlier). We see that \( \delta_\eta > \delta_m \) implies \( R < \sqrt{s} \), i.e., insider trading reduces the informativeness of accounting alone. To get an explicit value, substitute the policies given by equations (2.25) and (2.26) to find

\[ R = \sqrt{s}(1 - c\bar{p}_1/(\chi\nu)) \]

\( R \) increases in \( \chi \) and \( \nu \) because higher regulatory stringency decreases earnings management. \( R \) decreases in the terminal value coefficient \( c \) because higher \( c \) increases the difference between economic income and accounting error from the perspective of insider trading induced earnings management. Finally, \( R \) decreases in the ERC \( \bar{p}_1 \) because a higher ERC increases the gains from insider trading and this motivates larger earnings management.

### 2.3 The Equilibrium Price

In this section, I solve for the equilibrium ERC \( \bar{p}_1 \). We have already seen that if the manager’s conjectured ERC \( \bar{p}_1 \) is everywhere nonzero then the manager’s optimal policies, conditional on \( I_1 \), are linear in his private information. The next proposition characterizes the linear equilibrium.
Proposition 1  If the manager’s conjectured price function $\hat{P}(I_1)$ is linear and investor conjectures concerning the manager’s policies are linear, then the price equilibrium is unique. The stable equilibrium value of $\bar{p}_1$ is determined by the equation

$$\bar{p}_1 = c\sqrt{s}\left(1 - \frac{\bar{p}_1^2}{\chi\nu}\right) \frac{R}{1 + R^2}$$

Proposition 1 follows from Bayesian updating and the fact that rational risk-neutral pricing implies $\bar{p}_1 = d(cE_{m_1,\eta,\epsilon}(m_1|I_1))/dI_1$. The equation determining $\bar{p}_1$ is quadratic and so, if a non-degenerate root exists, exactly one root is stable. The following Corollary identifies this unique stable root.

Corollary 1  The equilibrium equation for the ERC $\bar{p}_1$ can be solved

$$\bar{p}_1 = c\frac{(s+1)b^2 + s - \sqrt{(s+1)^2b^4 - 2s(s-1)b^2 + s^2}}{2s}$$

where

$$b = \frac{\sqrt{\chi\nu}}{c}$$

The ERC $\bar{p}_1$ increases in both accounting precision $s$ and relative regulatory stringency $b$ (for all $s$ and $b$) and $\lim_{s \to \infty} \bar{p}_1 = c$ for all $b$.

Recall that $\bar{p}_1 < \sqrt{\chi\nu} = bc$ (from the second order condition) and $R = 1 - c\bar{p}_1/\chi\nu = 1 - \bar{p}_1/(cb^2)$. Thus, only if $b < 1$ can $R$ vanish in equilibrium. This, suggests taking $b$ as the measure of relative regulatory stringency. The parameter $b$ trades off the strength of regulatory stringency $\sqrt{\chi\nu}$ (which acts against insider trading and earnings management) with the gain from earnings management through insider trading (as measured by the terminal value coefficient $c$).
The relative regulatory stringency parameter $b$ will be central in the following analysis.

This section has determined that there is a unique stable equilibrium $\bar{p}_1$ and computed its value as a function of accounting precision $s$, relative regulatory stringency $b$, and terminal value coefficient, $c$. I now determine the implications of the equilibrium for informativeness.

### 2.4 Behavior of the Price Equilibrium

The key observation for understanding how the equilibrium varies with relative regulatory stringency $b$ and accounting precision $s$ is the following:

**Lemma 3** If $\bar{p}_1$ is to remain finite as $s \to \infty$, there are two possibilities:

$$\lim_{s \to \infty} R = \begin{cases} \infty & \text{or,} \\ 0 & \end{cases}$$

To understand why the lemma holds, note that from equation (2.32) obeyed by the equilibrium ERC $\bar{p}_1$, finite $\bar{p}_1$ as $s \to \infty$ requires that the Bayesian factor $R/(1 + R^2)$ vanish as $1/\sqrt{s}$ as $s \to 0$.

This lemma may appear innocuous and perhaps trivial. In particular, the possibility that $\lim_{s \to \infty} R = \infty$ is intuitive: as accounting error tends to zero, one expects investor inference to become increasingly accurate. However, it has significant implications if the possibility $\lim_{s \to \infty} R = 0$ also occurs since $R$ determines investor information. As will be seen, $\lim_{s \to \infty} R = 0$ indeed happens for both the basic model considered so far as well as for models considered in the extension appendix. Moreover, as shown in the next proposition, the decrease in the signal to noise ratio $R$ with respect to $s$ can begin at quite small values of $s$. But first, an important lemma:
Lemma 4 For all parameter values, \( \bar{p}_1 \) is positive and increasing in both accounting precision \( s \) and relative regulatory stringency \( b \). For fixed \( s \), the anti-smoothing of accounting noise \( \eta \) (i.e., \( \delta_\eta \)) is maximized at a relative regulatory stringency value \( b_0(s) \) given by

\[
b_0(s) = \sqrt{\frac{s}{s + 1}}.
\]

Finally, the earnings management coefficient \( \delta_m < 0 \) decreases in \( s \) (towards \(-1\)) if \( b < b_0(s) \).

Proposition 2 The signal to noise ratio \( R \) decreases in \( s \) if \( b < b_0(s) \) and vice versa.

This proposition demonstrates and quantifies the picture of insider trading-motivated earnings management painted in the introduction. The signal to noise ratio \( R \) as well as the two earnings management coefficients (\( \delta_m \) and \( \delta_\eta \)) all have different behavior depending on whether \( b < b_0(s) \) or \( b > b_0(s) \). In fact, the smoothing of economic income \( m_1 \) drives the behavior of \( R \) and \( \delta_\eta \).

The proposition implies that, so long as \( b < 1 \), if \( s > b_0^{-1}(b) = b^2/(1 - b^2) \), the signal to noise ratio \( R \) decreases in \( s \). But Lemma 3 implies that \( \lim_{s \to \infty} R = 0 \). This is formalized in the following corollary.

Corollary 2 As \( s \to \infty \), the unique stable equilibrium has the property that

\[
\lim_{s \to \infty} R = \begin{cases} 
\infty & \text{If } b > 1 \\
0 & \text{If } b < 1 
\end{cases}
\]

To understand how smoothing of economic income \( m_1 \) leads to Proposition 2, recall that the goal of smoothing \( m_1 \) is to disguise economic income so as to either buy (if \( m_1 > 0 \)) or sell (if \( m_1 < 0 \)) shares. For large relative regulatory
stringency, the gain from smoothing economic income falls with ERC because of a direct effect: higher ERC reduces the difference between the first period price and second period terminal value. Consequently, accounting informativeness increases in the ERC when regulatory stringency is high. In contrast, for low regulatory stringency, the gain to smoothing of economic income increases in the ERC due to an indirect effect: insider trading and earnings management are complements. This follows from the fact that \( \partial^2 W_2 / \partial e \partial \delta = \bar{p}_1 \) and so larger earnings management \( \delta \) increases the marginal benefit of insider trading \( e \) and vice versa. Consequently, informativeness decreases in the ERC when regulatory stringency is low. Since the ERC always increases with increasing accounting precision, (first part of Proposition 2) the behavior of smoothing with \( s \) follows.

The fact that the earnings management coefficient \( \delta_m \) increases (decreases) in accounting precision \( s \) if relative regulatory stringency \( b > b_0(s) \) (\( b < b_0(s) \)), explains the behavior of \( R \) as a function of \( s \) and \( b \). For relative regulatory stringency \( b > b_0(s) \), smoothing of economic income becomes negligible at large accounting precision \( s \) and so, since the anti-smoothing of accounting error is finite, \( R \) increases as \( \sqrt{s} \). For relative regulatory stringency \( b < b_0(s) \), smoothing becomes near total (i.e., \( \delta_m \rightarrow -1 \)) at large accounting precision \( s \) and so \( R \) eventually decreases with \( s \).

The fact that the earnings management coefficient \( \delta_m \) increases (decreases) with accounting precision \( s \) if \( b > b_0(s) \) (\( b < b_0(s) \)) also explains the behavior of the earnings management coefficient \( \delta_m \) as a function of \( s \) and \( b \). The gain from anti-smoothing accounting error always increases in the ERC. Offsetting the gain, is the cost from regulatory stringency. For \( b > b_0(s) \), smoothing of economic income is not yet significant. Consequently, the fall in the ERC as \( b \) is reduced is more
than offset by lowered regulatory costs and so anti-smoothing of accounting error increases as regulatory stringency decreases. For \( b < b_0(s) \), smoothing of economic income is significant and so the ERC falls rapidly as \( b \) is reduced. This lower gain from anti-smoothing accounting error more than offsets lowered regulatory costs and so anti-smoothing of accounting error now decreases as relative regulatory stringency decreases.

2.4.1 Comparing Informativeness

I close by comparing the combined report informativeness with and without insider trading. I show that if (as has been assumed) insider trading occurs after the accounting report, then upon release of both the accounting and insider trading reports, all residual uncertainty in the terminal value \( TV \) is resolved for investors, but there remains investor uncertainty concerning economic income, \( m_1 \). I then compare this result with that obtained by reversing the order of insider trading and the accounting report\(^4\).

Recall that the informativeness of a report is the difference in the precisions held before and after the report. Before any reports, \( \text{Prec}(m_1) = 1 \) and \( \text{Prec}(TV) = (c^2 \text{Var}(m_1) + (1 - \phi_2)^2 \text{Var}(z))^{-1} = 1/(c^2 + H) \), where \( H = (1 - \phi_2)^2 h^2 \) is the variance in \( TV \) arising from the shock \( z \). Thus, I define the following two measures of report informativeness,

\[
(2.38) \quad \text{Inf}_{IT}(m_1|I_1,e) = \text{Prec}_{IT}(m_1|I_1,e) - 1
\]
\[
(2.39) \quad \text{Inf}_{IT}(TV|I_1,e) = \text{Prec}_{IT}(TV|I_1,e) - 1/(c^2 + H)
\]

We have already seen insider trading reduces the informativeness of the ac-\(^4\)Specifically, with insider trading now taking place in stage 1b and reported in stage 1c, and the accounting report released at stage 2c.
counting report alone. The next Proposition describes what happens to investor information after both reports are released (but before $TV$ is revealed to investors).

**Proposition 3** When insider trading occurs after the accounting report, the total informativeness of the combined accounting and insider trading reports for economic income $m_1$ and terminal value $TV$ are, respectively,

\[
\text{Inf}_{IT}(m_1|I_1, e) = R^2 + \frac{(1 + p)^2}{h^2}
\]

\[
\text{Inf}_{IT}(TV|I_1, e) = \infty
\]

Moreover, $\text{Inf}_{IT}(m_1|I_1, e) > \text{Inf}_{NIT}(m_1|I_1) = 1 + s$ if and only if $s < \tau \equiv \frac{(1 + p)^2}{h^2}$ or $b > b_1(s)$ where

\[
b_1(s) \equiv \frac{\sqrt{s(s - \tau)^{3/4}}}{\sqrt{\left(\sqrt{s + s\sqrt{s - \tau}}\right)\left(\sqrt{s} - \sqrt{s - \tau}\right)}}
\]

The expression for $\text{Inf}_{IT}(m_1|I_1, e)$ follows from Bayesian updating. $R^2$ is the informativeness of the accounting report, and $\frac{(1 + p)^2}{h^2}$ that of the insider trading report. The result $\text{Inf}(TV|I_1, e) = \infty$ follows because the manager has observed $TV$ at the time his trade choice has been made. Since his insider trading profits depend only on $P(I_1)$ and $TV$, his insider trading policy depends only on $I_1$ and $TV$. Thus, after investors observe both $I_1$ and the trade volume $e$, they can infer $TV$.

If relative regulatory stringency $b < b_1(s)$ then total report informativeness concerning $m_1$ is lower with insider trading than without. While the lower informativeness of accounting alone is mitigated by the information provided by the manager’s trade volume, the precision gain from the insider trading report is independent of $s$ but the informativeness loss from the accounting report (due to
insider trading) increases with $s$. Thus, for large enough $s$ the loss in accounting informativeness eventually dominates.

I now consider the effect of having insider trading precede the accounting report. This is an important difference because of the presumption that insider trading before the report allows the manager to exploit non-public material information. Consequently, as documented by Seyhun (1992), tighter insider trading regulatory stringency has indeed significantly reduced insider trading before reports, but insider trading after reports has increased as has insider profits. However, the intuition underlying relative tolerance of insider trading after accounting reports fails to consider the effects future insider trading will have on the quality of reports. The previous proposition shows that this effect is significant. For insider trading, first define

\begin{align}
\text{Inf}_{IT\text{First}}(m_1 | I_1, e) &= \text{Prec}_{IT\text{First}}(m_1 | I_1, e) - 1 \\
\text{Inf}_{IT\text{First}}(TV | I_1, e) &= \text{Prec}_{IT\text{First}}(TV | I_1, e) - \frac{1}{c^2 + H}
\end{align}

The next proposition considers the case where insider trading precedes the accounting report.

**Proposition 4** When insider trading occurs before the accounting report, the total informativeness of the combined accounting and insider trading reports for $m_1$ and $TV$ are, respectively,

\begin{align}
\text{Inf}_{IT\text{First}}(m_1 | I_1, e) &= \infty \\
\text{Inf}_{IT\text{First}}(TV | I_1, e) &= \frac{1}{\frac{1}{H} + \frac{1}{c^2}}
\end{align}

To explain the results for insider trading before the accounting report, note that insider trading before the report implies that we are essentially back to the no-
insider trading case, since the trade is “sunk” by the time the report is issued (albeit, with the initial equity holdings $\epsilon$ reduced by any sales $\epsilon$ to $\epsilon - \epsilon$). Without insider trading, Lemma 1 found that the manager’s earnings management policy was independent of his private information and the same finding applies now. In particular, the manager’s earnings management and insider trading policies are independent of his new information $z$. This implies that investors now have two reports ($I_1$ and $e$) that depend on only two pieces of private information (economic income and accounting error) and this allows the private information to be unraveled. On the other hand, since the policies contain no dependence on $z$, the uncertainty in $z$ is still described by the unconditional variance $h^2$ of $z$. Then $\text{Prec}_{ITFirst}(TV|I_1, e) = 1/h^2$ and the result for $\text{Inf}_{ITFirst}(TV|I_1, e)$ follows by subtracting $1/(c^2 + H)$.

Together, these final propositions show that insider trading before the report always has a positive impact on the investor inference of both economic income $m_1$ and terminal value $TV$ relative to the case without insider trading. However, insider trading after the report does even better with respect to information about $TV$ but, for high enough accounting precision, is worse than even the case without insider trading for information about $m_1$.

Finally, I note that if the manager can trade both before and after the accounting report, then both $TV$ and $m_1$ will be revealed. However, the informativeness of the accounting report alone is still lower than without insider trading.

2.5 Robustness and Extensions

It is important to determine the robustness of the claim that accounting informativeness falls with increasing accounting precision when relative regulatory strin-
ergency is low. Possible extensions of my model include: unobserved productive actions (or real earnings management as in Ewert and Wagenhofer (2005)); incomplete reversal of accounting error; strategic trading; multi-dimensional economic information and reports; nonlinear accounting bonus; multiple periods; non-quadratic disutilities and/or nonlinear technology. I find (proofs in Appendix B) that, except for the last two effects, these extensions create no qualitative changes. For low enough relative regulatory stringency, lim_{s \to \infty} \rho_s = 0. The case of multiple-periods is studied in Chapter 3. Non-quadratic disutilities and/or nonlinear technology are not considered.

In the above, by "strategic trading" I mean situations in which the manager's trading volume can influence the price. I consider two cases: first, market microstructure effects (e.g., Kyle (1985)); second, a rational expectations equilibrium in which investors also observe private signals predicting the firm's terminal value (e.g., Grossman and Stiglitz (1980) or Grundy and McNichols (1989)). With the exception of multiple periods (not studied here) and nonlinearity (discussed next), the results of the basic model are qualitatively unchanged.

2.6 Conclusion

I have shown that if relative regulatory stringency is below a threshold, then increased accounting precision lowers investors' information deduced from accounting reports. Moreover, if accounting precision is high enough, even total the total information investors deduce from the combined accounting and insider trading reports is lower than that they deduce from accounting alone when insider trading is impossible. This has two implications. First, if relative regulatory stringency is low, then accounting error cannot be ignored even if there is reason to believe
it is small. This is because investor uncertainty increases in accounting precision for low regulatory stringency whereas if there were no accounting error, and hence the manager observes a single private signal, earnings management would be unraveled and investor uncertainty would be zero. Thus, the situation with zero accounting precision is a singular limit of models with non-zero accounting noise. Second, the number of situations in which relative regulatory stringency is too low may increase if efforts to increase accounting precision involve greater (harder to audit) managerial discretion.

My work also provides further insight into smoothing vs. anti-smoothing earnings management. Healy (1985) argues that smoothing and anti-smoothing of earnings may coexist, but at different times.

I find that, if insider trading is feasible, smoothing and anti-smoothing can take place simultaneously and that anti-smoothing is an independent activity, not just a tool abetting future smoothing. Anti-smoothing is done only on accounting errors and is largest at intermediate relative regulatory stringency. Smoothing is done exclusively on economic income and is most important if relative regulatory stringency is low. My work thus indicates that smoothing behavior can be a signature of poor “earnings quality.”

It is commonly assumed that one almost always sees anti-smoothing in a big-bath, i.e., a large negative earnings report and not the positive earnings analog. This is usually attributed to conservatism in accounting. However, as shown here, this asymmetry may also relate to insider trading since insider short-selling is

\[^5\text{Note that this does not contradict the possibility that firms smooth to signal their quality because only high quality firms can do it consistently. None of the features required for this picture (managerial motive for signalling quality, multi-periods, or why investors associate smoothed earnings with quality) are modeled here, and so the validity cannot be addressed.}\]
not feasible and restrictions on selling, both explicit and implicit (e.g., risk of lawsuits), are tougher than those on buying. Moreover, while ostensibly buying is made difficult because it requires cash outlay, this may be mitigated (or even eliminated) by company loans or option repricing. Option repricing requires no cash outlay and mimics equity purchase because the new options, assuming that they are granted at a lower strike price, are more sensitive to the movements in the underlying equity value. Together, the asymmetry of the ease of buying vs. selling implies that a manager may find it easier to take advantage of anti-smoothing behavior by buying when prices are low\textsuperscript{6}. Thus, insider trading profits and the earnings management they motivate provide an alternative explanation for the fact that only big-baths are observed.

\textsuperscript{6}As implied by Healy (1985), asymmetric anti-smoothing can also arise from bonus floors and caps. A bonus floor implies compensation is convex at low levels, and so increases the tendency for anti-smoothing at these low levels. A bonus cap has exactly the opposite implication. In short, this picture is not contradicted by the results here, but augmented by new possibilities.
Appendix A

Proofs

A.1 Proof of Lemma 1

The manager’s policies without insider trading are determined from the optimization

\[ U^* = \max_\delta U. \]  

The first order condition is

\[ 0 = (1 - \epsilon)(\phi_1 - \phi_2) - \nu \delta, \]  

and so the optimal \( \delta^* \) is given by

\[ \delta^{\text{NoIT}} = \frac{1 - \epsilon}{\nu} (\phi_1 - \phi_2). \]

A.2 Proof of Lemma 2

In the text, I have already shown that, conditional on conjectures \( \hat{p}_0 \) and \( \hat{p}_1 \), the manager’s optimal policies are

\[ e^* = \frac{\hat{p}_0 + \hat{p}_1 I_1 - cm_1 - (1 - \phi_2)z}{\chi} \]
\[(A.5)\]
\[
\delta^* = \frac{1}{\chi \nu - \hat{p}_1^2} \left( \chi (1 - \epsilon) (\phi_1 - \phi_2) + \hat{p}_1 (\hat{p}_0 + \hat{p}_1 \eta + (\hat{p}_1 - c)m_1) \right).
\]

Recalling the linear policy definitions of \(\delta_0\), etc.

\[(A.6)\]
\[
\delta^* = \delta_0 + \delta_m m_1 + \delta_\eta \eta,
\]

and using \(\mathcal{X}_1 = \delta_0 + (1 + m_1)\delta_m m_1 + (1 + \delta_\eta)\eta,\)

\[(A.7)\]
\[
\delta_0 = \frac{1}{\chi \nu - \hat{p}_1^2} \left( \chi (1 - \epsilon) (\phi_1 - \phi_2) + \hat{p}_0 \hat{p}_1 \right)
\]

\[(A.8)\]
\[
\delta_m = \frac{\hat{p}_1 (\hat{p}_1 - c)}{\chi \nu - \hat{p}_1^2}
\]

\[(A.9)\]
\[
\delta_\eta = \frac{\hat{p}_1^2}{\chi \nu - \hat{p}_1^2}.
\]

Next, from the form of \(e^*\),

\[(A.10)\]
\[
e_0 = \frac{\hat{p}_1 + \hat{p}_1 \delta_0}{\chi}
\]

\[(A.11)\]
\[
e_z = -\frac{1 - \phi_2}{\chi}
\]

\[(A.12)\]
\[
e_m = \frac{\hat{p}_1 (1 + \delta_m) - c}{\chi}
\]

\[(A.13)\]
\[
= \nu \frac{\hat{p}_1 - c}{\chi \nu - \hat{p}_1^2}
\]

\[(A.14)\]
\[
= \frac{\nu}{\hat{p}_1} \delta_m
\]

\[(A.15)\]
\[
e_\eta = \frac{\hat{p}_1 (1 + \delta_\eta)}{\chi}
\]
A.3 Proof of Proposition 1

We have two steps to prove: the equilibrium equation and uniqueness.

A.3.1 The Equilibrium Equation

We use the manager’s policies derived in the previous lemma for the first half of the equilibrium, the manager’s “response function” (his policies). For the second half (the investors’ “response function,” their price), we proceed as follows. First, we assume investor conjectures \( \delta_0, \delta_m, \) and \( \delta_\eta. \) Assuming risk-neutral investors, the value of the firm at the end of period one with respect to the information set \( \mathcal{I}_1 = m_1 + \eta + \hat{\delta} \) is just

\[
P(\mathcal{I}_1) = \mathbb{E}(m_1 + m_2 - \omega_1 - \omega_2 | \mathcal{I}_1, \hat{u}).
\]

Now, I assume that investors have the conjecture that the manager chooses his earnings management according to the linear policy \( \hat{\delta} = \delta_0 + \delta_m m_1 + \delta_\eta \eta, \) with the coefficients \( \delta_0, \delta_m, \) and \( \delta_\eta. \) Since, under this assumption, \( \mathcal{I}_1 = \delta_0 + (1 + \delta_m)m_1 + (1 + \delta_\eta)\eta, \) solve for \( \eta \) in terms of \( m_1 \) to find

\[
\eta = \mathcal{R}_1 - \frac{R}{\sqrt{a}} m_1
\]

\[
\mathcal{R}_1 = \frac{I_1 - \delta_0}{1 + \delta_\eta}.
\]
Now $m_1 \sim N(0, I)$, $\eta \sim N(0, Is)$, $E(m_2|m_1) = \rho m_1$, $E(z|I_1, \hat{u}) = 0$, and $\omega_t = \theta_t + \phi_t I_t$. Then

\[ P(I_t) = cE(m_1|R_1, \hat{u}) - ((\theta_1 + \theta_2) + (\phi_1 - \phi_2)I_1) \tag{A.21} \]

\[ c = (1 + \rho)(1 - \phi_2). \tag{A.22} \]

The expectation of $m_1$ can be computed as

\[ E(m_1|R_1, \hat{u}) = \frac{1}{\sqrt{2\pi}} \int m_1 e^{-\frac{1}{2}m_1^2} e^{-\frac{1}{2}(Rm_1 - \sqrt{\hat{u}R})^2} dm_1 \tag{A.23} \]

\[ = \sqrt{\frac{\hat{u}R}{1 + R^2}} R_1. \tag{A.24} \]

We thus find, in particular,

\[ \bar{p}_0 = -\frac{c}{1 + \delta_\eta} \frac{\sqrt{sR}}{1 + R^2} \hat{\eta} - (\theta_1 + \theta_2) \tag{A.25} \]

\[ p_1 = \frac{c}{1 + \delta_\eta} \frac{\sqrt{sR}}{1 + R^2} - (\phi_1 - \phi_2). \tag{A.26} \]

Using $\tilde{p}_1 = p_1 + \phi_1 - \phi_2$, $c = (1 + \rho)(1 - \phi_2)$, $b = \sqrt{\chi\nu}/c$, and $\nu = b^2 c^2$, these can be written as equations for $\bar{p}_0$ and $\bar{p}_1$:

\[ \bar{p}_0 = -c\sqrt{s}(1 - \bar{p}_1^2/(b^2 c^2)) \frac{R}{1 + R^2} \hat{\eta} \tag{A.27} \]

\[ \bar{p}_1 = c\sqrt{s}(1 - \bar{p}_1^2/(b^2 c^2)) \frac{R}{1 + R^2} = f(\bar{p}_1). \tag{A.28} \]

To obtain the equation for the equilibrium, I equate conjectures and responses, i.e., I set $\tilde{p}_1 = \bar{p}_1$, etc. and $\hat{\delta}_0 = \delta_0$, etc.

### A.3.2 Existence and Uniqueness

To prove existence, start with equation (A.28) that defines it. Now $f(0) = cs/(1 + s) > 0$ while $f(bc) = 0$. Thus, there is at least one down-crossing of $\bar{p}_1$ by $f(\bar{p}_1)$.
for \( \bar{p}_1 \in (0, bc) \). Since a down-crossing equilibrium is stable, there is always at least one stable equilibrium. But the equation for the equilibrium (after clearing denominators) is quadratic (next proof), implying at most one stable equilibrium. Thus, there always exists a unique equilibrium.

### A.4 Proof of Corollary 1

The equation for the equilibrium \( \bar{p}_1 \) is (after clearing denominators and subtracting the rhs from the lhs) second order in \( \bar{p}_1 \) (it would be third order, but the cubic terms cancel). It can be written \( g(\bar{p}_1) = 0 \) with

\[
0 = -b^2 c^2 s + c(s + (1 + s)b^2)\bar{p}_1 - sp_1^2.
\]

The two solutions to the quadratic equation \( g(\bar{p}_1) = 0 \) are

\[
\bar{p}_1^\pm = c \frac{(s + 1)b^2 + s \pm \sqrt{(s + 1)^2b^4 - 2(s - 1)sb^2 + s^2}}{2s}.
\]

Stability is determined by \( g'(\bar{p}_1) \), which must be positive. Now

\[
g'(\bar{p}_1) = c(s + (1 + s)b^2) - 2sp.
\]

Evaluating \( g'(\bar{p}_1) \) at the two equilibria,

\[
g'(\bar{p}_1^\pm) = \mp c\sqrt{(s + 1)^2b^4 - 2(s - 1)sb^2 + s^2}.
\]

Thus, \( \bar{p}_1^- \) is selected. It is also the only root that is finite as \( s \to 0 \).

### A.5 Proof of Lemma 3

I recall equation (A.28) obeyed by the equilibrium value of \( \bar{p}_1 \):

\[
\bar{p}_1 = c\sqrt{s(1 - \bar{p}_1^2/(\chi\nu))} \frac{R}{1 + R^2}.
\]
If the ERC $\bar{p}_1$ is to be finite as $s \to \infty$, the right hand side must be finite as $s \to \infty$. We can recast the equilibrium equation as

$$R = \frac{c\sqrt{s}(1 - \bar{p}_1^2/(\chi\nu)) \pm \sqrt{sc^2(1 - \bar{p}_1^2/(\chi\nu))^2 - 4\bar{p}_1^2}}{2p}. \tag{A.34}$$

These two possible values of $R$ are both positive and have limits as $R \to \infty$ of,

- $R_+ = \frac{1 - \bar{p}_1^2/(\chi\nu)}{2p} c\sqrt{s} \tag{A.35}$
- $R_- = 1/R_+ \tag{A.36}$

The lemma thus holds, since $1 - \bar{p}_1^2/(\chi\nu) > 0$ by assumption and $\lim_{s \to \infty} R_+ = \infty$ and $\lim_{s \to \infty} R_- = 0$.

### A.6 Proof of Lemma 4

I first prove that the earnings management coefficient $\delta_\eta$ has a maximum at $b_0$. Using the solution for the equilibrium ERC $\bar{p}_1$ given by equation (A.30) and equation (A.9) for $\delta_\eta$,

$$\frac{\partial \delta_\eta}{\partial s} = -\frac{2bs^2(b^2(s + 1) - s)}{((s + 1)^2b^4 - 2(s - 1)sb^2 + s^2)^{3/2}}. \tag{A.37}$$

This derivative vanishes at

$$b_0(s) = \sqrt{\frac{s}{1 + s}}. \tag{A.38}$$

and is positive for $b < b_0(s)$, negative otherwise. Thus, $\delta_\eta$ has a unique maximum with respect to $b$ at $b = b_0(s)$. 

A.7 Proof of Proposition 2

Now consider the behavior of $R$. Using the solution for the equilibrium ERC $\tilde{p}_1$ given by equation (A.30) and the fact that $R = 1 - \tilde{p}_1/(cb^2)$,

$$R = \frac{(s - 1)b^2 - s + \sqrt{(s + 1)^2b^4 - 2(s - 1)s b^2 + s^2}}{2b^2 \sqrt{s}}. \tag{A.39}$$

Setting $\partial R/\partial s = 0$ gives, once again, $b = b_0(s)$. If $b < b_0(s)$, $\partial R/\partial s < 0$ while if $b > b_0(s)$, $\partial R/\partial s > 0$.

A.8 Proof of Corollary 2

Of course, this has already been proved by Proposition 2 and Lemma 3. However, I now offer analytical confirmation. Using the solution for the equilibrium ERC $\tilde{p}_1$ given by equation (A.30) and the fact that $R = 1 - \tilde{p}_1/(cb^2)$, asymptotically

$$\lim_{s \to \infty} R = \frac{b^2 + \sqrt{(b^2 - 1)^2 - 1}}{2b^2} \sqrt{s} + \frac{b^2 - \sqrt{(b^2 - 1)^2 + 1}}{2(b^2 - 1)^2} \sqrt{s} + ... \tag{A.40}$$

$$= \frac{b^2 - 1}{b^2} - \sqrt{s} + \frac{1}{(b^2 - 1) \sqrt{s}} + ... \quad \text{If } b > 1 \tag{A.41}$$

$$= \frac{b^2}{(1 - b^2) \sqrt{s}} \quad \text{If } b < 1. \tag{A.42}$$

A.9 Proof of Proposition 3

I first prove the results for insider trading after the accounting report and then insider trading before the report.

A.9.1 Insider Trading after the Accounting Report

I perform the same calculation as done in the proof of Lemma 1 (done in section A.3) for the case with two reports. I first compute the mean and precision of $m_1$.
and then of $TV$.

From the two reports $I_1$ and $e$ together with the linear equilibrium policies, I can now solve for $\eta$ and $z$

\begin{align*}
    \eta &= \mathcal{R}_1 - \frac{R}{\sqrt{s}} m_1 \\
    z &= \mathcal{R}_2 - \theta m_1,
\end{align*}

where

\begin{align*}
    \mathcal{R}_2 &= \frac{e - e_0 - e_\eta \mathcal{R}_1}{e_z} \\
    \theta &= \frac{e_m - e_\eta R/\sqrt{s}}{e_z} \\
    &= -(1 + \rho).
\end{align*}

We compute two estimates: That of $m_1$ and that of $T$. We find for $m_1$

\begin{align*}
    \mathbb{E}(m_1 | I_1, e, \hat{u}) &= \frac{1}{2\pi} \int m_1 e^{-\frac{1}{2}m_1^2} e^{-\frac{1}{2}(Rm_1 - \sqrt{s}\mathcal{R}_1)^2 - \frac{1}{2h^2}(\mathcal{R}_2 - \mathcal{R}_1)^2} dm_1 \\
    &= Q_1 \mathcal{R}_1 + Q_2 \mathcal{R}_2 \\
    Q_1 &= \frac{\sqrt{s}R}{1 + R^2 + 1/h^2} \\
    Q_2 &= \frac{\theta}{h^2(1 + R^2) + \theta^2}.
\end{align*}

The precision of the estimate of $m_1$ is

$$\text{Prec}_{TLast}(m_1 | I_1, e, \hat{u}) = 1 + R^2 + \theta^2/h^2.$$ 

Finally, the variance of the estimate of $TV$ is

\begin{align*}
    \text{Var}(TV | I_1, e, \hat{u}) &= \text{Var}(cm_1 + (1 - \phi_2)z | I_1, e, \hat{u}) \\
    &= (1 - \phi_2)^2 \text{Var}((1 + \rho - \theta)m_1 + \mathcal{R}_2 | \mathcal{R}_1, \mathcal{R}_2, \hat{u})
\end{align*}
\[(A.54)\] 
\[= (1 - \phi_2)^2 \text{Var}(R_2|R_1, R_2, \hat{u})\]

\[(A.55)\] 
\[= 0,\]

and so

\[(A.56)\] 
\[\text{Prec}(TV|I_1, c, \hat{u}) = \infty.\]

Finally, to prove the last part, I recall that we have the solution

\[(A.57)\] 
\[\bar{p}_1 = c \frac{(s + 1)b^2 + s - \sqrt{(s + 1)b^4 - 2s(s - 1)b^2 + s^2}}{2s}\]

and

\[(A.58)\] 
\[R = 1 - \bar{p}_1/(bc^2).\]

Informativeness with insider trading is \(R^2 + \tau\) with \(\tau \equiv (1 + \rho)^2/h^2\). Informativeness without insider trading is \(s\). Thus, using the first two equations, equality of the two informativeness terms requires \(R^2 = s - \tau\). If \(s < \tau\), then equality is impossible for real \(R\) and so informativeness with insider trading is always greater than informativeness without insider trading. Otherwise, \(R^2 = s - \tau\) can be solved for \(b\) to give

\[(A.59)\] 
\[b = b_1(s)\]

\[(A.60)\] 
\[= \frac{\sqrt{s}(s - \tau)^{1/4}}{\sqrt{ \left( \sqrt{s} + s\sqrt{s - \tau} \right) \left( \sqrt{s} - \sqrt{s - \tau} \right) } }\]

Now consider \(b > b_1(s)\). I claim that \(\partial R/\partial b > 0\) which implies \(R^2 + \tau > s\) if \(b > b_1(s)\) (which completes the proof). To see this, calculate directly

\[(A.61)\] 
\[\frac{\partial R}{\partial b} = \frac{-(s(1 - b^2) + b^3) + \sqrt{(2s + 1)b^4 + 2sb^2 + s^2(1 - b^2)^2}}{b^3\sqrt{(2s + 1)b^4 + 2sb^2 + s^2(1 - b^2)^2}}\]
where \( X = b^2 + s(1 - b^2) \).

**A.10 Proof of Proposition 4**

Now, consider the manager's policies with insider trading before the accounting report is released. I first compute the manager's policies. To this end, I perform the analog of the calculation of Section 2 with the difference that now it is \( \delta \) that is chosen after \( z \) is known. We find

\[
\delta(\bullet) = \frac{(1 + \epsilon - \epsilon)(\phi_1 - \phi_2)}{\chi},
\]

Next, go to the stage 1b optimization. We substitute the policy for \( \delta \) into \( U \) and take expectation over \( z \). The first order condition for \( e \) then leads to

\[
e_m = \chi(1 - \phi_1 + \rho(1 - \phi_2)) \quad \text{and} \quad \nu \chi - (\phi_2 - \phi_1)^2.
\]

\[
\delta_m = \frac{(\phi_2 - \phi_1)(1 - \phi_1 + \rho(1 - \phi_2))}{\nu \chi - (\phi_2 - \phi_1)^2}
\]

\[
\delta_\eta = \delta_z = e_\eta = e_m = 0.
\]

The second order condition for the optimization is just \( \nu \chi > (\phi_2 - \phi_1)^2 \).

The fact that \( e \) depends only on \( m_1 \) implies that by observing \( e \) and deducing \( e_0 \), investors can infer \( m_1 \) exactly. Thus, \( \text{Var}(m_1 | I_1, e) = 0 \). On the other hand, since investors learn nothing about \( z \) from either \( I_1 \) or \( e \), \( \text{Var}(TV) = \text{Var}((1 - \phi_2)z) = (1 - \phi_2)^2 h^2 = H \). We thus find

\[
\text{Inf}_{ITFirst}(m_1 | I_1, e) \equiv \text{Prec}_{ITFirst}(m_1 | I_1, e) - 1
\]
\begin{align*}
(A.69) & \quad = \infty \\
(A.70) & \quad \text{Inf}_{\text{First}}(TV|I_1, e) = \text{Prec}_{\text{First}}(TV|I_1, e) - 1/(c^2 + H) \\
(A.71) & \quad = \frac{1}{H} - \frac{1}{c^2 + H} \\
(A.72) & \quad = \frac{1}{H(1 + H/c^2)}.
\end{align*}
Appendix B

Extensions

The relative ease of the preceding analysis stems from the simplicity of the model. Thus, it is important to determine the robustness of the claim that accounting informativeness falls with increasing accounting precision when relative regulatory stringency is low. Possible extensions include: unobserved productive actions (or real earnings management as in Ewert and Wagenhofer (2005)); incomplete reversal of accounting error; strategic trading; multi-dimensional economic assets and reports; nonlinear accounting bonuses; and multiple periods. Here, by “strategic trading” I mean situations in which the manager’s trading volume can influence the price. I consider two cases: market microstructure effects (e.g., Kyle (1985)); a rational expectations equilibrium in which investors also observe private signals predicting the firm’s terminal value (e.g., Grossman and Stiglitz (1980) or Grundy and McNichols (1989)). With the exception of multiple periods (not studied here) and nonlinearity, I find that these extensions include no qualitative changes: for low enough relative regulatory stringency, \( \lim_{s \to \infty} = 0. \)
B.1 Incomplete Reversal and Real Earnings Management

I first consider accounting error that does not fully reverse together with real earnings management. Let the manager be able to engage in real earnings management $a$. The real earnings management increases the first period reported earnings by $g_1a$ at a personal cost $a^2/2$ and cost to the firm of $\kappa a$. Provided the persistence of economic income and "accounting error" are not equal\(^1\), non-reversing accruals have no effect as $s \to \infty$, and real earnings management leads to the result

\[(B.1) \quad \lim_{s \to \infty} R = \begin{cases} 0 & \text{If } b < b_u, \\ \infty & \text{otherwise} \end{cases}\]

where

\[(B.2) \quad b_u = \sqrt{1 + \nu(g - \kappa \phi_2/c)^2}.\]

Thus, the threshold relative regulatory stringency as $s \to \infty$, $b_u$, is never smaller than that found without real earnings management (which was $b = 1$). Since $b_u$ depends on $\nu$ beyond the dependence of $b$, the single parameter $b$ is no longer sufficient to completely describe relative regulatory stringency if real earnings management is possible. Finally, for cases in which $cg = \kappa \phi_2$, the threshold is as without real earnings management, $b_u = 1$. This follows from the fact that the marginal benefit of the manager’s real earnings management $a_1$ to his insider trading profits (i.e., $\partial^2\mathbb{E}(W_2)/(\partial e \partial a_1)$) is $g_1p - \kappa \phi_2$. But, in the case without real earnings management (where $b = 1$ was the threshold), the value of the ERC $\tilde{p}_1$

\(^1\)If the persistence’s are equal, $m_1$ and $\eta$ only affect the manager’s utility and the investors’ information via their sum, and the model is equivalent to one in which the manager learns only the sum $m_1 + \eta$. 
at $b = 1$ as $s \to \infty$ is $\bar{p}_1 = c$. Thus, if $c_0 = \kappa \phi_2$, the marginal benefit of real earnings management to insider trading profits vanishes at $b = 1$.

B.2 Dependence on Stage 2c Price

Consider the assumption of having the manager liquidate any shares remaining after insider trading at the terminal price. What if a fraction $\varphi$ (of remaining shares) is liquidated at the market price that obtains before liquidation but after the stage 2b insider trading is reported in stage 2c. The fraction $1 - \varphi$ of shares are liquidated at the terminal value. In equilibrium, the stage 2c price is (up to a constant independent of the accounting report $I_1$ or trade volume report $e$)

$$\bar{p}_1(I_1, e) = \beta I - \alpha e.$$  
Let the relative regulatory stringency parameter $b$ be as in the base model ($b = \chi \nu / c$). We then have

\[
\lim_{s \to \infty} R = \begin{cases} 
0 & \text{if } b < b_\varphi, \\
\infty & \text{otherwise}
\end{cases}
\]

where

\[
b_\varphi = \frac{1 - \varphi}{1 + \alpha \varphi}.
\]

This result follows because the effect of $\varphi > 0$ reduces the impact of insider trading on informativeness through two effects. First, the manager will trade strategically because his trade reveals information that affects the stage 2c price. This changes this model's stage 1c price equilibrium (again obtained after investors observe $I_1$ only and characterized by the ERC $\bar{p}_1$) from that of the base model, but this change is entirely captured by replacing the insider trading stringency parameter $\chi$ with $\chi + \alpha \varphi$. Since $\alpha > 0$ (more selling is bad news), the strategic effect of trading mimics increased insider trading stringency. Second, the only source of
differential treatment of economic income $m_1$ and accounting error $\eta$ vis. a vis. earnings management is due to the dependence of the manager's utility on the terminal value. This dependence is now reduced by a factor $1 - \varphi$ and the result follows.

**B.3 Multi-Dimensional State and Reports**

What happens when the report of accounting income (sum of economic income $m_1$ plus accounting error $\eta$) is replaced by a multi-component financial statement? I examine this question by generalizing the basic model as follows. Assume that the value relevant economic assets are described by the $N$-vector $m_1$ and the report $T_1$ is an $M$-vector $T_1 = Hm_1 + \eta$ with $H$ and $M \times N$ matrix and $\eta$ and $M$-vector of accounting errors. Without loss of generality, I assume $M \leq N$ and $\text{Rank}(H) = M$. The cum-wage terminal value is $H'm_1$ with $H$ an $N$ vector. Wlog, I can choose $m_1 \sim N(0, I)$ and $\eta \sim N(0, \Sigma)$ for some $M \times M$ matrix $\Sigma$. The audit disutility $\nu$ is now an $M \times M$ matrix. Finally, the wage coefficients $\phi_1$ and $\phi_2$ are $M$-vectors, $\rho$ is an $N \times N$ matrix, and economic income is given by $E_t = Fm_t$, $t = 1, 2$ with $F'$ and $N$-vector.

The analog of the signal to noise ratio $R$ is now an $M \times N$ matrix and the ERC is now an $M$-vector. The analog of infinite accounting precision, $s \to \infty$, is $\Omega \to \infty$ (all eigenvalues of $\Omega$ become infinite). Let $c' = (F - \phi_2)'(1 + \rho)$, Then, if $\chi \frac{c'H'\nu Hc}{c'c} < 1$, then as $\Omega \to \infty$, the equilibrium is uninformative about the terminal value, i.e., $\text{Var}(c'm_1|T) = c'c$, the same as before the report $T_1$. 
Chapter 3

Infinite Horizon Model

3.1 Introduction

The prior chapter considered the informational efficiency of insider trading when earnings management is possible. The key cause of the loss of information when insider trading is possible was the fact that, in equilibrium, the manager always smooths economic income. In turn, this followed from the fact that the market, knowing that accounting income contains both persistent and fully reversing components, applies an average persistence factor to earnings and so undervalues the persistent component. However, in reality, the firm does not generally liquidate when the manager sells his remaining shares. Consequently, the price received by the manager when he liquidates remaining unsold shares is endogenous and so presumably also undervalues economic income. Moreover, the manager’s liquidation price also includes the informational effect of his prior insider trading and this impels him to trade strategically, further mitigating his opportunism. Does the manager’s incentive to smooth income survive in a multi-period setting?

I use an infinite-horizon, multiple-generations, rational-expectations, reporting model to study the effect of insider trading-induced earnings management on
investor informedness. I use investor uncertainty as to the firm’s true value immediately after the release of the insider-trading report as the benchmark of investor informedness (which biases in favor of insider trading).

In my model, each period (or generation, e.g., a quarter of a year) consists of a single manager’s tenure and has three stages. Investors process information using a Kalman filter and price shares rationally, both activities contingent on conjectures as to the manager’s reporting strategy.

In stage one (e.g., the first third of the manager’s tenure), the value of economic assets and economic flows is drawn. Accounting assets consist of the auditors’ valuation of those economic assets that are capitalized, using an accounting valuation formula and accounting flows are differences in accounting assets. This formula values assets as a function of current economic assets, past accounting assets, and some noise (which I call accounting error). My conclusions do not depend on the details of this valuation, but some specific examples (e.g., fair-value accounting) are discussed in Section 2.

In my model, economic assets are sufficient statistics for determining current operating cash flows as well as future economic assets. While the auditors do not observe economic assets, the manager observes true economic assets and flows as well as the auditors’ valuation of accounting assets and flows. For simplicity, in my basic model, I assume that the manager cannot change the evolution of economic assets.¹

In stage one, the manager is required to issue a report of accounting assets.

¹I use the term “economic asset” for this state since it is economically relevant. It is not to be confused with notions such as “economic value” which depends on many factors, e.g., the information structure. In particular, while economic assets are unchanged (by assumption) as I vary the reporting and insider-trading environment, economic value is not.
The manager’s report of accounting assets may include an additional intentional distortion, i.e., reported assets equals accounting assets plus earnings management. The manager can report any values, but the auditors induce a disutility that increases in deviations from the auditors’ valuation.²

In stage two (e.g., the second third of the manager’s tenure), after the economic state has changed with the passage of time, the manager engages in insider trading. He is assumed to be a price-taker and size of the trade is reported (with noise) to investors after it is consummated (the source of this noise is discussed in Section 2). As his trade size increases, it becomes more likely that the manager’s trade is based on material non-public information as opposed to liquidity trading. Consequently, I include a regulatory disutility that increases in the trade volume. I use the term “regulatory stringency” to denote the combined stringency of audits and enforcement of the ban on insider trading.

In stage three (e.g., the third month of the quarter), after the economic state has again changed with the passage of time, investors receive a new, independent, and unbiased, analyst report. Immediately afterwards, the manager shares not sold in stage one are liquidated at the post-analyst-report price.

There are two main drivers of my results. First, there are two potentially opposing prerequisites for the manager to profit from insider trading. One prerequisite is that the manager must possess private information that has predictive value for future prices. A second prerequisite is that before the manager reaps the benefit of any trade initiated earlier (e.g., sells shares previously bought), some of this private information must now be incorporated into price, i.e., it must no

²This disutility can proxy for a number of different effects. These include the effort required to convince auditors to accept the manager’s desired numbers, potential costs of future litigation, termination, etc.
longer be private. In a multi-period setting, all things equal, these two require-
ments are in competition (more information revelation leads to less informational
advantage in the next round).

Second, I show that, as the precision of accounting approaches infinity, there
are two possible equilibria. The first, unique for sufficiently high regulatory strin-
gency, has full revelation of the manager's private information in his financial
report. Because he trades after learning new information, he still has non-zero
trade volume, and so investors are always better informed when this revealing
equilibrium holds.

The second equilibrium (in the limit of perfect accounting) is fully conceal-
ing. The manager releases none of his private information in the financial report.
Despite this, investors could still be better informed than in the situation with no
insider trading if the insider-trading report has low noise. Since (as I show) the
limit of high accounting precision is regular, I focus on determining the proper-
ties of the equilibrium in this limit. I am concerned, in particular, with how the
equilibrium behaves as a function of two exogenous parameters: the precision of
the analyst report and the precision of the insider-trading report.

The two equilibria in the limit of perfect accounting are explained as follows.
In my model, the manager's information advantage consists of knowing both eco-
nomic and accounting assets. With high accounting precision, the manager has
only a small information advantage remaining if he releases a truthful report of
accounting assets. Because investors are rational, and so can infer the manager's
earnings-management policies, the only successful earnings management will be
one in which the price-relevant component of the accounting report is essentially
noise (i.e., accounting error). If regulatory stringency is high (with regulatory
stringency defined as the stringency of audits and enforcement of the ban on insider-trading), the effort of earnings management is not worth the small benefit, reports are truthful, and the manager (with no information advantage) does no insider trading. If regulatory stringency is low, reports completely suppress the price-relevant component of the accounting report.

As long as the equilibrium type is unchanged, increasing the noise in either the insider trading or analyst reports will always reduce investor informedness. However, changing the levels of noise can also change the equilibrium type.

With a change in the noise in the insider-trading report, the manager’s need for private information overrides his need for post-trade price efficiency. At low levels of insider-trading-report noise, the revealing equilibrium is unique, at high levels the concealing equilibrium is unique and at intermediate levels, both can exist. Consequently, increasing noise in the insider-trading report always degrades investor informedness, albeit there may be hysteresis effects if the noise level fluctuates past the region where both equilibria are possible.

With regard to changing the noise in the analyst report, the informational trade-off becomes salient. In particular, the threshold level of insider-trading report noise above which the revealing equilibrium becomes untenable is maximized for intermediate levels of analyst-report noise. At low levels of analyst-report noise, post-trade price reflects well the manager’s pre-trade private information, and this increases his incentive to trade and thus distort. At high levels of analyst-report noise, the manager’s informational advantage is higher and this fact dominates poor revelation.

The reason this interior maximum is not seen with respect to noise in the

\(^3\)When both exist, the revealing equilibrium always has higher investor informedness.
insider-trading report is that the insider-trading signal is also the manager’s trading volume. Thus, from the manager’s perspective, insider trading confounds an informational effect with a monetary gain effect.

The rest of this Chapter is organized as follows. Section 2 introduces the model. Sections 3-6 then uses partial-equilibrium methods to study the model. Section 3 solves the investors’ filtering problem as a function of her conjecture concerning the manager’s policies. Section 4 determines the manager’s earnings management policy given conjectures about the investors’ filter and pricing function. Section 5 determines the investors’ pricing function given her filter and conjecture concerning the manager’s policies. Section 6 studies the problem in the limit of perfect accounting and shows that a critical value of a regulatory-stringency parameter divides two behaviors. Section 7 derives the dependence of the perfect accounting equilibrium as a function of the precisions of the insider-trading report and the analyst report. Section 8 considers the effects of more general nonlinear technologies and utilities. Section 9 concludes.

3.2 Model

The model is infinite-horizon, discrete-time, with three stages per period. The time line is as follows:

1. At time $t$, $t = 1, 2, 3...$, the new manager is hired and:

   (a) Observes a vector $\eta_t^0$ of accounting assets and flows (e.g., revenues) and the privately observed economic state, a vector $m_t$.

   (b) Chooses an additional vector of account distortions, $\delta_t$ (before the report is issued), at a cost $\delta_t^t / 2$ and releases the accounting report
\[ \eta_t = \eta_t^0 + \delta_t. \]

(c) Investors update their estimate of \( m_t \) to reflect information gleaned from report \( \eta_t \). This state estimate is denoted \( \hat{m}_t \) and is common knowledge.

(d) Risk-neutral investors rationally price the shares using the endogenous function \( P_0(\hat{m}_t, \eta_t) \) (the subscript 0 indicates this is the price function at the start of the manager’s tenure).

\[ \eta_t. \]

2. At time \( t + 1/3 \):

(a) The new manager observes \( m_{t+1/3} \).

(b) Investors estimate \( m_{t+1/3} \), denoted by \( \hat{m}_{t+1/3} \) (the superscript denotes that this is the state estimate obtaining immediately before the insider-trading report which is made right after the trade).

(c) The new manager sells \( e_{t+1/3} \) shares (a choice variable) at price \( P_{t+1/3}(\hat{m}_{t+1/3}, \eta_t) \) at a personal cost \( \chi e_{t+1/3}^2/2 \) (negative choices of \( e \) denote buying and short sales are allowed).

(d) Immediately after the sale, the sale’s volume \( e_{t+1/3} \) is reported with some level of noise, i.e., investors receive a report \( \hat{e}_t = e_{t+1/3} + z_{t+1/3} \), \( z_{t+1/3} \sim N(0, k^2) \).

(e) Investors update their estimate of \( m_{t+1/3} \) to reflect information gleaned from report \( e_{t+1/3} \). This state estimate is denoted \( \hat{m}_{t+1/3} \).

3. At time \( t + 2/3 \):

...
(a) Investors receive an exogenous unbiased public signal (from analysts),

\[ y_{t+2/3}^p \sim N(m_{t+2/3}, \Omega_p) \]

concerning the firm's true economic state \( m_{t+2/3} \)

(b) Investors use the report to update their estimate of \( m_{t+2/3} \), denoted by \( \hat{m}_{t+2/3} \)

(c) Immediately after, the manager liquidates any remaining shares at price

\[ P_{2/3}(\hat{m}_{t+2/3}, \eta_t) \]

I now discuss these stages in more detail.

### 3.2.1 Firm Economics

The vector \( m_t \) (with the unconditional mean \( \mathbb{E}(m_t) = 0 \)) represents the true economic assets (or state) of the firm, in the following sense. First, \( m_t \) is a sufficient statistic for predicting the next period's economic state \( m_{t+1} \). Second, up to i.i.d. shocks, \( m_t \) determines the current period's "operating results" (revenues and operating expenses).

I decompose the evolution of \( m_t \) into three stages:

1. \( m_{t+1/3} = \rho_1 m_t + z_t^1 \)
2. \( m_{t+2/3} = \rho_2 m_{t+1/3} + z_t^2 \)
3. \( m_{t+1} = \rho_3 m_{t+2/3} + z_t^3 \).

In the above, the exogenous economic shocks \( z_t^c \) are independent and distributed as \( z_t^c \sim N(0, \Sigma_c) \), \( c = 1, 2, 3 \). The dynamic matrices \( \rho_c \) represent a combination of economic depreciation together with non-discretionary new investment. I assume that the \( \rho_c \) are all invertible and stable, i.e., all eigenvalues of \( \rho_c \) lie in the unit disk and none are zero.
3.2.2 Accounting

The accounting report is represented by a vector $\eta_t$ which contains both period reports (revenues and costs) together with balance-sheet items. In normal circumstances, the accounting report is influenced by the economic state $m_t$ only via the income statement (or simplicity, I lump the income and cash flow statements. This is because $m_t$ is an input into the firm’s ‘production function’ and this is reported in the income statement.

In contrast, the accounting balance sheet initially values items at cost, not economic value. Moreover, accounting depreciation is a cost allocation, a reflection of past not current economics. However, GAAP provides a mechanism for revaluing assets, and the manager can use his private knowledge of the economic values in $m_t$ to do so. While US GAAP is asymmetric (due to conservatism, only write-downs are allowed except in unusual circumstances, e.g., takeovers), in my model I assume symmetry. An extreme example of revaluation is fair-value accounting, about which more shortly.

With the above considerations, I model the evolution of the accounting state $\eta_t$ by

$$\eta_{t+2/3} = \eta_{t+1/3} = \eta_t$$

(3.4)

$$\eta_{t+1} = \psi_0 m_{t+1} + g_0 \eta_t + \Gamma (\delta_t + n_t)$$

(3.5)

and so

$$\eta_{t+1} = \psi_0 \rho_3 m_{t+2/3} + g_0 \eta_t + \Gamma (\delta_t + n_t) + \psi_0 z^3.$$  

(3.6)

\footnote{Of course, depreciation schedules are usually determined to reflect expected economics, but they usually adjust slowly or not at all for deviations from expectations.}
The matrix $\psi_0$ includes the operating results (period revenues and costs) as well as any revaluations. If accounting rules specified fair-value accounting, then the current value of accounting assets would depend only on the present value of economic assets and not past values of accounting assets. In this case, $g_0$ would be zero, and accounting assets would have zero persistence conditional on economic assets. In practice, many assets are not valued this way, but rather mechanically depreciated over time (with occasional revaluations, as needed). In this case, $g_0$ would not be zero.

To facilitate comparisons and without loss of generality, I choose $\psi_0$ to be have non-negative singular values. Finally, the accounting error vector, $n_t$, is modeled as i.i.d. noise, $n_t \sim N(0, \Omega_0)$.

The accounting observation matrix $\psi_0$ plays a crucial role in the analysis. In the absence of any earnings management $\delta_t$, the publicly observable difference $n_{t+1} - g_0n_t$ is just $\psi_0m_{t+1} + n_t$, i.e., a noisy signal of the true economic assets $m_t$. Thus, in the absence of earnings management, it is this signal that investors input into their Kalman filter to update estimates of the economic state $m_t$. I will show that the manager's earnings management has the informational effect of changing $\psi_0$ and $\Omega_0$ to new matrices $\psi$ and $\Omega$, respectively (un-subscripted $\psi$ and $\Omega$ will refer to an insider trading firm's post earnings management parameters).

### 3.2.3 Investors

Investors have two roles in the model. First, they interpret information. Second, they price the firm's equity using their interpretation.

To interpret information, investors construct a dynamic estimate of the manager's private information, $m_t$. This has two steps. First, they must conjecture
the manager’s policies to construct a model of the firm and combined evolution of the economic state \( m_t \) and accounting values \( \eta_t \). Second, using this model, they estimate \( m_t \) using the stationary Kalman filter appropriate for the three reports investors receive: accounting at time \( t \), insider trading at time \( t + 1/3 \), and independent research at time \( t + 2/3 \). Associated with these three reports are respective Kalman filter gains, \( \beta, K, \) and \( \gamma \) which are just the responses of the respective estimates \( \hat{m}_t, \hat{m}_{t+1/3}, \) and \( \hat{m}_{t+2/3} \) to the respective information signals \( \eta_t, e_{t+1/3}, \) and \( y_{t+2/3} \).

I assume that investors’ estimate of \( m_t \) at the start of the firm (at \( t = 0 \)) is normal. It follows that investors’ estimates for \( m_t \) remain normal for all subsequent periods\(^5\). Consequently, it suffices to keep track of the mean and covariance (this covariance is termed the “error covariance”) of the investors’ distribution. My notation for the ensuing means and covariances is as follows: Start with the investor estimate \( m_{t+1/3} \sim N(\hat{m}_{t+1/3}, S_t) \) immediately before release of the accounting report. Then investors have the estimate \( m_t \sim N(\hat{m}_t, T_t) \) immediately after the accounting report \( \eta_t \) has been observed by investors. Likewise, investors have the estimate \( m_{t+1/3} \sim N(\hat{m}_{t+1/3}, S_{t+1/3}) \) immediately before release of the insider-trading report (if any), \( m_{t+1/3} \sim N(\hat{m}_{t+1/3}, T_{t+1/3}) \) immediately after, \( m_{t+2/3} \sim N(\hat{m}_{t+2/3}, S_{t+2/3}) \) immediately before release of the independent analyst report, and \( m_{t+2/3} \sim N(\hat{m}_{t+2/3}, T_{t+2/3}) \) immediately after. Stationary values of the covariances will be denoted \( T_0, T_{1/3}, T_{2/3}, S_0 \) etc. for a insider-trading firm and \( T_{0,0}, T_{0,1/3}, T_{0,2/3}, S_{0,0}, \) etc., for a no-insider-trading firm.

At any given time, the investor information set is given by the observed

\(^5\)This holds true even when insider trading induces insider trading since, for the equilibrium I study, the earnings-management policy is linear in \( m_t \).
report history \( \{\eta_t, y_{t+2/3}^P\} \) without insider trading, \( \{\eta_t, e_{t+1/3}, y_{t+2/3}^P\} \) with). In equilibrium, when investor conjectures are correct, this a sufficient state is just the latest accounting report \( \eta_t \) together with investors’ most recent estimate of \( m \). Let \( P_t^- \) denote the cum-dividend price just before \( \eta_t \) is observed. Let \( P_t \) denote the ex-dividend price just after \( \eta_t \) is observed (so that the dividend is assumed to paid with the accounting report). Let \( P_{t+1/3}^- \) denote the price just before the insider trading volume \( e_{t+1/3} \) is observed. Let \( P_{t+1/3} \) denote the price just after \( e_{t+1/3} \) is observed. Finally, let \( P_{t+2/3}^- \) denote the price just before the analyst report \( y_{t+2/3}^P \) is observed and \( P_{t+2/3} \) denote the price just after \( y_{t+2/3}^P \) is observed.

Arbitrage-free prices are determined by

\[
\begin{align*}
P_t^- &= D_t + R_1 \mathbb{E}(P_{t+1/3}|\eta_{t-1}, e_{t+1/3-1}, \hat{m}_t^-) \\
P_t &= \mathbb{E}(P_{t+1/3}|\eta_t, e_{t+1/3-1}, \hat{m}_t) \\
P_{t+1/3}^- &= R_2 \mathbb{E}(P_{t+1/3}|\eta_t, e_{t+1/3-1}, \hat{m}_{t+1/3}^-) \\
P_{t+1/3} &= \mathbb{E}(P_{t+1}^-|\eta_t, e_{t+1/3}, \hat{m}_{t+1/3}) \\
P_{t+2/3}^- &= R_3 \mathbb{E}(P_{t+2/3}|\eta_t, e_{t+1/3}, \hat{m}_{t+2/3}^-) \\
P_{t+2/3} &= \mathbb{E}(P_{t+1}^-|\eta_t, e_{t+1/3}, \hat{m}_{t+2/3}).
\end{align*}
\]

with \( D_t \) the dividend and the \( R_1, R_2, \) and \( R_3 \) the respective risk-free discount rates over \( (t, t+1/3), (t+1/3, t+2/3), \) and \( (t+2/3, t+1) \) respectively.\(^6\) The dividend policy is introduced later.

### 3.2.4 The Manager

I first describe the manager’s program. I then discuss the manager’s insider trading in more detail.

\(^6\)Because I assume the firm never liquidates, some dividends are necessary to yield a non-zero price. Since I am interested in stationary behavior, I assume a stationary dividend policy.
The Manager’s Program

I assume that the manager is risk-neutral and obtains utility from terminal wealth. For simplicity, I assume the manager’s compensation is entirely from insider-trading profits. Including an accounting bonus or initial equity grant make no difference to investor informedness (assuming, as I do, no short sale restrictions).

The two stages of his program can be written:

\[ U^* = \max_{\delta_t} \mathbb{E}(w_1 + R_1 V^* + DU_1) \quad \text{at time } t \]

\[ V^* = \max_{e_{t+1/3}} (e_{t+1/3} P_{t+1/3} (\hat{\eta}_t, \bar{m}_{t+1/3}) + R_2 w_2 + DU_2) \quad \text{at time } t + 1/3, \]

where

\[ w_1 = \epsilon D_t \]

\[ w_2 = -e_{t+1/3} P_{t+2/3} (\eta_t, \bar{m}_{t+2/3}) \]

\[ DU_1 = -\frac{1}{2} \delta_t \nu \delta_t \]

\[ DU_2 = -\frac{1}{2} \chi \epsilon_{t+1/3}. \]

Insider Trading

Here, I discuss the two features of insider trading that I model. The first is the fact that the manager’s trades are reported with noise. The second is the source of the manager’s insider-trading disutility.

Noise in the manager’s trade report models numerous unobservable effects that mask the manager’s true information but are otherwise unaddressed in the model studied here. These include:
• Trades are often reported with delay.

• The manager experiences consumption shocks (e.g., a house purchase).

• The timing of a manager’s trading is often exogenous (e.g., he hires a new financial advisor who suggests he diversify).

• Purchases are often made via company loans which are later forgiven and investors thus know neither whether the purchase was really discretionary nor the true cost.\(^7\)

• The manager has less overt means to effectively trade stock. These include option repricing and the choice of equity/cash mix to receive in compensation.

• Market microstructure effects. For example, a limit order may be partially unfilled and so investors will not observe the manager’s true intent.

Trading on non-public material information is proscribed in U.S. markets. If this ban were strictly enforceable, then this paper would be moot. Thus I implicitly assume that managers can get away with some insider trading. However, the larger the trade size, the more likely that regulators, potential litigants, shareholders, and the board of directors may become alerted and take some action against the manager. Thus, I assume the manager experiences disutility increasing in the magnitude of his trading volume.

\(^7\)Prior to the Sarbanes-Oxley act, more than 75 firms lent money to executives. \(\ldots\) discuss both below market loan rates as well as loan forgiveness.
3.2.5 Equilibrium

Each stage of the game involves investors and an incumbent manager. For the equilibrium, the manager optimizes his utility given his conjectures on how his actions impact investor information and prices. Investors interpret the manager’s report using their own conjectures as to the manager’s policies and set prices accordingly. With policy conjectures, they can construct the Kalman filter needed to form a dynamic estimate, $\hat{m}$, of the economic state of the firm, $m_t$, at all relevant times.

**Definition 3** A stationary perfect Bayesian equilibrium is one in which the following hold. First, the current manager’s policies $v^* = (e^*, \delta^*)$ are optimal given conjectures about the investors filter gains and investors price functions. Second, the investors’ filter gains are stationary and Bayesian given their conjectures of the manager policies. Third, the investors’ price function is stationary and rational given their policy conjectures. Finally, in equilibrium, all conjectures are correct.

The reason the manager must conjecture as to the investors’ filter gains is because these gains depend, in turn, on the investors’ conjecture as to the manager’s earnings management and insider trading policies.

Let $\hat{I} = (\hat{m}, \eta)$ denote the investors information set at any given time and let $I = (m, \hat{m}, \eta)$ denote the manager’s information set. Let $\hat{\beta}$ represent the current manager’s conjecture concerning the investors filter gain for his accounting report, and likewise for the insider trading gain $\hat{K}$ and analyst signal gain $\hat{\gamma}$. $\hat{P} \equiv (\hat{P}^- (\hat{I}_{t+1/3}), \hat{P}_{t+2/3}(\hat{I}_{t+2/3}))$ represent the manager’s conjecture concerning the prices obtaining just before the insider trading and just after the analyst
report, respectively (i.e., the insider trading and liquidation prices). Finally, let \( \hat{u} \) represent the investors conjecture concerning the current manager's policies. Then, the linear equilibrium satisfies

\[
\delta^*_t = \arg\max_\delta \mathbb{E}_t(U(u_t, m_t, \eta_t) | \hat{K}, \hat{\beta}, \hat{\gamma}, \hat{P}, I_t)
\]

\[
e^*_t = \arg\max_e (V(u_t, m_{t+1/3}, \eta_t) | \hat{K}, \hat{\beta}, \hat{\gamma}, I_t),
\]

the investor price function obeys (3.7)-(3.12), and

\[
(3.19) \quad \hat{P}(\bullet) = P(\bullet|\hat{u}) \quad \hat{u} = u^*
\]

\[
(3.20) \quad \delta = \delta_0 + \delta'_m m_t + \delta'_n \eta^0_t + \delta_n n_t + \delta'_{\hat{m}} \hat{m}_t + \delta'_{\eta} \eta_{t-1}
\]

\[
(3.21) \quad e_{t+1/3} = e_0 + e'_m m_{t+1/3} + e'_n \eta_t + e'_{\hat{m}} \hat{m}_{t+1/3}
\]

\[
(3.22) \quad \hat{P}_t = \hat{P}(\hat{m}_t, \hat{\eta}_t|\hat{u}, \hat{\beta}(\hat{u}), K(\hat{u}), \gamma(\hat{u}))
\]

\[
(3.23) \quad = \hat{p}^0 + \hat{p}^1 \hat{m}_t + \hat{p}^2 \eta_t
\]

\[
(3.24) \quad \hat{P}_{t+1/3} = \hat{p}^0_{1/3} + \hat{p}^1_{1/3} \hat{m}_{t+1/3} + \hat{p}^2_{1/3} \eta_t,
\]

and likewise for the other four price functions \( \hat{P}_t, \hat{P}_{1/3}, \hat{P}_{2/3}, \) and \( \hat{P}_{2/3}. \) The next sections describe computing the respective manager and investor response functions and ends with proof that a linear equilibrium always exists.

---

8Stationarity is implicit from the fact that the policy coefficients \( \delta_0, \) etc., and price coefficients \( P_0, \) etc., do not depend on time.
3.3 Filtering

This section describes the investors' Kalman filter. For filtering, investors are assumed to make the conjecture that the manager's policies for his choice vector \((\delta, e)\) are linear in his private information state \(m_t\). As in the last section, assume investors start with the distribution \(m_t \sim N(\hat{m}_t, T_t)\) that obtains immediately after \(\eta_t\) has been observed by investors. The investors' estimate of \(m_t\) evolves with new information as

**Lemma 5 Without insider trading**

\[
\begin{align*}
\hat{m}_t &= \hat{m}_{t-} + \beta^0_t (y_t - \psi \hat{m}_{t-}) \\
\hat{m}_{t+1/3} &= \rho_1 \hat{m}_t \\
\hat{m}_{t+1/3} &= \hat{m}_{t+1/3} \\
\hat{m}_{t+2/3} &= \rho_2 \hat{m}_{t+1/3} \\
\hat{m}_{t+2/3} &= \hat{m}_{t+2/3} + \gamma^0_t (y_{t+2/3} - H \hat{m}_{t+2/3}) \\
\hat{m}_{t+1} &= \rho_3 \hat{m}_{t+1/3},
\end{align*}
\]

where

\[
y_t = \eta_t - g_0 \eta_{t-1}.
\]

With insider trading, the filtering equations are similar in form, but with different filter gains. In addition, we have a estimation update for the manager's trade report \((\text{reftrade below})\) that is obviously not present without insider trading. The with-insider-trading equations are

\[
\hat{m}_t = \hat{m}_{t-} + \beta_t (y_t - \psi \hat{m}_{t-})
\]
\begin{align}
\hat{m}_{t+1/3} &= \rho_1 \hat{m}_t \\
\hat{m}_{t+1/3} &= \hat{m}_t + K(y^e_{t+1/3} - \varepsilon m\hat{m}_{t+1/3}) \\
\hat{m}_{t+2/3} &= \rho_2 \hat{m}_{t+1/3} \\
\hat{m}_{t+2/3} &= \hat{m}_{t+2/3} + \gamma_t(y^p_{t+2/3} - H\hat{m}_{t+2/3}) \\
\hat{m}_{t+1} &= \rho_3 \hat{m}_{t+1/3} \\
y_t &= \eta_t - g \eta_{t-1} - \pi_\eta \hat{m}_t \\
y^e_{t+1/3} &= \varepsilon_{t+1/3} - \varepsilon \eta_t - \varepsilon \hat{m}_{t+1/3}.
\end{align}

The filter gains $\beta^0$, $\beta$, $\gamma^0$, $\gamma$, and $K$ are given in the proof. Both these results are just the respective Kalman filter appropriate for the policy conjectures (if any). In addition to entailing a new report, insider trading affects the filtering by changing some of the parameters. As will be seen, the relevant change is captured by $\psi_0 \rightarrow \psi$ (due to the effect of insider trading on earnings management). This is why it is $\psi$ and not $\psi_0$ that appears in the equations given above. The changes $g_0 \rightarrow g$ and introduction of a nonzero $\pi_\eta$, do not affect the investors’ information directly because the terms they introduce are fully observed by investors. Of course, this assumes that investor conjectures concerning $\pi_\eta$ et. al. are correct, but this is so in equilibrium.

The parameters $\psi$, $g$, and $\pi_\eta$, $\Omega$, are derived from the investors’ earnings-management conjecture

\begin{align}
\delta_t &= \hat{\delta}_0 + \hat{\delta}_m m_t + \hat{\delta}_\eta \eta_t + \hat{\delta}_n \hat{m}_t, \\
\psi &= (1 + \Gamma \hat{\delta}_\eta) \psi_0 + \Gamma \hat{\delta}_m
\end{align}
(3.42) \[ g = (1 + \Gamma \delta_\eta) g_0 \]
(3.43) \[ \pi_\eta = \Gamma \delta \hat{m} \]
(3.44) \[ \Omega = (I + \Gamma \delta_\eta) \Omega_0 (I + \Gamma \delta_\eta)' \]

(see the proof of 3 for the derivation).

### 3.4 The Manager’s Program

In this section, I derive the manager’s optimal polices as a function of his (linear) conjectures as to the investor price function and filter gains. As stated previously, the manager optimizes his utility for given conjectures on how his actions impact investor information and prices. The manager’s program is:

(3.45) \[ U^* = \max_{a_t, \delta_t} \mathbb{E}(R_t V^* + DU_1) \text{ at time } t \]
(3.46) \[ V^* = \max_{e_{t+1/3}} (e_{t+1/3} P_{t+1/3}(\hat{m}_{t+1/3}, \eta_t) + R_2 w_2 + DU_2) \text{ at time } t, \]

where

(3.47) \[ w_2 = -e_{t+1/3} P_{t+2/3}(\eta_t, \hat{m}_{t+2/3}) \]
(3.48) \[ DU_1 = -\frac{1}{2} \delta_t \nu \delta_t \]
(3.49) \[ DU_2 = -\frac{1}{2} \lambda e_{t+1/3}^2. \]

When insider trading is impossible, so long as the manager’s price conjecture is linear in the state variables $\hat{m}_t^-$, $\eta_t^-$, and $m_t$, his optimization yields $\delta_m = \delta_\nu = e_m = e_\eta = 0$. When insider trading is possible, the policies are not independent of the state variables $\hat{m}_t^-$, $\eta_t^-$, and $m_t$. I find
Proposition 5 Assume insider trading is allowed and the manager makes linear price and filtering conjectures. Then the manager's optimal earnings-management and insider-trading policies are linear in the state variables $\hat{m}_t$, and

\begin{align*}
(3.50) & \quad e_m = -\frac{1}{\bar{\chi}} v_2 = -\epsilon_m \\
(3.51) & \quad e_\eta = 0 \\
(3.52) & \quad e_{\hat{m}} = \epsilon_m,
\end{align*}

where

\begin{align*}
(3.53) & \quad v'_1 = R_2 p'_1 H (1 - \gamma) \rho_2 \rho_1, \quad v'_2 = R_2 p'_1 H \gamma \rho_2 \rho_1.
\end{align*}

with the scalar $\bar{\chi}$ defined in the proof. Moreover, the accounting persistence is unchanged (i.e., $g = g_0$), the signal matrix $\psi$ is reduced relative to the signal without insider trading (i.e., $0 < \psi < \psi_0$) and the accounting noise is increased relative to the noise without insider trading (i.e., $\Omega > \Omega_0$).

The second-order condition for the insider trading optimization is $\chi - K'E_m > 0$. This equation is endogenous, and will be reconsidered shortly.

The results concerning the manager's earnings-management policy are straightforward. Since there are no new accounting reports between the time of the manager's purchase and sale, prices are efficient with respect to accounting between these two dates (1/3 and 2/3) and so the manager ignores accounting. The fact that $0 < e_{\hat{m}} = E_m = -\epsilon_m$ means that the manager trades to exploit investor error $\epsilon_{t+1/3} = m_{t+1/3} - \hat{m}_{t+1/3}$ (i.e., buys if investors undervalue the firm, sells otherwise). In order to facilitate this information advantage, the manager uses earnings management to increase investor error. He does this by decreasing
the signal-to-noise ratio by reducing the signal (as modeled by $\psi$) and increasing the noise. Finally, because policies do not depend on accounting, the persistence of accounting is unchanged. If the manager were required (or allowed) to hold shares until after the release of the next accounting report (by the next manager), this situation would not obtain and $e_n$ would no longer be zero and $g$ no longer $g_0$.

### 3.5 Pricing

In this section, I compute the investors' price function as a function of their conjecture as to the manager's policies. I assume there are no arbitrage opportunities using public information. Thus, investors set the current ex-dividend price to be the discounted expected value of the next period's price. I derive and solve a recursion for the ex-dividend price, $P_t^-$, that obtains just before the new manager's accounting report is released.

For simplicity, I assume that the firm has a policy of paying out a dividend, $D_t$, that is an exogenous linear function of the manager's information set:

$$D_t = \hat{D}_0 + \hat{D}_1 m_t + \hat{D}_2 \eta_t + \hat{D}_3 m_t + z^D_t,$$

with $z^D_t \sim N(0, \sigma^2_D)$ and the $\hat{D}$ exogenous constants. I assume that the dividend is part of the accounting report, so the signal of $m_t$ that it contains (if $\hat{D}_3 \neq 0$) need not be considered separately.

Recall the investors' information vector $\hat{I}_t = (\hat{m}_t, \eta_{t-1})$ and note that (from the investors' perspective) $\hat{I}_t^- = (\hat{m}_t^-, \eta_{t-1})$ evolves as

$$\hat{I}_{t+1}^- = g \hat{I}_t^- + \text{noise},$$
where

$$G = \begin{pmatrix} p_3p_2p_1 & 0 \\ \psi + \pi_\eta & g \end{pmatrix},$$

and the noise component is just the filter error innovation (a zero-mean i.i.d. sequence when the filter is stationary).

Anticipating linear pricing, define the price coefficients

$$P_t^0 = p^0 + p^{-1}m_t^- + p^{-2}\eta_{t-1}$$

Just before the accounting report

$$P_t^0 = p^0 + p^1\hat{m}_t + p^{-2}\eta_t$$

Just after the accounting report

$$P_{t+1/3}^- = p_{1/3}^- + p_{1/3}^-\hat{m}_{t+1/3}^- + p_{1/3}^-\eta_t$$

Just before the insider-trading report

$$P_{t+1/3}^0 = p_{1/3}^0 + p_{1/3}^0\hat{m}_{t+1/3} + p_{1/3}^0\eta_t$$

Just after the insider-trading report

$$P_{t+2/3}^- = p_{2/3}^- + p_{2/3}^-\hat{m}_{t+2/3}^- + p_{2/3}^-\eta_t$$

Just before the analyst report

$$P_{t+2/3}^0 = p_{2/3}^0 + p_{2/3}^0\hat{m}_{t+2/3} + p_{2/3}^0\eta_t$$

Just after the analyst report.

Then

**Lemma 6** For any given investor conjecture of linear manager policies, the price $P_t$ is linear in the investor state estimate $\hat{m}$ and $\eta$. Moreover, the stationary price coefficients are unique and given by

$$p^1 = (\hat{D}_1 + \hat{D}_3)(I - Rp_3p_2p_1)^{-1} + \hat{D}_2R(I - Rg)^{-1}(\psi + \pi_\eta)(I - Rp_3p_2p_0^0)^{-1}$$

$$p_{1/3}^1 = R_3p_1$$

$$p_{2/3}^1 = R_2p_{2/3}p_2$$

$$p^2 = \hat{D}_2(I - Rg)^{-1}$$

$$p_{1/3}^2 = R_2p_{2/3}^2 = R_2R_3p^2.$$
The above are straightforward consequences from the rational price recursion
\[ P_{t}^- = (\hat{D}_1 + \hat{D}_3) t^- + \hat{D}_2 \eta_t + RE(P_{t+1}) \]
which has the formal solution
\[ \hat{P}_{t}^- = (\hat{D}_1, 0) (I - RG)^{-1}. \]

I have shown that policies are unique given price conjectures. Moreover, the investors' filtering is unique given policy conjectures. However, as will be seen shortly, there are two different conjectural equilibria, and both may be possible for some parameter values.

### 3.6 The Equilibrium with Accurate Accounting

I consider the equilibrium that obtains in the limit of perfectly accurate accounting. To define this limit precisely, recall the matrix \( \Omega \) characterizes the information content of the accounting report. Then, define the matrix \( \omega \) and scalar \( s \) by \( \Omega = s \omega \) with \( \text{Det}(\omega) = 1 \) and take the limit \( s \to \infty \) to denote perfectly accurate accounting. I show that two distinct behaviors can result depending on whether the audit and insider-trading disutilities are high or low compared to the manager's insider-trading profit.

Assume that the vector \( \nu' = E'_m \rho_1 \) lies in the span of \( \psi_0 \). Then, let
\[ \nu_p = \frac{\nu' \psi^{\dagger} \nu^{-1} \psi^{\dagger} \nu}{\nu'_2 / \nu^{-1} \nu'_2 / 3}, \]
\[ Q = 1 - \nu'_2 / \nu'_2 / 3 \psi^{\dagger} \psi^{\dagger} \nu^{-1} \psi^{\dagger} \nu, \]
with \( \psi^{\dagger} \) the pseudo-inverse of \( \psi \). If \( \nu' \) does not lie in the span of \( \psi_0 \), set \( b = \infty \).

\( ^9 \)From the contraction mapping theorem, the Kalman filter always converges to a unique stable value because the matrices \( \rho_c \) are constant and stable.
The parameter $b$ is similar to $b^2$ defined in Chapter 2 (recall $b^2 = \chi \nu$ there). In Chapter 2, it was shown that $b = 1$ was a critical value with different behaviors for $b$ greater or less than this value. Here, we will find similar behavior for $Q$ greater than or less than one. In fact, it is an immediate analogue of the results of Chapter 2 that

**Proposition 6**

\[(3.63) \quad \lim_{s \to \infty} \psi = \psi_0 \quad \text{if } Q < 0 \]
\[(3.64) \quad \lim_{s \to \infty} v'S_0\psi' = 0 \quad \text{if } Q \geq 0. \]

The intuition behind the theorem is exactly as in Chapter 2. If the relative regulatory-stringency parameter $Q < 0$ (tough regime), then as accounting precision $s \to \infty$ the manager’s earnings management vanishes. In contrast, if $Q > 0$, then as $s \to \infty$ the manager’s earnings management causes the report to be completely uninformative concerning the vector $v'S_0$. However, in contrast to the case of Chapter 2, now the critical value $Q = 0$ involves endogenous quantities (in Chapter 2, the analogue $1 = \chi \nu$ involved only the exogenous audit and insider trading stringency parameters $\nu$ and $\chi$).

To see why the vector $v$ is important, note that $v'm_t = E'_{m_t} \rho_1 m_t = E(E'_m m_{t+1/3} | m_t)$ is the manager’s private information that leads to his insider-trading profits. Note also that if $v'S_0\psi' = 0$, then investors gain no new information concerning the future value of $v'm_t$ from the manager’s report. To see this, recall that investors’ uncertainty in $v'm_t$ prior to the report is $v'T_0v$ and after the report it is $v'T_0v$. But $v'T_0v = v'(S_0 - S_0\psi'(\Omega + \psi S_0\psi')^{-1}\psi S_0)v = v'S_0v$ since $v'S_0\psi' = 0$.

In the following, I specialize to the case in which the analyst report $y_{l+2/3}$
includes a valuation signal that is uncorrelated with any other information in the report. Precisely, I assume that the quantity

\[ p_{2/3}^2 \chi H = p_{2/3}^2 S_{2/3} H'(\Omega + HS_{2/3}H')^{-1}HS_{2/3} \]

reduces to the simpler form

\[ p_{2/3}^2 \chi H \rightarrow \frac{p_{2/3}^2 S_{2/3} p'_{2/3}}{\omega_p + p_{2/3}^2 S_{2/3}} p_{2/3} S_0. \]

I also assume that the dividend coefficient, \( d = D_1 + D_3 \), is proportional to the price coefficient, \( p_0^1 \). Note that \( d \) represents the change of the investor estimate of the next periods dividends as their state estimate \( \tilde{m} \) changes. Since

\[ p_0^1 = d'(I - \sqrt{\rho})^{-1}, \]

with \( r = R_1^2 R_2^2 R_3^2 \), \( \rho = \rho_3 \rho_2 \rho_1 \), the requirement \( p_1^0 = \Delta d \) implies that \( d \) is an eigenvector of \( \rho \). The above assumptions will greatly simplify the following analysis, but do entail loss of generality. The usefulness of the approximation is that it implies

\[ R_1 p_{1/3}^1 = p_0' - d' \]

\[ = R p_0' \]

where \( R = (1 - 1/\Delta)^2 \). Note that \( \rho \) stable implies that \( R < r \). Finally, in the following, since the accounting-dependent price coefficients \( p_0^2 \), etc., play no further role in my analysis, I drop the superscript 1 in the price coefficients, e.g., \( p_0^1 \rightarrow p_0 \).

\[ ^{10} \Delta \text{ is less than or equal to the largest eigenvalue of } (I - \sqrt{\rho})^{-1} \text{ which is less than } 1/(1 - \sqrt{r}) \text{ since } \rho \text{ is stable. Thus, } R < (1 - 1/(1 - \sqrt{r}))^2 = r. \]
3.7 Behavior of the Equilibrium with Accurate Accounting

In this section, I consider how the equilibrium properties vary with the analyst and insider-trading variances ($\Omega_p$ and $k^2$, respectively) in the limit in which $\Omega \rightarrow 0$. In this limit, we know that the accounting report is either fully revealing or fully concealing, and so I refer to the equilibria as revealing and concealing, respectively.

I first obtain the properties of the equilibrium conditional on assuming either $Q < 0$ (revealing) or $Q > 0$ (concealing). Since $Q$ is endogenous, I next determine when the two possibilities obtain, i.e., when the equilibrium $e^*$ computed assuming $Q > 0$ actually satisfies $Q > 0$, and likewise for $Q < 0$.

I will use $e$ (defined below) as the informational-performance measure regarding the efficiency of insider-trading vs. no insider trading. The quantity $e$ is proportional to the variance of the investors' estimate of the firm's value immediately after the insider-trading report. Since it is after release of the insider-trading report that the informational effect of insider trading is most beneficial to investors, this measure is biased in favor of insider trading.

Define the endogenous variables

\[
e = R_1p_1^{1/3}T_{1/3}p_{1/3}, \quad \bar{e} = p_1^{1/3}S_{1/3}p_{1/3}, \quad z = \frac{\bar{x}kR_1}{\sqrt{e R_2 x(e)}}
\]

where

\[
x(e) = \frac{m_2 + e}{o + m_2 + e}
\]
with \( o \) defined shortly (note that \( p_{2/3} \) is a function of exogenous variables, in equilibrium). Define also the exogenous variables

\[
\begin{align*}
\sigma_1 &= p_{1/3}' \Sigma_1 p_{1/3}, & \sigma_2 &= p_{2/3}' \Sigma_2 p_{2/3}, & \sigma_3 &= p_0' \Sigma_3 p_0, & q &= \chi k \\

m_1 &= R_1^2 \sigma_1, & m_2 &= R_1^2 R_2^2 \sigma_2, & m_3 &= R_1^2 R_2^2 R_3^2 \sigma_3, & o &= R_1^2 R_2^2 \Omega_p.
\end{align*}
\]

Finally, the condition can be written \( Q(e) > 0 \), where

\[
(3.74) \quad Q(e) = 1 - b \sqrt{ez}/(qR_1x(e)).
\]

The variable \( o \) will play an important role in the following analysis. It is just the noise-covariance of the analyst report with a factor \( R_1^2 R_2^2 \) included to simplify the subsequent expressions.

### 3.7.1 Equilibrium Assuming \( Q(e^*) > 0 \) or \( Q(e^*) < 0 \)

In this subsection, I assume either \( Q(e^*) > 0 \) or \( Q(e^*) < 0 \), with \( e^* \) the equilibrium \( e \). I begin by computing equilibria assuming the sign of \( Q(e) \). I denote the equilibrium value of \( e \) when assuming \( Q(e) > 0 \) as \( e_* \) and \( \bar{e}_* \) when assuming \( Q(e) < 0 \). If indeed \( Q(e_*) > 0 \), then \( e^* = e_* \) is a valid equilibrium, and likewise for \( \bar{e}_* \) if \( Q(\bar{e}_*) < 0 \). However, this self-consistency is not considered until the next subsection.

**Lemma 7** With the above definitions, the equilibrium when \( Q(e) > 0 \) is determined by the intersection of the two curves \( z_1(e) \) and \( z_2(e) \). The equilibrium when \( Q(e) < 0 \) is determined by the intersection of the two curves \( z_1(e) \) and \( \bar{z}_2(e) \). The curves \( z_1(e) \), \( z_2(e) \), and \( \bar{z}_2(e) \) are given by

\[
(3.75) \quad z_1(e) = C(e) \left( 1 + \sqrt{1 - \frac{1 - R_2 x(e)}{R_2 x(e) C(e)^2}} \right)
\]
\[ z_2(e) = \frac{\alpha + Rx(e)\sigma}{\alpha + Rx(e) - Re}, \quad \tilde{z}(e) = \sqrt{\frac{m_1}{m_1 - e}} \]

\[ C(e) = \frac{qR_1}{2R_2x(e)\sqrt{e}}, \quad \alpha = R_2(m_3 + rm_1). \]

For future reference, I define \( Q_1(e) = 1 - b\sqrt{e}z_1(e)/(qR_1x(e)) \), \( Q_2(e) = 1 - b\sqrt{e}z_2(e)/(qR_1x(e)) \), and \( \tilde{Q}_2(e) = 1 - b\sqrt{e}\tilde{z}_2(e)/(qR_1x(e)) \).

The expression for \( z_1(e) \) is obtained from the consistency condition for \( \tilde{\chi} \). The expression for \( z_2(e) \) (\( \tilde{z}_2 \)) comes from the recursion for the stationary error covariance \( T_{1/3} \) under the assumption \( Q(e) > 0 \) (\( Q(e) < 0 \)), i.e., that the equilibrium is concealing (revealing). The recursion for \( T_{1/3} \) is greatly simplified when only the scalar \( e = R^2p'_1T_2p_1/3 \) is needed rather than the full matrix covariance \( T_{1/3} \).

Now \( z_1(e) \) is monotonically decreasing over its range. Reality of \( z_1(e) \) requires \( e < e_0 \) where \( e_0 \) satisfies \( 1 = R_2x(e_0)(1 + C(e_0)^2) \) and the minimum value of \( z_1(e) \) at this \( e_0 \) is given by \( z_1(e_0) = \sqrt{(1 - x(e_0))/x(e_0)} \). Finally, \( z_1(e) \) is infinite at \( e = 0 \) (all these additional facts are proved in the proof of the above lemma). In contrast, both \( z_2(e) \) and \( \tilde{z}_2 \) are monotonically increasing over their range and become infinite at finite \( e \).\(^{11}\)

The form of \( \tilde{z}_2(e) \) (valid for \( Q(e) < 0 \)) is immediate from the fact that without insider trading or with insider trading but \( Q(e) < 0 \) there is no earnings management when \( s \to \infty \). Since \( p_0 \) is contained in the span of \( \psi_0 \), the report fully reveals \( p'_0m_0 \), i.e., \( p'_0S_0p_0 = 0 \). Thus, the recursion for \( e \) is simpler and the form of \( \tilde{z}_2 \) vs. \( z_2 \) reflects this.

\(^{11}\)This divergence of \( z_2(e) \) and \( \tilde{z}_2(e) \) signifies that the analyst report has no information, which must be the case when \( e \) rises to the level of uncertainty existing after the insider-trading report.
Decreasing $z_1(e)$ and increasing $z_2(e)$ (or $\dot{z}_2(e)$) implies that the equilibrium $e^*$ at which $z_1(e^*) = z_2(e^*)$ (or $z_1(e^*) = \dot{z}_2(e^*)$) is unique if it exists. However, since $z_1(e) > z_1(e_0) = \sqrt{(1 - R_2x(e_0))/(R_2x(e_0))}$, there will be no equilibrium if $z_2(e_0) < z_1(e_0)$. More precisely:

Lemma 8 Assume $Q(e) > 0$. Define $e_0$ and $q_0$ by

\begin{align*}
(3.79) & \quad z_2(e_0) = \sqrt{1 - R_2x(e_0) \\
(3.80) & \quad q_0 = 2 \sqrt{R_2e_0x(e_0)(1 - R_2x(e_0))}.
\end{align*}

Then, there is no equilibrium if $q < q_0$

An identical lemma holds assuming $Q(e) < 0$ for analogues $\tilde{e}_0$ and $\tilde{q}_0$. I henceforth assume that $q > \max(q_0, \tilde{q}_0) = \tilde{q}_0$.\footnote{The later equality comes from the fact that $\tilde{e}_* < e_*$, proved in the next lemma.}

Recall that $q = k\tilde{x}$ with $k$ the noise in the manager’s insider-trading report. Recall also that the manager benefits by having information revealed after his trade. Thus, the more accurate the insider-trading signal is for investors, the greater the manager’s incentive to trade. In turn, the greater the manager trade volume, the more accurate the signal to investors. This positive feedback induces instability once $q$ falls below $q_0$. Of course, the failure of equilibrium existence may also be a sign that the original quadratic cost assumption is not an acceptable approximation rather than an event of economic significance. Finally, note that if $q > \tilde{q}_0$, then $\tilde{x} > 0$ and the second-order condition for the manager’s optimization is satisfied.

The next lemma provides some qualitative information regarding the equilibrium that will help lead to my main results.
Lemma 9 So long as the sign of \(Q(e)\) does not change, then

\[
\frac{\partial e_*}{\partial \Omega_p} > 0, \quad \frac{\partial e_*}{\partial q} > 0, \quad \frac{\partial \hat{e}_*}{\partial \Omega_p} > 0, \quad \frac{\partial \hat{e}_*}{\partial q} > 0
\]

Moreover, we have, assuming \(Q(e) > 0\)

\[qR_1 \leq \lim_{e \to 0} Q(e) \leq qR_1/R_2, \quad \lim_{p \to \infty} e = \frac{\alpha}{r - \mathcal{R}}\]

\[1 - b/R_2 \leq \lim_{p \to \infty} Q(e) \leq 1 - b, \quad \lim_{p \to \infty} Q(e) < 0\]

\[
\frac{\partial Q_1(e)}{\partial e} > 0, \quad \lim_{e \to 0} Q_1(e) > 0.
\]

The first two results simply state that, conditional on the equilibrium being the same, more noise in either the analyst or insider-trading reports (i.e., higher \(\Omega_p\) or \(q\)) leads to more noise in the investors' estimate of \(m_{t+1/3}\) after the insider-trade report i.e., higher \(e\). Thus, all non-intuitive effects of insider trading must come through changing the equilibrium (which is either revealing, \(Q(e) < 0\) or non-revealing, \(Q(e) > 0\)). Regarding the limits, note that the limit \(\Omega_p \to 0\) is just a situation in which the analyst report gives the firm's value without error. The result for \(Q(e)\) with \(\Omega_p \to 0\) (as \(R_2 \to 1\)) reproduces that found in Chapter 2.

At first glance, the limits \(\lim_{e \to 0} Q_1(e) > 0\) and \(\lim_{p \to \infty} Q(e) < 0\) may appear incompatible given that \(e\) increases in \(o\). However, the curve \(Q_1(e)\) shifts to the right as \(o\) increases, and is eventually always less than zero for \(e < \alpha/(r - \mathcal{R})\), which is the largest \(e\) can get.

### 3.7.2 Which Equilibrium?

The analysis in the above two subsections assumed \(Q(e) > 0\) or \(Q(e) < 0\). However, given either assumption, does the computed equilibrium \(Q(e_*)\), respectively, \(Q(\hat{e}_*)\), actually satisfy the assumed inequality?
First consider the equilibrium \( e_* \) that would obtain if \( Q(e_*) > 0 \). Let \( q_1(e) \) denote that \( q \) at which \( Q_1(e) = 0 \) and likewise for \( q_2(e) = \sqrt{e_2(e)/(R_1x(e))} \). Note that \( q_1 \) is implicitly a function of \( o, m_1, \) etc., which are assumed held fixed in the following discussion. The key observation is the following:

**Observation 1** If \( q > q_1(e) \) then \( Q_1(e) < 0 \) and vice versa. If \( q > q_2(e) \) then \( Q_2(e) > 0 \) and vice versa.

Note also that \( z_1(e) > z_2(e) \) if \( e < e_* \) implies both \( Q_1(e) < Q_2(e) \) and \( q_1(e) < q_2(e) \) when \( e < e_* \). This implies \( Q_1(e) \) and \( Q_2(e) \) have the same sign if and only if \( q \) lies between the two curves \( q_1(e) \) and \( q_2(e) \). Since \( Q_1(e) = Q_2(e) \) for any equilibrium \( e_* \), all possible non-revealing equilibria lie for \( q \) between these two curves. Moreover, since non-revealing equilibria require \( Q(e) > 0 \), it follows that such equilibria can exist only for \( e_* \) larger than the point of intersection of \( q_1(e) \) and \( q_2(e) \).

Thus, define \( e_q(\pi) \) by \( q_1(e_q(\pi), \pi) = q_2(e_q(\pi), \pi) \), where I am now explicitly including all other exogenous parameters, \( \pi = (b, o, m_1, m_2, .. R_1, ..) \) than \( q \). Since this implies \( z_1(e_q(\pi), \pi) = z_2(e_q(\pi), \pi) \), \( e_q \) is an equilibrium for which \( Q(e_q(\pi), \pi) = 0 \):

**Lemma 10** There exists a unique critical equilibrium \( e_q(\pi) \) at which \( Q(e_q(\pi), \pi) = 0 \) defined by \( q_1(e_q(\pi), \pi) = q_2(e_q(\pi), \pi) \).

This follows immediately from the facts that \( Q_1(e) < Q_2(e) \) if \( e < e_* \) and \( Q_1(e) > Q_2(e) \) if \( e > e_* \). Together, these facts imply that there is a unique \( e \) at which the curves \( Q_1 \) and \( Q_2 \) cross, i.e., \( Q_1(e) = Q_2(e) \). That we choose \( q \) so that \( Q_1(e) = Q_2(e) = 0 \) is just a special case.
Exactly the same analysis applies to the equilibrium candidate $\tilde{e}_*$ assuming $Q(e) < 0$, but now the equilibrium is valid for $q < q_1(\tilde{e}_q(\pi), \pi)$. Since $\tilde{e}_* < e_*$, it follows that $q_1(\tilde{e}_q(\pi), \pi) < q_1(e_q(\pi), \pi)$. We now have enough information to characterize the equilibrium properties.

**Proposition 7** If $q > q_1(e_q(\pi), \pi)$ then there is a revealing equilibrium and it is unique. If $q_2(e_q(\pi), \pi) \leq q < q_1(\tilde{e}_q(\pi), \pi)$ then there is a concealing equilibrium and it is unique. If $q_1(e_q(\pi), \pi) < q < q_1(\tilde{e}_q(\pi), \pi)$ then there is both a revealing and a concealing equilibrium.

### 3.7.3 The Role of the Analyst Report

The last subsection focused on the noisiness of the insider-trading report, $q$, as the driver of the equilibrium properties. I now consider the influence of the precision of the analyst report, $\Omega_p$, on the equilibrium. I first consider the effect on the possibility of revealing equilibria, and then the effect on the possibility of non-revealing equilibria.

Since a revealing equilibria always improves investor informativeness compared to the situation with no insider trading, the main issue with revealing equilibria is whether they are possible. Since this is determined by the requirement $q < q_1(e_q)$, I now consider the behavior of the threshold $q_1(e^*)$ (recalling that $e_q$ is also an equilibrium). We have

\begin{equation}
q_1(e^*) = \sqrt{\frac{e(1 - R_2(e))}{x(b - R_2x^2)}}.
\end{equation}

Then

\begin{equation}
\frac{d \ln(q)}{de} = \frac{de}{do} \frac{txu + y(1 - x)(ux(1 - u) - t)}{x^2t^2}
\end{equation}
(3.87) \[ t = b - R_2 x^2, \quad u = 1 - R_2 x, \quad y = \frac{e}{e + m_2 + o}, \]

and, for small \( o \)

(3.88) \[ \lim_{x \to \sqrt{bR_2}} \frac{d\ln(q)}{de} = \infty, \]

while

(3.89) \[ \lim_{o \to \infty} x^2 \frac{d\ln(q)}{de} = - \frac{y}{b} \frac{de}{do} < 0, \]

where I have used \( de/do > 0 \). Thus, \( q_i(e^*) \) has an interior maximum with respect to \( o \). In other words, the threshold regulatory stringency below which a revealing equilibrium cannot exist is maximal for intermediate values of the analyst report noise, \( o \). At larger analyst noise, while the sensitivity of the price to the manager’s earnings management is small, the manager’s information advantage is large and the latter effect dominates. At low \( o \), the manager’s information advantage is small, but the price response to earnings management is large and the latter effect dominates.

### 3.8 Conclusions

I have studied how investor informedness depends on regulatory stringency, the noise in insider-trading reports and the noise in independent-analyst reports in the limit of perfect accounting. I find that revealing equilibria can exist only if the insider-trading report noise is below a threshold, and this threshold has an interior maximum with respect to analyst noise. These results suggest that accounting
precision and analyst precision may be poor proxies for investor informedness, price efficiency, etc. when insider trading is significant.

A corollary to my results is that, for earnings management to be effective in the manner described above, the manager’s accounting report must be a sufficient statistic for predicting the analysts’ report. In particular, if analysts are valuing firms using non-GAAP accounting information, then managers wishing to use earnings management to increase their profit from insider trading need to include non-GAAP accounting information into their reports.

Another aspect of the rational expectations equilibrium is that, because the market is efficient with respect to public accounting and analyst reports, the manager’s earnings management is independent of accounting. Consequently, the persistence of accounting is unchanged by insider trading.\(^\text{13}\)

### 3.9 Glossary

- \(m_t, m_{t+1/3}, m_{t+2/3}\): the true economic assets of the firm at times \(t, t + 1/3, t + 2/3\).
- \(\eta_t\): the accounting economic assets of the firm.
- \(\hat{m}_t, \hat{m}_{t+1/3}, \hat{m}_{t+2/3}\): the investor’s estimate of the true economic assets of the firm just before the accounting, insider trading, and analyst reports, respectively.
- \(\hat{m}_t^+, \hat{m}_{t+1/3}^+, \hat{m}_{t+2/3}^+\): the investor’s estimate of the true economic assets

\(^{13}\)By persistence of accounting, I mean the correlation between past and future accounting assets conditional on economic assets. At one extreme, if fair-value accounting is used, then the persistence of accounting assets is zero because current accounting values depend only on current economic values, not on past accounting values. In any event, this persistence is unchanged by earnings management.
of the firm just after the accounting, insider trading, and analyst reports, respectively.

- $z_t, z_{t+1/3}, z_{t+2/3}$: a shock to economic assets at times $t, t + 1/3, t + 2/3$.

- $n_t$: a shock to accounting assets.

- $\delta_t$: the manager's choice of earnings management.

- $\Gamma$: matrix describing the effect of earnings management on the accounting report.

- $e_{t+1/3}$: the manager's choice of insider trading volume.

- $\rho_i, i = 1, 2, 3$: the persistence of the economic assets.

- $\gamma_0$: the persistence of the accounting assets.

- $\psi_0$: the information matrix for the accounting report.

- $\psi$: the information matrix for the accounting report modified by insider trading.

- $H$: the information matrix for the analyst report.

- $\beta_0$: The Kalman Gain for the accounting report with no insider trading.

- $\beta$: The Kalman Gain for the accounting report with insider trading.

- $\gamma$: The Kalman Gain for the analyst report.

- $K$: The Kalman Gain for the insider trading report.
• \( \pi_i \): change in accounting assets due to earnings management that depends on the investor estimate \( \hat{m}_t \).

• \( \Omega_0 \): the pre-earnings management accounting noise matrix.

• \( \Omega \): the post-earnings management accounting noise matrix.

• \( P_t^-, P_{t+1/3}^-, P_{t+2/3}^- \): the equity price before the accounting, insider trading, and analyst reports, respectively.

• \( P_t^+, P_{t+1/3}^+, P_{t+2/3}^+ \): the equity price after the accounting, insider trading, and analyst reports, respectively.

• \( S_t, S_{t+1/3}, S_{t+2/3} \): the investor error covariance before the accounting, insider trading, and analyst reports, respectively.

• \( T_t, T_{t+1/3}, T_{t+2/3} \): the investor error covariance after the accounting, insider trading, and analyst reports, respectively.

• \( p_i^\delta \): price coefficients, e.g., \( P_t^- = p^{-0} + p^{-1}\hat{m}_t^- + p^{-2}\eta_t \).

• \( \delta_m \): the manager’s earnings management policy coefficients, e.g., \( \delta_t = \delta_0 + \delta_m m_t + \delta_\eta \eta_t + \delta_\hat{m} \hat{m}_t^- \).

• \( R_1, R_2, R_3 \): discount rates.

• \( k \): standard deviation of the noise in the manager’s insider trading report.

• \( \chi \): scales the manager’s personal disutility incurred by insider trading.

• \( \nu \): scales the manager’s personal disutility incurred by earnings management.
\( e = R_1^2 p_1^{1/3} T_1^{1/3} P_1^{1/3}, \quad \bar{e} = p_1^{1/3} S_1^{1/3} P_1^{1/3}, \quad z = \frac{\sqrt{k_{R_0}}}{\sqrt{e R_0 x(e)}}, \quad \sigma_1 = p_1^{1/3} S_1^{1/3} P_1^{1/3}, \quad \sigma_2 = p_2^{1/3} S_2^{1/3} P_2^{1/3}, \quad \sigma_3 = p_0^{1/3} S_0^{1/3} P_0^{1/3}, \quad q = \chi k, \quad m_1 = R_1^2 \sigma_1, \quad m_2 = R_1^2 R_2^2 \sigma_2, \quad m_3 = R_1^2 R_2^2 R_3^2 \sigma_3, \quad o = R_1^2 R_2^2 \omega_p, \quad x(e) = \frac{(m_2 + e)}{(m_2 + e + o)}: \text{scaled parameters.} \)
Appendix A

Proofs

A.1 Proof of Lemma 5

I use the matrix $T_t$, etc., to denote the investors’ error covariance of the entire stacked history, $m_t$, and $T_{ts}$ to denote the error covariance of a single pair $m_t, m_s$.

First, let there be no insider trading. Using a conjecture about the manager’s policies, investors posit the evolution equations for $m_t$ and $\eta_t$ are

$$m_{t+1/3} = \rho_1 m_t + z^1_t$$
$$m_{t+2/3} = \rho_2 m_{t+1/3} + z^2_t$$
$$m_{t+1} = \rho_3 m_{t+2/3} + z^2_t$$
$$\eta_{t+2/3} = \eta_{t+1/3} = \eta_t$$
$$\eta_{t+1} = \psi_0 m_{t+1} + g\eta_t + n_t$$

Observing $\eta_{t+1}$ implies the observation equation $y_{t+1} = \eta_{t+1} - g\eta_t$ has the form

$$y_t = \psi_0 m_{t+1} + n_t$$

Start with the distribution $m_t \sim N(\hat{m}_t, T_{0,t})$ that obtains immediately after $\eta_t$ has been observed by investors. Then investors have subsequent estimates
\( m_{t+2/3} \sim N(\hat{m}_{t+2/3}, S_{0,t+2/3}) \) immediately before release of the analyst report and
\( m_{t+2/3} \sim N(\hat{m}_{t+2/3}, T_{0,t+2/3}) \) immediately after. Continuing, investors estimate
\( m_{t+1} \sim N(\hat{m}_{t+1}, S_{0,t+1}) \) immediately before release of the accounting report and
\( m_{t+1} \sim N(\hat{m}_{t+1}, T_{0,t+1}) \) immediately after. Standard Kalman filtering gives the
results that

\[
\hat{m}_{t+2/3} = \rho_0 \rho_1 \hat{m}_t \\
S_{0,t+2/3} = \Sigma_2 + \rho_2 (\Sigma_1 + \rho_1 T_t \rho_1') \rho_2' \\
\hat{m}_{t+2/3} = \hat{m}_{t+2/3} + \gamma_t' (y_{t+2/3} - H \hat{m}_{t+2/3}) \\
T_{0,t+1} = (S_{0,t+1}^{-1} + L_p)^{-1} \\
\hat{m}_{t+1} = \rho_3 \hat{m}_{t+2/3} \\
S_{0,t+1} = \Sigma_3 + \rho_3 T_{t+2/3} \rho_3' \\
\hat{m}_{t+1} = \hat{m}_{t+1} + \beta_t' (y_t - \psi \hat{m}_{t+1}) \\
T_{0,t+1} = (S_{0,t+1}^{-1} + L_0)^{-1}
\]

where

\[
\beta_t' = T_{0,t+1}^{-1} \psi_0 \Omega^{-1} \\
L_0 = \psi_0' \Omega^{-1} \psi_0, \quad L_p = \psi_0' \Omega_p^{-1} \psi_0
\]

Finally, at time \( t + 1/3 \), \( m_{t+1/3} \sim N(\hat{m}_{t+1/3}, T_{t+1/3}) \) with

\[
\hat{m}_{t+1/3} = \rho_1 \hat{m}_t \\
T_{0,t+1/3} = \Sigma_1 + \rho_1 T_0 \rho_1'
\]

With insider trading investors posit that the evolution equations for \( m_t \) and
\( \eta_t \) are

\[
m_{t+1/3} = \rho_1 m_t + 1/3 \eta_t + \pi_1 m_t + z_t
\]
\[ m_{t+2/3} = \rho_2 m_{t+1/3} + z_t^2 \]
\[ m_{t+1} = \rho_3 m_{t+2/3} + z_t^2 \]
\[ \eta_{t+2/3} = \eta_{t+1/3} = \eta_t \]
\[ \eta_{t+1} = \psi m_{t+1} + g \eta_t + \pi \bar{m}_t + n_t \]

Observing \( \eta_t \) implies the observation equation \( y_{t+1} = \eta_{t+1} - g \eta_t - \pi \bar{m}_t \) has the form

\[ y_t = \psi m_{t+1} + n_t \]

With insider trading equilibrium also requires that the investors conjecture the manager insider-trading policy is \( e_{t+1/3} = e_0 + e'_m m_{t+1/3} + e'_m \bar{m}_{t+1/3} + e'_\eta \eta_t + z_{t+1/3}^e \). Let \( y_{t+1/3}^e = e_{t+1/3} - e'_m \eta_t - e'_x \bar{m}_{t+1/3} \). Once again, start with the distribution \( m_t \sim N(\bar{m}_t, T_t) \) that obtains immediately after \( \eta_t \) has been observed by investors. Standard Kalman filtering gives the results that

\[ \hat{m}_{t+1/3} = \rho_1 \hat{m}_t + 1/3 \eta_t + \pi_{m} \bar{m}_t \]
\[ S_{t+1/3} = \Sigma_1 + \rho_1 T_t \rho'_1 \]
\[ \hat{m}_{t+1/3} = \hat{m}_t + K(y_{t+1/3} - e_m \bar{m}_t + 1/3) \]
\[ T_{t+1/3} = (S_{t+1/3}^{-1} + L_e)^{-1} \]
\[ \hat{m}_{t+2/3} = \rho_2 \hat{m}_{t+1/3} \]
\[ S_{0,t+2/3} = \Sigma_2 + \rho_2 T_{t+1/3} \rho'_2 \]
\[ \hat{m}_{t+2/3} = \hat{m}_{t+2/3} + \gamma_0 \tilde{y}_{t+2/3} - H \hat{m}_{t+2/3} \]
\[ T_{0,t+1} = (S_{0,t+1}^{-1} + L_p)^{-1} \]
\[ \hat{m}_{t+1} = \rho_3 \hat{m}_{t+2/3} \]
\[ S_{t+1} = \Sigma_3 + \rho_3 T_{t+1/3} \rho'_3 \]
\[ \hat{m}_{t+1} = \hat{m}_{t} + \beta_t (y_t - \psi \hat{m}_{t+1}) \]

\[ T_{t+1} = (S_{t+1}^{-1} + L)^{-1} \]

where

\[ \beta_t = T_{t+1/3}^{-1} \psi \Omega^{-1}, \quad K = k^{-1}T_{t+1/3}e_m \]

\[ L = \psi' \Omega^{-1} \psi, \quad L_e = k^{-2} e_m e'_m \]

Now, consider the case in which insider trading is possible. Let \( \eta_0^t \) denote the value of \( \eta \) before it has been managed, i.e., \( \eta_0^t = \eta^t - \Gamma \delta_t \).

When insider trading is possible the manager's program can be written:

\[ U^* = \max_{\delta_t} \mathbb{E}(w_1 + R_1 V^* + DU_1 | \mathcal{I}_t) \text{ at time } t \]

\[ V^* = \max_{e_{t+1/3}} \mathbb{E}(e_{t+1/3} P_{t+1/3} (\hat{m}_{t+1/3}, \eta_t) + R_2 w_2 + DU_2 | \mathcal{I}_{t+1/3}) \text{ at time } t \]

where \( \mathcal{I}_t \) is the manager's information set at time \( t \), similarly for \( \mathcal{I}_{t+1/3} \), and

\[ w_1 = \epsilon D_t \]

\[ w_2 = \theta_2 + \phi_2 \eta_{t+1} + (\epsilon - e_{t+1/3}) P_{t+2/3} (\eta_t, \hat{m}_{t+2/3}) \]

\[ DU_1 = - \frac{1}{2} \delta_t \nu \delta_t \]

\[ DU_2 = - \frac{1}{2} \chi e_{t+1/3}^2 \]

\[ \eta_t = \gamma \eta_{t-1} + \psi m_t + \Gamma (\delta + n_t) \]

\[ \hat{m}_t = \hat{m}_t + \beta (\eta_t - \hat{e} \eta_{t-1} - (\psi + \hat{\pi}) \hat{m}_t) \]

\[ \hat{m}_{t+1/3} = \hat{\rho}_1 \hat{m}_t \]

\[ \hat{m}_{t+1/3} = \hat{m}_{t+1/3} + K (\hat{y}_{t+1/3} - (\hat{e} \hat{m} + \hat{e}_m) \hat{m}_{t+1/3}) \]

\[ \hat{m}_{t+1} = \rho_3 (I - \gamma H) \rho_2 \hat{m}_{t+1/3} + \rho_3 \gamma (H \hat{m}_{t+2/3} + z_{t+2/3}) \]

\[ \hat{m}_{t+2/3} = \rho_2 \hat{m}_{t+1/3} \]
\[ \hat{m}_{t+2/3} = \hat{m}_{t+2/3}^\gamma + \gamma^0 y_{t+2/3}^\gamma - H \hat{m}_{t+2/3}^\gamma \]
\[ m_{t+1/3} = \rho_1 m_t + z_t^1 \]
\[ m_{t+2/3} = \rho_2 m_{t+1/3} + z_t^2 \]

where \( \hat{g}, \) etc., denote investor conjectures, policy.

With some algebra,

\[ V^* = V_0 + \max_{e_{t+1/3}} \left( \beta' \eta_t + \beta' m_{t+1/3} - E_{m} \right) \]

where \( V_0 \) is an irrelevant constant independent of all decision or state variables, and

\[ \xi = \hat{e}_m + \hat{e}_n \]
\[ \hat{P}_t \equiv p_{1/3}^2 - Rp_{2/3} \]
\[ = 0 \]
\[ v_1' = Rp_{2/3} (I - \gamma H) \rho_2 \]
\[ v_2' = Rp_{2/3} \gamma H \rho_2 \]

Performing the optimization over \( e_{t+1/3} \) leads to

\[ V^* = V_0 + \frac{1}{2\hat{\chi}} (e' \eta_t + e' m_{t+1/3}) - E_{m}^2 \]
\[ e^* = \frac{1}{\hat{\chi}} (e' \eta_t + e' m_{t+1/3}) - E_{m} \]
\[ \hat{\chi} \equiv \chi - v_1' \]

with the indentifications

\[ e_m = -E_{m} = \frac{1}{\hat{\chi}} v_2, \quad e_\eta = \frac{1}{\hat{\chi}} v_1' \hat{e}_\eta' \]
\[ e_m = \frac{1}{\bar{\chi}} (p_{t+1/3}^1 - \nu'_1(I - K\zeta)) \]

Requiring the equilibrium conditions \( \dot{e}_\eta = e_\eta \) and \( \dot{e}_m = e_m \), I find

\[ e_\eta = 0 \]
\[ e_m = E_m \]

Now, since \( K = S_{1/3}E_m/(k^2 + E'_mS_{1/3}E_m) \), the equation for \( E_m \) is implicit, because it depends on \( \bar{\chi} = \chi - \nu'_1K \). We have

\[ ((k^2 + E'_mS_{1/3}E_m)\chi - \nu'_1S_{1/3}E_m)E'_m = (k^2 + E'_mS_{1/3}E_m)\nu'_2 \]

and so

\[ (A.1) \quad E'_m = \frac{\nu'_2}{\bar{\chi}} \]

where \( \bar{\chi} \) satisfies

\[ 0 = (\bar{\chi}^2k^2 + \nu'_2S_{1/3}\nu_2)(\chi - \bar{\chi}) - \bar{\chi}v'_1S_{1/3}\nu_2 \]

Next, using

\[ E(m_{t+1/3}|I_t) = \rho_t m_t \]
\[ \eta_t = \eta_t^0 + \Gamma \delta_t \]

the first period optimization over \( \delta \) can be written

\[ U^* = \max_{\alpha_t, \delta_t} \left( \frac{\bar{\chi}}{2} \left( \nu'_4\Gamma(\delta_t + n_t) + \nu'_0(\dot{m}_t - m_t) - \nu'_4g\eta_{t-1} \right)^2 - \frac{1}{2} (\delta_t'\nu\delta_t) \right) \]

where

\[ (A.2) \quad \nu'_0 = E'_m\rho_1(I - \beta\psi) \]
\[ v_4' = E_m^T \hat{p}_1 T \]

Performing the first period optimization over \( \delta \)

\[ \delta^* = \bar{c} \nu^{-1} v_4' \Gamma v_4 \quad (v_4' \Gamma n_t + v_0' \hat{m}_t - v_0' m_t - v_4' g n_{t-1}) \]

\[ \bar{c} = \frac{1}{\bar{\chi} - v_4' \Gamma \nu^{-1} \Gamma v_4} \]

from which I identify (using the matrix \( z = \bar{c} \Gamma \nu^{-1} \Gamma' \))

\[ \Gamma \delta_m = -z v_4 v_0' \quad \Gamma \delta_n = z v_4 v_4' \]

\[ \Gamma \delta_n = -z v_4 v_0' \quad \Gamma \delta_m = z v_4 v_0' \]

The second-order condition for the optimization is just \( \bar{c} > 0 \). Finally, note that \( z \) is exogenous save for its dependence on \( \bar{c} \).

**A.2 Proof of Lemma 6**

From no arbitrage, the price evolves according to (3.7)-(3.12). Because of linear evolution of \( \sigma_t \) and linear dependence of \( D_t \) on \( \hat{I}_t \), it follows that \( P_t \) is linear in \( \hat{I}_t \):

\[ P_t^{-0} = p_t^{-0} + p_t^{-1} \hat{m}_t^{-} + p_t^{-2} \eta_{t-1} \] etc. I consider the case with insider trading only as that without insider trading is not needed and, in any event, easily obtained as a special case.

To see how to form expectations, note that (from the innovations property of the prediction error), when there is insider trading, \( E(y_t - \psi_0 \hat{m}_t | \hat{m}_t^{-}, \eta_{t-1}) = 0 \) and so

\[ E(\eta_t | \hat{m}_t^{-}, \eta_{t-1}) = g \eta_{t-1} + \psi_0 \hat{m}_t^{-} \]
When the insider-trading report comes out, there is no update to $\eta_t$. Next, for all reports, there is no expected change in the state estimate:

$$E(\hat{m}_t | \hat{m}_t, \eta_{t-1}) = \hat{m}_t^-$$
$$E(\hat{m}_{t+1/3} | \hat{m}_{t+1/3}, \eta_t) = \hat{m}_{t+1/3}^-$$

Finally,

$$\hat{m}_{t+1/3}^- = \rho_1 \hat{m}_t, \quad \hat{m}_{t+1}^- = \rho_2 \hat{m}_{t+1/3}^-$$

Define $\tilde{P}_t^- = (P_t^{-1}, P_t^{-2})$, etc. Substituting for the price the linear conjecture, using the above expectations, and matching coefficients of powers of $\sigma_t \equiv (\hat{m}_t, \eta_t)$, the stationary price coefficients obeys the following:

$$\tilde{P}_t^- = \tilde{P}_t \begin{pmatrix} I & 0 \\ \psi + \pi & g \end{pmatrix} + d$$

where

$$d = \begin{pmatrix} \hat{D}_1 + \hat{D}_3 \\ \hat{D}_2 \end{pmatrix}$$

Finally, note that with insider trading, the above can be combined into a recursion for $\tilde{P}_t^-:

$$\tilde{P}_t^- = d + R\tilde{P}_{t+1}^- G$$
\[ G = \begin{pmatrix} \rho_3 \rho_2 \rho_1 & 0 \\ \psi + \pi_\eta & g \end{pmatrix} \]

with formal (stationary) solution

\[ \tilde{P}^- = d(I - RG)^{-1} \]

which leads to

(A.3) \[
\begin{equation}
\begin{aligned}
p^1 &= (\hat{D}_1 + \hat{D}_3)(I - R\rho_3 \rho_2 \rho_1)^{-1} + \hat{D}_2 R(I - Rg)^{-1} (\psi + \pi_\eta)(I - R\rho_3 \rho_2 \rho_1^0)^{-1} \\
p_{2/3}^1 &= R_3 p^1 \rho_3 \\
p_{1/3}^1 &= R_2 p_{2/3}^1 \rho_2 \\
p^2 &= \hat{D}_2 (I - Rg)^{-1} \\
p_{1/3}^2 &= R_2 p_{2/3}^2 = R_2 R_3 p^2
\end{aligned}
\end{equation}
\]

This formal solution is valid provided \( RG \) is a stable matrix, in which case \( RG \) is a contraction and so the solution is unique.

### A.3 Proof of Proposition 6

Now, the dynamics for \( m_t \) and \( \eta_t \) can be written in two forms. The first uses dependence on the manager’s action \( \delta_t \), the second assumes a policy for this actions. Thus,

\[ \eta_{t+1}^0 = g_0 \eta_t + n_t \]

\[ \eta_t = g_0 \eta_{t-1} + \psi m_t + \Gamma(\delta_t + n_t) \]

\[ = \hat{g} \eta_{t-1} + \hat{\psi} m_t + \hat{\pi}_\eta \hat{n}_t^- + G \Gamma n_t \]
from which I identify

\[ \hat{\psi} = \psi_0 + \Gamma \hat{\delta}_m \]
\[ = \psi_0 - zv_4 v'_0 \]
\[ \hat{g} = g_0 + \Gamma \delta_n^- \]
\[ = g_0 - zv_4 v'_4 \hat{g} \]
\[ G = I + \Gamma \delta_n \]
\[ = I + zv_4 v'_4 \]
\[ \Omega = (I + \Gamma \hat{\delta}_n) \Omega_0 (I + \Gamma \hat{\delta}_n)' \]
\[ = (I + zv_4 v'_4 \Gamma) \Omega_0 (I + \Gamma' v_4 v'_4)' \]
\[ \hat{\pi}_n = \Gamma \hat{\delta}_m \]
\[ = zv_4 v'_4 \]

Finally, for \( g \), I find

\[ g = (I + zv_4 v'_4) g_0 - zv_4 v'_4 g \]

which implies

\[ g = g_0 \]

This proves the results \( g = g_0 \).

For the results about \( \psi \), first note that \( \psi + \pi_n = \psi_0 \). It follows that the price coefficients \( p^1 \), which depends on endogenous quantities only through the sum \( \psi + \pi_n \), does not depend on \( \beta \) or \( \psi \). It follows that the vector \( v \) defined by

\[ v' = E_m p_1 \]
depends on endogenous variables only through \( \gamma \). Now, look at \( v' = \nu' \beta \) and \( v'_0 = \nu' - v'_4 \psi \). Using \( \psi = \psi_0 - zv_4 v'_0 \) we have

\[
v'_0 = \nu' - v'_4 \psi_0 + c_1 v'_0, \quad c_1 = v'_4 zv_4
\]

and so

\[
v'_0 = \frac{\nu' - v'_4 \psi_0}{1 - c_1}
\]

To study \( v'_4 \), begin with

\[
\beta = (S_0^{-1} + \psi' \Omega^{-1} \psi)^{-1} \psi' \Omega^{-1} = S_0 \psi' (\Omega + \psi S_0 \psi')^{-1}
\]

from which we obtain the consistency equation for \( v_4 \)

\[
v'_4 \Omega = (\nu' - v'_4 \psi) S_0 \psi' = v'_0 S_0 \psi'
\]

Now consider the limit \( \Omega_0 \to 0 \). Since \( \Omega = (1 + zv_4 v'_4) \Omega_0 (I + v_4 v'_4) \) and \( zv_4 v'_4 \) is bounded (by the second-order condition), \( \Omega_0 \to 0 \) implies \( \Omega \to 0 \). Thus, in this limit we must either have

\[
v'_0 = 0
\]

or

\[
0 = v'_0 S_0 \psi'
\]
I now investigate these two possibilities. To this end, recall that

\[ z = \frac{\mu}{\bar{X} - c_0} \]

where

\[ \mu \equiv \Gamma \nu^{-1} \Gamma', \quad c_0 = v'_4 \mu v_4 \]

and note that this implies

\[ c_1 = \frac{c_0}{\bar{X} - c_0} \]

**A.3.1 Case 1: \( v'_0 = 0 \)**

Let

\[ w'_0 = v' - v'_4 \psi_0 \]

Then, the equation for \( v_0 \) is

\[ v'_0 = w'_0 + v'_4 z v_4 v'_0 \]

Then, using \( v'_4 z v_4 = c_0/(\bar{X} - c_0) \), we find

\[ v_0 = c_2 w_0, \quad c_2 = \frac{\bar{X} - c_0}{\bar{X} - 2c_0} \]

and so the condition \( w = 0 \) becomes \( w'_0 = 0 \). Thus, we must have

\[ v' = v'_4 \psi_0 \]

But the second-order condition requires

\[ \bar{X} > v'_4 \mu v_4 \]
with $\psi_0^\dagger$ the pseudo inverse of $\psi_0$. Thus, this solution is possible only if $\mu$ is small enough, i.e., the audit disutility matrix $\nu$ is large enough.

### A.3.2 Case 2: $v'_0 S_0 \psi' = 0$

As the proof of the first case showed, if $v_0 \neq 0$, $v'_0 S_0 \psi' = 0$ implies $w'_0 S_0 \psi' = 0$.

Thus, Now, with $v_0 = c_2 (v - \psi'_0 v_4)$,

\begin{equation*}
0 = w'_0 S_0 \psi' = (v' - v'_4 \psi_0) S_0 (\psi'_0 - c_2 (v - v_4 \psi_0) v'_4 x)
\end{equation*}

from which I find

\begin{equation*}
v'_4 = v'_0 \psi'_0 (c_1 \mu + \psi_0 S_0 \psi'_0)^{-1}
\end{equation*}

where the scalar $c_1$ is given by

\begin{equation*}
c_1 = \frac{c_2}{\lambda - c_0} (v' - v'_4 \psi_0) S_0 (v - \psi'_0 v_4)
\end{equation*}

\begin{equation*}
c_1 = \frac{1}{\lambda - 2c_0} (v' - v'_4 \psi_0) S_0 (v - \psi'_0 v_4)
\end{equation*}

Define

\begin{equation*}
B = S_0^{-1/2} \psi_0^\dagger \mu \psi_0^\dagger S_0^{-1/2}, \quad y' = v'_0 S_0^{1/2}
\end{equation*}

From the solution for $v_4$, we obtain the consistency equations for $c_0$ and $c_1$:

\begin{equation*}
c_0 = v'_4 \mu v_4 = y'(c_1 B + I)^{-1} B(c_1 B + I)^{-1} y
\end{equation*}
and, for $c_1$

$$(\bar{x} - 2c_0)c_1 = y'(I - (c_1B + I)^{-1})(I - (c_1B + I)^{-1})y$$

or

(A.8) $$(\bar{x} - 2c_0) = c_1y'B(c_1B + I)^{-2}By$$

Combining the two, we find

$$\bar{x} = y'B(c_1B + 2I)(c_1B + I)^{-2}y$$

$$= y'B((c_1B + I)^{-2} + (c_1B + I)^{-1})y$$

The right hand side of this equation is monotonically decreasing in $c_1$ with maximal value $y'By = v'\psi_0^t\mu\psi_0^{t'\psi}v$. Thus, a solution is possible only if $\bar{x} \leq v'\psi_0^t\mu\psi_0^{t'\psi}v$.

Note also that the prior case ($v_0' = 0$) was found to be possible only if the inequality were reversed. Thus, only one solution is possible for any set of parameters.

To show that a solution for case two is possible when $\bar{x} < v'\psi_0^t\mu\psi_0^{t'\psi}v$, note that if so, then the last equation has a unique solution for $c_1$. Using (A.8), we find which gives $2c_0 = \bar{x} - y'B^2(c_1B + I)^{-1}(c_1B + I)^{-1}y < \bar{x}$ and $> 0$, and so a valid solution.

A.4 Proof of Lemma 7

The equilibrium is defined by the recursions for the covariances $T$ and $S$ together with expressions for $E_m$ and $\gamma$. As will be seen, because I am considering the case $s \to \infty$, the accounting report gain $\beta$ does not play a role. As will be seen, for the special case of inner products with the price vectors $p_0$, etc., the filtering recursions simplify greatly.
I first work out the recursions for $T$ and $S$ for the assumption $Q(e) > 0$, and then $Q(e) < 0$. I next determine the consistency condition for $\tilde{\chi}$.

Define

$$p = \frac{p'_{2/3}S_{2/3}p_{2/3}}{\Omega_p}$$

$$e = R_1'p_{1/3}T_{1/3}p_{1/3}, \quad \bar{e} = p'_{1/3}S_{1/3}p_{1/3}, \quad z = \frac{\chi k R_1(1 + p)}{\sqrt{e} R_2 p}$$

$$\sigma_1 = p'_{1/3}\Sigma_1 p_{1/3}, \quad \sigma_2 = p'_{2/3}\Sigma_2 p_{2/3}, \quad \sigma_3 = p'_{0}\Sigma_3 p_{0}$$

$$q = \chi k, \quad r = (R_1 R_2 R_3)^2$$

$$m_1 = R_1'^2\sigma_1, \quad m_2 = R_1'^2 R_2'^2 \sigma_2, \quad m_3 = R_1'^2 R_2'^2 R_3'^2 \sigma_3, \quad o = R_1'^2 R_2'^2 \Omega_p$$

$$z = \frac{\chi k R_1}{\sqrt{e} R_2 x(e)}, \quad x(e) = \frac{m_2 + e}{o + m_2 + e}$$

Note that since $p_{1/3} = R_2 p_{2/3} p_2$ and $S_{2/3} = \Sigma_2 + \rho_2 T_{1/3} \rho_2'$, we have

$$x(e) = (\sigma_2 + e/(R_1'^2 R_2'^2))/\Omega_p$$

$$= (m_2 + e)/o$$

Assume $Q(e) > 0$, so that $p_0 T_0 p_0 = p_0' S_0 p_0$. Begin with two of the filtering recursions for $S$ and $T$

$$S_0 = \Sigma_3 + \rho_3 T_{2/3} \rho_3$$

$$S_{1/3} = \Sigma_1 + \rho_1 T_0 \rho_1'$$

which implies

$$\bar{e} = \sigma_1 + p'_{1/3} p_1' T_0 p_0' / R_1^2$$

$$= \sigma_1 + R p_0' T_0 p_0 / R_1^2.$$
where \( \mathcal{R} = (1 - 1/\Delta)^2 \), and I have used \( p'_0 = \delta d' = d' + p'_{1/3} \rho_1 \), from which \( p'_{1/3} \rho_1 = (1 - 1/\Delta)p'_0 \). Since, assuming \( Q(e) > 0 \), earnings management implies \( p'_0 T_0 p_0 = p'_0 S_0 p_0 \), we have

\[
\bar{e} = \sigma_1 + \mathcal{R} p'_0 S_0 p_0 / R_1^2 \\
= \sigma_1 + \mathcal{R} p'_0 (\Sigma_3 + \rho_3 T_{2/3} p'_3 p_0) / R_1^2 \\
= \sigma_1 + \mathcal{R} \sigma_3 / R_1^2 + \mathcal{R} p'_{1/3} T_{2/3} p_{1/3} / (R_1^2 R_3^2) \\
= \sigma_1 + \mathcal{R} \sigma_3 / R_1^2 + \mathcal{R} x(e) \Omega_p / R_1^2 R_3^2 \\
= \frac{1}{R_1^2} (r m_1 + \mathcal{R} m_3 + \mathcal{R} o x(e)).
\]

Now, assume \( Q(e) < 0 \), so that \( p'_0 S_0 p_0 = 0 \) and \( \bar{e} = p'_{1/3} S_{1/3} p_{1/3} = \sigma_1 \), and no further work is needed.

Next, we use

\[
e = R_1^2 p'_{1/3} \left( S_{1/3} - \frac{S_{1/3} E_m E'_m S_{1/3}}{k^2 + E'_m S_{1/3} E_m} \right) p_{1/3}
\]

Now

\[
E'_m = \frac{1}{\chi} \nu'_2 \\
= R_2 p'_{2/3} \gamma H \rho_2 \\
= \frac{R_2 p'_{2/3} S_{2/3} p_{2/3} \rho_2}{\Omega_p + p'_{2/3} S_{2/3} p_{2/3}} \\
= x(e) R_2 p'_{1/3}
\]

Thus,

\[
e = R_1^2 \bar{e} - \frac{R_1^2 R_3^2 x(e)^2 \bar{e}^2}{\chi^2 k^2 + R_3^2 x(e)^2 \bar{e}}
\]

Solving for \( \bar{e} \), we find, using the definition of \( z \)

\[
\bar{e} R_1^2 = \frac{e z^2}{z^2 - 1}
\]
Note that non-negativity of $\tilde{e}$ and $e$ requires $z > 1$. Now, above I found two other expressions for $\tilde{e}$, depending on whether I assumed $Q(e) > 0$ or $Q(e) < 0$. For $Q(e) > 0$,

$$\tilde{e} = \frac{1}{rR_1^2} (r\sigma_1 + \sigma_3 + ox(e))$$

Eliminating $\tilde{e}$ and solving for $z$ gives $z_2(e)$

$$z_2^2(e) = 1 + \frac{re}{rm_1 + Rm_3 + Rox(e) - re}$$

Assuming $Q(e) < 0$, using $e = \tilde{e}R_1^2(1 - 1/z^2)$, and $\tilde{e} = \sigma_1$, I find

$$z_2^2(e) = 1 + \frac{e}{m_1 - e}$$

Finally, we have the equation for $\tilde{x}$, which will give us $z_1(e)$.

$$\tilde{x} = x - K'v_1$$
$$= x - \frac{E'_m S_{1/3}(1 - x(e)R_2)p_{1/3}}{\tilde{x}k^2 + R_2^2x(e)^2\tilde{e}}$$
$$= x - \frac{x(e)eR_2/R_1^2}{\tilde{x}k^2} (1 - x(e)R_2)$$

Using $q = k\chi$ and the definition of $z$, one finds

$$0 = z^2 - 1 + \frac{1}{x(e)R_2\sqrt{e}} \left( \sqrt{\tilde{e} - R_1qz} \right)$$

The solution of this equation is just $z_1(e)$. 

(A.9)
A.5 Proof of Lemma 8

If \( q < q_* \) then \( z_1(e) \) becomes imaginary for some \( e_0 < e_* \). Since \( z_1 \) is decreasing in \( e \) and \( z_2 \) increasing, we have the chain of inequalities \( z_1(e_0) > z_1(e_*) = z_2(e_*) > z_2(e_0) \) where the middle equality is just the definition of \( e_* \). But \( z_1(e_0) > z_2(e_0) \) implies there is no intersection for \( e < e_0 \) and hence no equilibrium.

A.6 Proof of Lemma 9

The proof uses the envelope theorem. Let

\[
eqn_1 = R_2x(e)(z^2 - 1) + 1 - \frac{R_1q}{\sqrt{e}}z
\]

\[
eqn_2 = -z^2 + \frac{rm_1 + m_3 + ax}{rm_1 + m_3 + ax(e) - re}
\]

I will also use \( eqn_1 = 0 \) to replace \( z^2 - 1 \) with an expression linear in \( z \), as needed.

Then, since in equilibrium \( eqn_1 = 0 = eqn_2 \), we have

\[
0 = \frac{\partial eqn_1}{\partial o} + \frac{\partial eqn_1}{\partial e} \frac{\partial e}{\partial o} + \frac{\partial eqn_1}{\partial z} \frac{\partial z}{\partial o}
\]

\[
0 = \frac{\partial eqn_2}{\partial o} + \frac{\partial eqn_2}{\partial e} \frac{\partial e}{\partial o} + \frac{\partial eqn_2}{\partial z} \frac{\partial z}{\partial o}
\]

Using

\[
\frac{\partial x(e)}{\partial o} = -p(e)x(e), \quad \frac{\partial x(e)}{\partial e} = p(e)^2 o
\]

where

\[
p(e) = \frac{1}{o + m_2 + e}
\]

I find

\[
\left( \frac{\partial z}{\partial o}, \frac{\partial e}{\partial o} \right) = x(e) \left( \frac{z^2 - 1}{\text{Det}(M)} \right) M \theta
\]
\[ M = \left( \frac{z^2 - 1}{e} (1 + (z^2 - 1)(1 - (1 - x)^2 / r)) \right) \quad - \left( \frac{gR_1 z}{2e^{3/2}} + p(e)(1 - x(e))(z^2 - 1) \right) \]
\[ \theta = \left( \frac{p(e)R_2 x}{x(e) \frac{e^{3/2} - 1}{re}} \right) \]

with \( \text{Det}(M) > 0 \). I thus find, ignoring the positive prefactor \( x(z^2 - 1) / \text{Det}(M) \),
\[
\frac{\partial z}{\partial \theta} = \frac{(z^2 - 1) \left( 2e^{3/2} p(e) R_2 (2x^2 - 3x + (r + (3 - 2x)x - 1)z^2 + 1) - qR_1 xz \right)}{2e^{5/2}r}
\]
\[
\frac{\partial e}{\partial \theta} = \frac{2p(e)R_2 e^{3/2} z + (z^2 - 1)(2R_2 \sqrt{e} x(e) z - qR_1)}{e^{3/2}r}
\]

Finally, this leads to (up to a positive constant)
\[
\frac{\partial Q(e)}{\partial \theta} = \frac{A - B}{e^2 q R_1}
\]
\[
A = e^{3/2} p(e) R_2 ((1 - x(e))(z^2 - 1)^2 - rz^4)
\]
\[
B = zx(e)(z^2 - 1)(\sqrt{e} R_2 x(e) z - qR_1)
\]

Now consider the limit \( \theta \to 0 \). Let \( g = z \sqrt{e}, \ g_1(e) = z_1(e) \sqrt{e}, \) etc. In this limit, \( x(e) \to 1 \) and so the equations for \( g \) and \( e \) can be written
\[
R_2 g^2 + (1 - R_2)e = qR_1 g,
\]
\[
g^2 = \frac{\alpha e}{\alpha - re}
\]

Eliminating \( e \), we obtain an equation for \( g \):
\[
0 = g(rg^2 (R_2 g - qR_1) + \alpha (g - qR_1))
\]
\[
\alpha + rg^2
\]

We always have \( g = 0 \) as a solution. If \( g \neq 0 \), the two terms multiplying \( g \) must have opposite signs. Since \( R_2 < 1 \), this requires \( qR_1 \leq g \leq qR_1 / R_1 \).

Finally, consider the limits \( \theta \to \infty \). Assume \( e \) is finite in this limit (which must be true, economically). Then, in this limit, \( x(e) \to 0, \alpha x(e) \to e \) and so
\[
z_1(e) \to \infty \]
\[ z_2(e) \to \sqrt{1 + \frac{re}{\alpha - (r - \mathcal{R})e}} \]

and so \( z_1(e) = z_2(e) \) requires

\[ e = \frac{\alpha}{r - \mathcal{R}} \]

confirming \( e \) is finite (since \( r > \mathcal{R} \)). Since both \( z \) and \( 1/x(e) \) diverge as \( o \to \infty \), while \( e > 0 \), it follows that \( r(e) \to -\infty \).
Bibliography


