First draft: June 2013
This draft: March 2014

A Five-Factor Asset Pricing Model

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Abstract

A five-factor model directed at capturing the size, value, profitability, and investment patterns in average stock returns is rejected on the *GRS* test, but for applied purposes it provides an acceptable description of average returns. The model's main problem is its failure to capture the low average returns on small stocks that invest a lot despite low profitability. The performance of the model is not sensitive to the specifics of the way its factors are defined, at least for the definitions considered here.

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There is much evidence that average stock returns are related to the book-to-market equity ratio, *B/M*. There is also evidence that profitability and investment add to the description of average returns provided by *B/M*. The logic for why these variables are related to average returns can be explained via the dividend discount model. The model says that the market value of a share of stock is the present value of expected dividends per share,

$$(1) \qquad m_t = \sum_{\tau=1}^{\infty} \mathrm{E}(d_{t+\tau}) / (1+r)^{\tau} .$$

In this equation m_t is the share price at time t, $\mathrm{E}(d_{t+\tau})$ is the expected dividend per share in period $t+\tau$, and r is (approximately) the long-term average expected stock return or, more precisely, the internal rate of return on expected dividends.

Equation (1) says that if at time t the stocks of two firms have the same expected dividends but different prices, the stock with a lower price has a higher expected return. If pricing is rational, the future dividends of the stock with the lower price must have higher risk. The predictions drawn from (3), here and below, center on the price, m_t, however, and the predictions are the same whether the price is rational or irrational.

With a bit of manipulation, we can extract the implications of equation (1) for the relations between expected return, and expected profitability, expected investment, and *B/M*. Miller and Modigliani (1961) show that that the time t total market value of the firm's stock implied by (1) is,

$$(2) \qquad M_t = \sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^{\tau} .$$

In this equation $Y_{t+\tau}$, is total equity earnings for period $t+\tau$ and $dB_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$ is the change in total book equity. Dividing by time t book equity gives,

$$(3) \qquad \frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} \mathrm{E}(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^{\tau}}{B_t} .$$

Equation (3) makes three statements about expected stock returns. First, fix everything in (3) except the current value of the stock, M_t, and the expected stock return, r. Then a lower value of M_t, or equivalently a higher book-to-market equity ratio, B_t/M_t, implies a higher expected return. Next, fix M_t and the values of everything in equation (3) except expected future earnings and the expected stock return. The equation then tells us that higher expected future earnings imply a higher expected return. Finally, for fixed values of B_t, M_t, and expected earnings, higher expected growth in book equity – investment – implies a lower expected return.

The research challenge posed by (3) has been to identify empirical proxies for expected future earnings and expected investments. A recent paper by Novy-Marx (2012) identifies a proxy for expected profitability that is strongly related to average return. Aharoni, Grundy, and Zeng (2013) document a weaker but statistically reliable relation between investment and average return. (See also, Haugen and Baker 1996, Cohen, Gompers, and Vuolteenaho 2002, Fairfield, Whisenant, and Yohn 2003, Titman, Wei, and Xie 2004, Fama and French 2006, 2008.)

These results and the motivation provided by (3) lead us to examine an augmented version of the three-factor model of Fama and French (FF 1993) that adds profitability and investment factors to the market, size, and *B/M* factors of the FF model. This paper examines the performance of the five-factor model and different versions of its factors.

A warning is in order. The five-factor model can leave lots of the cross-section of expected stock returns unexplained. For example, the discount rate r in (3) is a constant, but the risks of net cash flows (earnings minus investment) can have a term structure that differs across firms and produces a term structure of expected returns that differs across stocks. As a result, stocks with the same values of all variables in (3) can have different expected returns one period ahead. Moreover, the measures of profitability and investment we use are simple proxies for the infinite sums of discounted expected earnings and investment. The inclusion of a size factor, which is not suggested by (3), in our five-factor model is an admission that (3) is an incomplete model of next period's expected return and that our empirical measures are imperfect.

We begin (Section I) with a brief discussion of the five-factor model. Section II examines the patterns in average returns the model is designed to explain. Sections III and IV present definitions and summary statistics for the factors. Summary statistics for the asset pricing tests are in Section V, with details in Section VI. The paper closest to ours is Hou, Xue, and Zhang (2012). We discuss their work inSection VII (Conclusions), where, with all our results in hand, contrasts with our work are easily described.

I. Empirical Asset Pricing Models

The FF three-factor model is an empirical asset pricing model. Standard asset pricing models work forward, from assumptions about investor tastes and portfolio opportunities to predictions about how risk should be measured and the relation between risk and expected return. Empirical asset pricing models work backward. They take as given the patterns in average returns, and propose models to capture them. The FF three-factor model is designed to capture the relation between average return and *Size* (market capitalization, price times shares outstanding) and the relation between average return and price ratios like the book-to-market ratio, which were the two well-known patterns in average returns at the time of our 1993 paper. The model's regression equation is,

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + e_{it}. \quad (4)$$

In this equation R_{it} is the return on security or portfolio i for period t, R_{Ft} is the riskfree return, R_{Mt} is the return on the value-weight (VW) market portfolio, SMB_t is the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks, HML_t is the difference between the returns on diversified portfolios of high and low *B/M* stocks, and e_{it} is a zero-mean residual. If the sensitivities b_i, s_i, and h_i to the portfolio returns in (4) capture all variation in expected returns, the intercept a_i is zero for all securities and portfolios i.

The valuation model summarized in equation (3) suggests that (4) may be an incomplete model for expected return because its three factors probably miss much of the relations between expected return and expected profitability and investment. Put differently, equation (3) suggests that *B/M* is a noisy proxy for expected return because the market value of the stock also reflects forecasts of profitability and investment. It thus, seems reasonable that we can better isolate the information in stock prices about expected returns by adding profitability and investment factors to the three-factor model,

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}. \tag{5}$$

In this equation RMW_t is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA_t is the difference between the returns on diversified portfolios of low and high investment stocks, which we call conservative and aggressive. If the sensitivities to the five factors, b_i, s_i, h_i, r_i, and c_i, capture all variation in expected returns, the intercept a_i is zero for all securities and portfolios i.

We suggest two ways to interpret the zero-intercept hypothesis. Leaning on Huberman and Kandel (1987), the first proposes that the mean-variance-efficient tangency portfolio, which prices all assets, combines the riskfree asset, the market portfolio, *SMB*, *HML*, *RMW*, and *CMA*. The more ambitious interpretation proposes (5) as a regression equation for a version of Merton's (1973) model in which up to four unspecified state variables lead to risk premiums that are not captured by the market factor. In this view, *Size*, *B/M*, *OP*, and *Inv* are not themselves state variables, and *SMB*, *HML*, *RMW*, and *CMA* are not state variable mimicking portfolios. Instead, in the spirit of Fama (1996), the factors are just diversified portfolios that provide different combinations of exposures to the unknown state variables. And, along with the market portfolio and the riskfree asset, the factor portfolios span the relevant multifactor efficient set. In this scenario, the role of the valuation model (3) is to suggest factors that allow us to capture the expected return effects of state variables without naming them.

II. The Playing Field

Our empirical tests examine whether the five-factor model and models that include subsets of its factors are able to explain returns on portfolios formed to produce large spreads in *Size*, *B/M*, profitability, and investment. We also look at whether performance is sensitive to the way factors are constructed.

The first step is to examine the *Size*, *B/M*, profitability, and investment patterns in average returns we seek to explain. Panel A of Table 1 shows average excess returns (returns in excess of the one-month

U.S. Treasury bill rate) for 25 value weight (VW) portfolios from independent sorts of stocks into five *Size* groups and five *B/M* groups. (We call them 5x5 *Size*-*B/M* sorts.) The *Size* and *B/M* quintile breakpoints use only NYSE stocks, but the sample is all NYSE, Amex, and NASDAQ stocks on both CRSP and Compustat with the data for *Size* and *B/M* and share codes 10 or 11. The period is July 1963 to December 2012. Fama and French (1993) use these portfolios to evaluate the three-factor model, and the patterns in average returns in Table 1 are like those in the earlier paper, with 21 years of new data.

In each *B/M* column of Panel A of Table 1, average return typically falls from small stocks to big stocks – the size effect. The first column (extreme growth stocks) is the only exception, and the glaring outlier is the low average return of the smallest (microcap) portfolio. For the other four portfolios in the lowest *B/M* column, there is no obvious relation between *Size* and average return.

The relation between average return and *B/M*, called the value effect, shows up more consistently in Table 1. In every *Size* row, average return increases with *B/M*. As is well-known, the value effect is stronger among small stocks. For example, for the microcap portfolios in the first row, average excess return rises from 0.19% per month for the lowest *B/M* portfolio (extreme growth stocks) to 1.11% per month for the highest *B/M* portfolio (extreme value stocks), a spread of 0.92%. In contrast, the average spread for the biggest stocks (megacaps) is only 0.16%.

Panel B of Table 1 shows average excess returns for 25 VW portfolios from independent sorts of stocks into *Size* and profitability quintiles. The details of these 5x5 sorts are the same as in Panel A, but the second sort is on profitability rather than *B/M*. For portfolios formed in June of year *t,* profitability (measured with accounting data for the fiscal year ending in *t*-1) is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses, all divided by book equity at the end of fiscal year *t*-1. We call this variable operating profitability, *OP*, but it is operating profitability minus interest expense. As in all our sorts, the *OP* breakpoints use only NYSE firms.

The patterns in the average returns of the 25 *Size*-*OP* portfolios in Table 1 are like those observed for the *Size*-*B/M* portfolios. Holding operating profitability roughly constant, average return typically falls as *Size* increases. The decline in average return with increasing *Size* is monotonic in the three middle quintiles of *OP*, but for the extreme low and high *OP* quintiles, the action with respect to *Size* is almost entirely due to lower average returns for megacaps.

The profitability effect identified by Novy-Marx (2012) and others is evident in Panel B of Table 1. For every *Size* quintile, extreme high operating profitability is associated with higher average return than extreme low *OP*. In each of the first four *Size* quintiles, the middle three portfolios have similar average returns, and the profitability effect is a low average return for the lowest *OP* quintile and a high average return for the highest *OP* quintile. In the largest *Size* quintile (megacaps), average return increases more smoothly from the lowest to the highest *OP* quintile.

Panel C of Table 1 shows average excess returns for 25 *Size*-*Inv* portfolios again formed in the same way as the 25 *Size*-*B/M* portfolios, but where the second variable is investment (*Inv*). For portfolios formed in June of year *t*, *Inv* is the growth of total assets for the fiscal year ending in *t*-1 divided by total assets at the end of *t*-1. In the valuation equation (3), the investment variable is the expected growth of book equity, not assets. We have replicated all tests using the growth of book equity, with results similar to those obtained with the growth of assets. The main difference is that sorts on asset growth produce slightly larger spreads in average returns. (See also Aharoni, Grundy, and Zeng 2013.) Perhaps the lagged growth of assets is a better proxy for the infinite sum of expected future growth in book equity in (3) than the lagged growth in book equity. The choice is in any case innocuous for all that follows.

In every *Size* quintile the average return on the portfolio in the lowest investment quintile is much higher than the return on the portfolio in the highest *Inv* quintile, but in the smallest four *Size* quintiles this is mostly due to low average returns on the portfolios in the highest *Inv* quintile. There is a size effect in the lowest four quintiles of *Inv*; that is, portfolios of small stocks have higher average returns than big stocks. In the highest *Inv* quintile, however, there is no size effect, and the microcap portfolio in the highest *Inv* group has the lowest average excess return in the matrix, 0.29% per month. The five-factor regressions will show that the stocks in this portfolio are like the microcaps in the lowest *B/M* quintile of Panel A of Table 1, specifically, their stock returns behave like those of firms that invest a lot despite low profitability. The low average returns of these portfolios are lethal for the five-factor model.

Equation (3) predicts that controlling for profitability and investment, *B/M* is positively related to average return, and there are similar conditional predictions for the relations between average return and profitability or investment. The valuation model does not predict that *B/M*, *OP*, and *Inv* effects show up in average returns without the appropriate controls. Moreover, Fama and French (1995) show that the three variables are correlated. High *B/M* value stocks tend to have low profitability and investment, and low *B/M* growth stocks – especially large low *B/M* stocks – tend to be profitable and invest aggressively. Because the characteristics are correlated, the *Size*-*B/M*, *Size*-*OP*, and *Size*-*Inv* portfolios in Table 1 do not isolate separate value, profitability, and investment effects in average returns.

To disentangle the characteristics, we would like to sort jointly on *Size*, *B/M*, *OP*, and *Inv*. Even 3x3x3x3 sorts, however, produce 81 poorly diversified portfolios that have low power in tests of asset pricing models. We compromise with sorts on *Size* and pairs of the other three variables. We form two *Size* groups (small and big), using the median market cap for NYSE stocks as the breakpoint, and we use NYSE quartiles to form four groups for each of the other two sort variables. Even though we have only 2x4x4 = 32 portfolios for each combination of variables, correlations between the characteristics cause an uneven allocation of stocks. For example, *B/M* and *OP* are negatively correlated, especially among big stocks, so portfolios of stocks with high *B/M* and high *OP* can be poorly diversified. In fact, when we sort stocks independently on *Size*, *B/M*, and *OP*, the portfolio of big stocks in the highest quartiles of *B/M* and *OP* is often empty before July 1974. To spread stocks more evenly, we condition the *B/M*, *OP*, and *Inv* sorts on *Size*, with separate NYSE breakpoints for small and big stocks.

Panel A of Table 2 shows average excess returns for the 32 *Size*-*B/M*-*OP* portfolios. For small and big stocks, there is a clear value effect in every profitability quartile. Holding operating profitability roughly constant, average return increases with *B/M*. Both *Size* groups also show a clear profitability effect in every *B/M* quartile. Holding *B/M* roughly constant, average return typically increases strongly with *OP*. Note the extreme low average excess return, -0.04% per month, on the portfolio of small stocks in the lowest *B/M* and *OP* quartiles. The five-factor regressions will again suggest that the stocks in this portfolio tend to share the low-profitability-high-investment combination that, at least for small stocks, is associated with low average returns left unexplained by the five-factor model.

The average excess returns for the *Size*-*B/M*-*Inv* portfolios of small stocks in Panel B of Table 2 also show a strong value effect. Average return increases with *B/M* in every *Inv* quartile. The pattern is weaker for big stocks. In every *Inv* quartile the highest *B/M* portfolio of big stocks has a higher average return than the lowest *B/M* portfolio, but the increase in average return is not always smooth or strong. For small and big stocks, the lowest *Inv* portfolio in every *B/M* quartile has a higher average return than the highest *Inv* portfolio, but for big stocks, the differences between the average returns of low and high *Inv* quartiles are modest for the lowest and highest *B/M* quartiles.

The 32 *Size*-*OP*-*Inv* portfolios in Panel C of Table 2 show rather strong profitability and investment patterns in average excess returns for small stocks. Among big stocks, the negative relation between *Inv* and average return is fairly strong in the lowest three profitability quartiles, but it is weak in the highest *OP* quartile. Among big stocks, the profitability effect is only clear in two of the four quartiles of *Inv*. Of special note is the low average excess return, -0.15% per month, for small stocks in the lowest *OP* and the highest *Inv* quartiles. In this case, we don't need five-factor slopes to infer that the small stocks in this portfolio invest a lot despite low profitability – the lethal combination noted earlier.

The portfolios in Tables 1 and 2 do not cleanly disentangle the value, profitability, and investment effects in average returns predicted by the valuation model (3), but we shall see that they expose variation in average returns sufficient to provide strong challenges in asset pricing tests.

III. Factor Definitions

We use three sets of factors to capture the patterns in average returns in Tables 1 and 2. The three approaches are described formally and in detail in Table 3. Here we provide a brief summary.

The first approach augments the three factors of Fama and French (1993) with profitability and investment factors defined like the value factor of that model. The *Size* and value factors use independent sorts of stocks into two *Size* groups and three *B/M* groups (independent 2x3 sorts). The *Size* breakpoint is the NYSE median market cap, and the *B/M* breakpoints are the 30^{th} and 70^{th} percentiles of *B/M* for NYSE stocks. The intersections of the sorts produce six VW portfolios. The *Size* factor, SMB_{BM}, is the average of the three small stock portfolio returns minus the average of the three big stock portfolio returns. The value factor *HML* is the average of the two high *B/M* portfolio returns minus the average of the two low *B/M* portfolio returns. Equivalently, it is the average of small and big value factors constructed with portfolios of only small stocks and portfolios of only big stocks.

The profitability and investment factors of the 2x3 sorts, *RMW* and *CMA*, are constructed in the same way as *HML* except the second sort is either on operating profitability (robust minus weak) or investment (conservative minus aggressive). Like *HML*, *RMW* and *CMA* can be interpreted as averages of profitability and investment factors for small and big stocks.

The 2x3 sorts used to construct *RMW* and *CMA* produce two additional *Size* factors, SMB_{OP} and SMB_{Inv}. The *Size* factor *SMB* from the three 2x3 sorts is defined as the average of $SMB_{B/M}$, SMB_{OP}, and SMB_{Inv}. Equivalently, *SMB* is the average of the returns on the nine small stock portfolios of the three 2x3 sorts minus the average of the returns on the nine big stock portfolios.

When we developed the three-factor model, we did not consider alternative definitions of *SMB* and *HML*. The choice of a 2x3 sort on *Size* and *B/M* is, however, arbitrary. To test the sensitivity of asset pricing results to this choice, we construct versions of *SMB*, *HML*, *RMW*, and *CMA* in the same way as in the 2x3 sorts, but with 2x2 sorts on *Size* and *B/M*, *OP*, and *Inv*, using NYSE medians as breakpoints for all variables (details in Table 3).

Since *HML*, *RMW*, and *CMA* from the 2x3 (or 2x2) sorts weight small and big stock portfolio returns equally, they are roughly neutral with respect to size. Since *HML* is constructed without controls for *OP* and *Inv*, however, it is not neutral with respect to operating profitability and investment. Similar comments apply to *RMW* and *CMA*. This means that with these factors, the regression slopes in the five-factor model together capture variation in returns related to *B/M*, *OP*, and *Inv*, but the separate regression slopes for *HML*, *RMW*, and *CMA* do not isolate exposures to the value, profitability, and investment effects in returns. This can make the slopes difficult to interpret. This problem is also inherent in the three-factor model of Fama and French (1993). Since *B/M*, *OP*, and *Inv* are correlated, the *HML* slope in that model is again an unknown mix of exposures to value, profitability, and investment.

The final candidate factors use four sorts to control jointly for *Size*, *B/M*, *OP*, and *Inv*. We sort stocks independently into two *Size* groups, two *B/M* groups, two *OP* groups, and two *Inv* groups using NYSE medians as breakpoints. The intersections of the groups are 16 VW portfolios. The *Size* factor *SMB* is the average of the returns on the eight small stock portfolios minus the average of the returns on the eight big stock portfolios. The value factor *HML* is the average return on the eight high *B/M* portfolios minus the average return on the eight low *B/M* portfolios. The profitability factor, *RMW*, and the investment factor, *CMA*, are also differences between average returns on eight portfolios (robust minus weak *OP* or conservative minus aggressive *Inv*). Though not detailed in Table 3, we can, as usual, also interpret the value, profitability, and investment factors as averages of small and big stock factors.

In the 2x2x2x2 sorts, *SMB* equal weights high and low *B/M*, robust and weak *OP*, and conservative and aggressive *Inv* portfolio returns. Thus, the *Size* factor is roughly neutral with respect to value, profitability, and investment, and this is what we mean when we later say that the *Size* factor controls for the other three variables. Likewise, *HML* is roughly neutral with respect to *Size*, *OP*, and *Inv*, and similar comments apply to *RMW* and *CMA*. We shall see, however, that neutrality with respect to characteristics does not imply low correlation between factor returns.

Since each of the 2x2x2x2 factors is constructed with controls for the other three, they are our best shot at isolating exposures to the different dimensions of returns. But best shot does not mean perfect. Lagged growth rates of profitability and investment are noisy proxies for the infinite sums of expected future values in the valuation model (3). Since *B/M*, expected profitability, and expected investment are surely correlated, it is likely that even with 2x2x2x2 sorts, sensitivities to the resulting factors capture unknown mixes of value, profitability, and investment effects in returns. The results to come suggest, however, that the mixing is stronger in the factors from 2x2 and 2x3 sorts than in the factors from the 2x2x2x2 sorts.

Finally, it is important to note that noisy proxies for the infinite sums of expected future values in the valuation equation (3) are not necessarily a problem in time-series tests of asset pricing models like (5). If the factor portfolios in (5) are well-diversified (multifactor minimum variance) and capture different combinations of exposures to the underlying variables that drive expected returns, they can work well in time-series tests, despite the noise in the sort variables used to construct them.

IV. Summary Statistics for Factor Returns

Table 4 shows summary statistics for the different versions of the factors. Summary statistics for returns on the portfolios used to construct the factors are in Appendix Table A1.

Average *SMB* returns are 0.29% per month for all three versions of the factors (Panel A of Table 4). The standard deviations of *SMB* are similar, 2.93% to 3.15%, and the correlations of the different versions of *SMB* are 0.98 and 1.00 (Panel B of Table 4). All this is not surprising since the *Size* breakpoint for *SMB* is always the NYSE median market cap, and the three versions of *SMB* use all stocks. The average *SMB* returns are more than 2.2 standard errors from zero.

The summary statistics for *HML*, *RMW*, and *CMA* depend more on how they are constructed. The results from the 2x3 and 2x2 sorts are easiest to compare. The standard deviations of the three factors are lower when only two *B/M*, *OP*, or *Inv* groups are used, due to better diversification. In the 2x2 sorts, *HML*, *RMW*, and *CMA* include all stocks, but in the 2x3 sorts, the stocks in the middle 40% of *B/M*, *OP*, and *Inv* are dropped. The 2x3 sorts focus more on the extremes of the three variables, and so produce larger average *HML*, *RMW*, and *CMA* returns. For example, the average *HML* return is 0.38% per month in the 2x3 *Size*-*B/M* sorts, versus 0.29% in the 2x2 sorts. Similar differences are observed in average *RMW* and *CMA* returns. The *t*-statistics (and thus the Sharpe ratios) for average *HML*, *RMW,* and *CMA* returns are, however, similar for the 2x3 and 2x2 sorts. The correlations between the factors of the two sorts (Panel B of Table 4) are also high, 0.97 (*HML*), 0.96 (*RMW*), and 0.95 (*CMA*).

Each factor from the 2x2 and 2x3 sorts controls for *Size* and one other variable. The factors from the 2x2x2x2 sorts control for all four variables and so produce cleaner evidence on the value, profitability, and investment premiums in expected returns. Joint controls have little effect on *HML*. The correlations of the 2x2x2x2 version of *HML* with the 2x2 and 2x3 versions are high, 0.94 and 0.96. The 2x2 and 2x2x2x2 versions of *HML*, which split stocks on the NYSE median *B/M*, have almost identical means and standard deviations, and both means are more than 3.2 standard errors from zero (Panel A of Table 4).

The correlations of *RMW* and *CMA* from the 2x2x2x2 sorts with the corresponding 2x3 and 2x2 factors are lower, 0.80 to 0.87, and joint controls produce an interesting result – a boost to the profitability premium at the expense of the investment premium. The 2x2x2x2 and 2x2 versions of *RMW* have the same standard deviation, 1.53% per month, but the 2x2x2x2 *RMW* has a larger mean, 0.26% (t = 4.10) versus 0.17% (t = 2.77). The standard deviation of *CMA* drops from 1.49 for the 2x2 version to 1.18 with four-variable controls, and the mean falls from 0.22 (t = 3.65) to 0.15% (t = 3.08). Thus, with joint controls, there is reliable evidence of an investment premium in expected returns, but the average value is about half the size of the other 2x2x2x2 factor premiums.

The value, profitability, and investment factors are averages of small and big stock factors. Here again, joint controls produce interesting changes in the premiums for small and big stocks (Panel A of Table 4). The factors from the 2x3 and 2x2 sorts confirm earlier evidence that the value premium is larger for small stocks (e.g., Fama and French 1993, 2012). For example, in the 2x3 *Size-B/M* sorts the average HML_S return is 0.55% per month (t = 4.10), versus 0.21% (t = 1.67) for HML_B. The evidence of a value premium in big stock returns is stronger if we control for profitability and investment. The average value of HML_B in the 2x2 and 2x3 sorts is less than 1.7 standard errors from zero, but more than 2.3 standard errors from zero in the 2x2x2x2 sorts. Controls for profitability and investment also reduce the spread between the value premiums for small and big stocks. The average difference between HML_S and HML_B falls from 0.25 (t = 3.11) in the 2x2 sorts to 0.15 (t = 1.84) in the 2x2x2x2 sorts.

For all methods of factor construction, there seem to be expected profitability and investment premiums for small stocks; the average values of RMW_S and CMA_S are at least 2.65 standard errors from zero. The average profitability premium is larger for small stocks than for big stocks, but the evidence that the expected premium is larger is weak. For the three definitions of *RMW*, the average difference between RMW_S and RMW_B is less than 1.3 standard errors from zero. The average value of RMW_B is 1.94 standard errors from zero in the 2x3 and 2x2 sorts, but with the boost to the premium provided by joint controls, the *t*-statistic rises to 3.47 in the 2x2x2x2 sorts, and the average difference between RMW_S *and* RMW_B is only 0.84 standard errors from zero.

In contrast, there is strong evidence that the expected investment premium is larger for small stocks. The average value of CMA_S is 4.61 to 5.43 standard errors from zero, but the average value of CMA_B is only 0.98 to 2.00 standard errors from zero, and it is more than 2.2 standard errors below the average value of CMA_S. In the 2x2x2x2 sorts that jointly control for *Size*, *B/M*, *OP*, and *Inv*, the average value of CMA_B is 0.06% per month (t = 0.98), and almost all the average value of *CMA* is from small stocks.

Panel C of Table 4 shows the correlation matrix for each set of factors. With 594 monthly observations, the standard error of the correlations is only 0.04, and most of the estimates are more than three standard errors from zero. The value, profitability, and investment factors are negatively correlated with both the market and the size factor. Since small stocks tend to have higher market betas than big stocks, it makes sense that *SMB* is positively correlated with the excess market return. Given the positive correlation between profitability and investment, it is perhaps surprising that the correlation between the profitability and investment factors is low, -0.19 to 0.15.

The correlations of the value factor with the profitability and investment factors merit comment. When *HML* and *CMA* are from separate 2x2 or 2x3 sorts, the correlation between the factors is about 0.70. This is perhaps not surprising given that high *B/M* value firms tend to be low investment firms. In the 2x2x2x2 sorts the correlation falls about in half, to 0.37, which also is not surprising since the factors from these sorts attempt to neutralize the effects of other factors.

The correlations between *HML* and *RMW* are surprising. When the two factors are from separate *Size*-*B/M* and *Size*-*OP* sorts, the correlation is close to zero, -0.04 for the 2x2 sorts and 0.08 for the 2x3 sorts. When the sorts jointly control for *Size*, *B/M*, *OP*, and *Inv*, the correlation between *HML* and *RMW* jumps to 0.63. There is a simple explanation. Among the 16 portfolios used to construct the 2x2x2x2 factors, the two with by far the highest return variances (small stocks with low *B/M*, weak *OP*, and low or high *Inv*) are held short in *HML* and *RMW*. Similarly, the portfolio of big stocks with the highest return variance is held long in the two factors, and the big stock portfolio with the second highest return variance is in the short end of both factors. The high correlation between *HML* and *RMW* is thus somewhat artificial, and it is a troubling feature of the factors constructed with joint controls.

V. Model Performance Summary

We turn now to our primary task, testing how well the three sets of factors explain average excess returns on the portfolios of Tables 1 and 2. We consider seven asset pricing models: (i) three three-factor models that combine $R_M - R_F$ and *SMB* with *HML*, *RMW*, or *CMA*; (ii) three four-factor models that combine $R_M - R_F$, *SMB*, and pairs of *HML*, *RMW*, and *CMA*; and (iv) the five-factor model.

With seven models, six sets of left hand side (LHS) portfolios, and three sets of right hand side (RHS) factors, it makes sense to restrict attention to models that fare relatively well in the tests. To judge the improvements provided by the profitability and investment factors, we always show results for the original three-factor model of Fama and French (1993), the five-factor model, and the three four-factor models. But we show results for alternative three-factor models only for the 5x5 sorts on *Size* and *OP* or *Inv* and only for the model in which the third factor – *RMW* or *CMA* – is aimed at the second LHS sort variable.

If an asset pricing model completely captures expected returns, the intercept is indistinguishable from zero in a regression of an asset's excess returns on the model's factor returns. Table 5 shows the *GRS* statistic of Gibbons, Ross, and Shanken (1989), which tests this hypothesis for combinations of LHS portfolios and factors. The *GRS* test easily rejects all models considered for all LHS portfolios and RHS factors. To save space, the probability, or *p*-value, of getting a *GRS* statistic larger than the one observed if the true intercepts are all zero, is not shown. We can report, however, that for four of the six sets of LHS returns, the *p*-values for all models round to zero to at least three decimals. The models fare best in the tests on the 25 *Size-OP* portfolios, but the *p*-values are still less than 0.04. In short, the GRS test says that all our models are incomplete stories for expected returns.

The *GRS* test compares the Sharpe ratios for the portfolio of the RHS portfolios that has the highest Sharpe ratio and the portfolio of the LHS and RHS portfolios that has the highest Sharpe ratio. The hypothesis that the RHS portfolios alone capture all variation in expected returns is rejected if adding the LHS assets produces a statistically reliable increase in the maximum Sharpe ratio. In solving for the maximum Sharpe ratios, no constraints on shortselling are imposed, and the weights on individual LHS and RHS portfolios are often wildly positive and negative (see Fama and French 2013). This is appropriate for tests of asset pricing models because we want to ferret out model problems in an unconstrained way. For applications, however, rejection on the *GRS* test may be irrelevant if due to small deviations of average returns from model predictions.

More important, asset pricing models are simplified propositions about expected returns that are rejected in tests with power. We are less interested in whether competing models are rejected than in their relative performance, which we judge using *GRS* and other statistics. We want to identify the model that is the best (but imperfect) story for average returns on portfolios formed in different ways.

We are interested in the improvements in the description of average returns provided by adding the profitability and investment factors to the original three-factor model. For All six sets of LHS portfolios, the five-factor model produces lower *GRS* statistics than the original three-factor model. The improvements are difficult to judge, however, since the *p*-values for the five-factor model also point to rejection, and one can quarrel with comparisons of GRS statistics for different sets of RHS variables. The average absolute intercepts, also shown in Table 5, are more amenable to comparison. For the 25 *Size-B/M* portfolios the five-factor model produces minor improvements, less than a basis point, in the average absolute intercept. The improvements are larger for the 25 *Size-OP* portfolios (2.0 to 4.1 basis points), the 25 *Size-Inv* portfolios (1.7 to 2.3 basis points), the 32 *Size-B/M-OP* portfolios (1.6 to 2.3 basis points), and the 32 *Size-B/M-Inv* portfolios (3.7 to 4.5 basis points).

Relative to the original three-factor model, the biggest improvements in the average absolute intercept (6.8 to 7.9 basis points per month) are produced by the five-factor model when applied to the 32 *Size-OP-Inv* portfolios. This is perhaps not surprising since these portfolios are formed on two variables (profitability and investment) not directly covered by the three-factor model (versus one such variable for the other LHS portfolios). These results suggest that the original three-factor model is likely to fare poorly when applied to portfolios that involve strong tilts toward combinations of profitability and investment.

Table 5 also shows two ratios that estimate the proportion of the cross-section of expected returns left unexplained by competing models. The numerator of each is a measure of the dispersion of the intercepts produced by a given model for a set of LHS portfolios; the denominator measures the dispersion of the LHS expected returns. Define $\bar{R}_i$ as the time-series average excess return on portfolio i, define $\bar{R}$ as the cross-section average of the average returns, and define $\bar{r}_i$ as portfolio i's deviation from the cross-section average, $\bar{r}_i = \bar{R}_i - \bar{R}$. The first estimate is $A|a_i|/A|\bar{r}_i|$, the average absolute value of the intercept a_i over the average absolute value of the deviation $\bar{r}_i$.

The results for $A|a_i|/A|\bar{r}_i|$ in Table 5 tell us that for different combinations of LHS portfolios and factor definitions, the five-factor model's average absolute intercept $A|a_i|$ ranges from 42% to 54% of $A|\bar{r}_i|$. Thus, measured in units of return, the five-factor model leaves 42% to 54% of the dispersion of average excess returns unexplained. The dispersion of average excess returns left unexplained by the three-factor model is higher, from 54% to 68%. Though not shown in Table 5, we can also report that when the CAPM is estimated on the six sets of LHS portfolios, $A|a_i|/A|\bar{r}_i|$ ranges from 1.25 to 1.56. Thus, CAPM intercepts are more disperse than LHS average returns, and this result persists no matter how we measure dispersion.

Measurement error inflates both the average absolute intercept $A|a_i|$ and the average absolute deviation $A|\bar{r}_i|$. The estimated intercept, a_i, is the true intercept, α_i, plus estimation error, $a_i = \alpha_i + e_i$.

Similarly, $\bar{r}_i$ is μ_i, portfolio *i*'s expected deviation from the grand mean, plus estimation error, $\bar{r}_i = \mu_i + \varepsilon_i$. We can adjust for measurement error if we focus on squared intercepts and squared deviations.

The cross-section average of the μ_i's is zero, so the average μ_i^2 is the cross-section variance of the portfolios' expected returns. Thus, $A(\alpha_i^2)/A(\mu_i^2)$ is the proportion of the cross-section variance of expected returns left unexplained by a model. Since α_i is a constant, the expected value of the square of an estimated intercept is the squared value of the true intercept plus the sampling variance of the estimate, $E(a_i^2) = \alpha_i^2 + E(e_i^2)$. Our estimate, $\hat{\alpha}_i^2$ of the square of the true intercept α_i^2, is the difference between the squared estimates of the regression intercept and its standard error. Similarly, our estimate of μ_i^2, $\hat{\mu}_i^2$, is the difference between the square of the realized deviation, $\bar{r}_i^2$, and the square of its standard error. The ratio of averages, $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$, is then a measure of the proportion of the variation in LHS expected returns left unexplained.

In part because it is in units of return squared and in part because of the corrections for sampling error, $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$ provides a more positive picture of the five-factor model than the ratio of average absolute values, $A|a_i|/A|\bar{r}_i|$. In the 5x5 sorts, the *Size-Inv* portfolios present the biggest challenge, but the estimates suggest that the five-factor model nevertheless leaves only around 30% of the cross-section variance of expected returns unexplained. The estimate drops to less than 25% for the 25 *Size-B/M* portfolios and to roughly 10% for the 25 *Size-OP* portfolios. These are all far less than the variance ratios produced by the original three-factor model, which are mostly greater than 50% for the *Size-Inv* and *Size-OP* portfolios and about 40% for the *Size-B/M* portfolios. For the 25 *Size-OP* portfolios, however, the five-factor model is not systematically better on any metric than the three-factor model that substitutes *RMW* for *HML*.

The estimates of variance left unexplained by the five-factor model are lower for the LHS portfolios from the 2x4x4 sorts. The $A(\hat{\alpha}_i^2)/A(\hat{\mu}_i^2)$ ratios for the 32 *Size-OP-Inv* portfolios suggest that only about 20% of the cross-section variance of expected returns is left unexplained, versus 60% to 68% for the original three-factor model. The five-factor estimates drop to 14% to 19% for the 32 *Size-B/M-Inv* portfolios and 10% to 17% for the *Size-B/M-OP* portfolios, and most are less than half the estimates produced by the three-factor model.

Two general results show up in the tests for each of the six sets of LHS portfolios. First, the factors from the 2x3, 2x2, and 2x2x2x2 sorts produce much the same results in the tests of a given model. Second, and more interesting, the five-factor model outperforms the original three-factor model on all metrics and it generally outperforms other models, with one major exception. Specifically, the five-factor model and the four-factor model that excludes *HML* are similar on all measures of performance, including the *GRS* statistic. We explore this result later.

We close this section with two final comments related to Table 5. First, the *GRS* test rejects every model as a complete description of expected returns, but some sets of LHS portfolios pose bigger challenges than others. For example, in the tests of the five-factor model on the 25 *Size-B/M* portfolios, the average absolute intercepts are between 0.090% and 0.097%. In the tests of the five-factor model on the 25 *Size-OP* portfolios, the average absolute intercepts are lower, 0.067% to 0.074%. Later we examine the common characteristics of the portfolios that pose the biggest challenges to our asset pricing models.

Second, to save space we do not show average values of R^2 in Table 5, but we can report that on average our models absorb a smaller fraction of return variance for the LHS portfolios from the 2x4x4 sorts than for the portfolios from the 5x5 sorts. For example, the average R^2 in the five-factor regressions is 0.91 to 0.93 for the 5x5 sorts, versus 0.85 to 0.89 for the 2x4x4 sorts. Average R^2 is lower with three sort variables because correlation between variables limits the diversification of some LHS portfolios. For example, the negative correlation between *OP* and *B/M* means there are often few big stocks in the top quartiles of *OP* and *B/M* (highly profitable extreme value stocks). Lower average R^2 reduces the power of the *GRS* test, but the *p*-values in the tests on the 2x4x4 portfolios are 0.01 or less. Despite strong rejections on the *GRS* test, however, small average absolute intercepts and rather low estimates of the proportion of the cross-section variance of expected returns left unexplained suggest that the five-factor model and the four-factor model that drops *HML* do well in the tests on the portfolios from the 2x4x4 sorts.

VI. *HML*: A Redundant Factor

We note above that the five-factor model never improves the description of average returns from the four-factor model that drops *HML*. The explanation is interesting. The average *HML* return is captured by the exposures of *HML* to other factors. Thus, in the five-factor model, *HML* seems to be redundant for explaining average returns.

The evidence is in Table 6, which shows regressions of each of the five factors on the other four. In the $R_M - R_F$ regressions, the intercepts (average returns unexplained by exposures to *SMB*, *HML*, *RMW*, and *CMA*) are around 0.75% per month, with *t*-statistics greater than 4.5. In the regressions to explain *SMB*, *RMW*, and *CMA*, the intercepts are more than three standard errors from zero. In the *HML* regressions, however, the intercepts are -0.03% (t = -0.40) for the 2x3 factors, 0.00% (t = 0.07) for the 2x2 factors, and 0.02% (t = 0.24) for the 2x2x2x2 factors.

In the spirit of Huberman and Kandel (1987), the evidence suggests that including *HML* does not improve the mean-variance-efficient tangency portfolio produced by combining the riskfree asset, the market portfolio, *SMB*, *RMW*, and *CMA*. Or, from the perspective of Merton (1973), perhaps the four factors are related to three rather than four unknown state variables that are the source of special risk premiums.

The four-factor model that drops *HML* seems to be as good a description of average returns as the five-factor model, but the five-factor model may be a better choice in applications. Though captured by exposures to other factors, there is a large value premium in average returns that is often targeted by money managers. Thus, in evaluating how investment performance relates to known premiums, we probably want to know the tilts of LHS portfolios toward the *Size*, *B/M*, *OP*, and *Inv* factors. And for explaining average returns, nothing is lost in using a redundant factor.

Finally, the slopes in the Table 6 regressions often seem counterintuitive. For example, in the *HML* regressions, the large average *HML* return is mostly absorbed by the slopes for *RMW* and *CMA*. The *CMA* slopes are strongly positive, which is in line with the fact that high *B/M* value firms tend to do little investment. But the *RMW* slopes are also strongly positive, which says that controlling for investment, value stocks behave like stocks with robust profitability, even though unconditionally value stocks tend to be less profitable.

VII. Regression Details

For more perspective on model performance we examine details of the regression results, specifically, intercepts and pertinent slopes. To keep the presentation manageable, we show only a limited set of results for factors from the 2x3 sorts, the original approach to factor formation of Fama and French (1993), and we do not show results for the 2x2 factors since they are similar to those for the 2x3 factors. We always show regression slopes for factors from the 2x2x2x2 *Size*-*B/M*-*OP*-*Inv* sorts since we shall see that they seem easier to interpret than slopes for factors from the 2x3 sorts. Finally, results for the 32 LHS portfolios formed on *Size*, *B/M*, and either *OP* or *Inv* are relegated to the Appendix since they just reinforce the results produced by other LHS portfolios.

The discussion of regression details is long, and a summary is helpful. Despite rejection on the *GRS* test, the five-factor model performs well: unexplained average returns for individual portfolios are almost all close to zero. The major exception, by far, is a portfolio that shows up in most sorts. The stocks in the offending portfolio are small and they have strong negative exposure to *RMW* and *CMA*, suggestive of firms that invest a lot despite low profitability. In each sort that produces such a portfolio, its five-factor intercept is so negative that, using Bonferroni's inequality, we can easily reject the model for the entire set of 25 or 32 LHS portfolios. Adding to the puzzle, big stocks that invest a lot despite low profitability pose no problem for the five-factor model.

A. 25 *Size-B/M* Portfolios

In the Table 5 tests on the 25 *Size-B/M* portfolios, adding profitability and investment factors to the original three-factor model that includes R_M-R_F, *SMB*, and *HML* improves the *GRS* statistic and other measures of performance. Examining the sources of the gains provides insights into well-known problems of the three-factor model.

Panel A of Table 7 reports intercepts from the regressions for the 25 *Size-B/M* portfolios using factors from the 2x3 sorts. As in Fama and French (1993, 2012), extreme growth stocks (left column of the intercept matrix) are a problem for the three-factor model. The portfolios of small extreme growth stocks produce negative three-factor intercepts and the portfolios of large extreme growth stocks produce positive intercepts. Microcap extreme growth stocks (upper left corner of the intercept matrix) are a huge problem. By itself, the three-factor intercept for this portfolio, -0.50% per month (t = -5.21), is sufficient to reject the three-factor model as a description of expected returns on the 25 *Size-B/M* portfolios.

Adding the profitability and investment factors, *RMW* and *CMA,* reduces these problems. The intercept for the microcap extreme growth portfolio rises from -0.50 (t = 5.21) in the three-factor model to -0.30 (t = -3.37) in the five-factor model, and the intercepts for three of the other four extreme growth portfolios shrink toward zero. But the pattern in the extreme growth intercepts – negative for small stocks and positive for large – survives. Skipping the details, we see the same behavior in the three- and five-factor intercepts when we use the 2x2x2x2 factors.

Panel B of Table 7 shows the five-factor slopes for *HML*, *RMW*¸ and *CMA* when we use the 2x3 and 2x2x2x2 versions of the factors to explain returns on the 25 *Size-B/M* portfolios. The market and *SMB* slopes are not shown. The market slopes are always close to 1.0, and the *SMB* slopes are always strongly positive for small stocks and slightly negative for big stocks. The market and *SMB* slopes are similar for different models, so they cannot account for changes in the intercepts observed when factors are added. To save space, here and later, we concentrate on *HML*, *RMW*, and *CMA* slopes, with special emphasis on *RMW* and *CMA* slopes, which are important for interpreting the intercepts.

Some of the slopes for the 2x3 factors in Table 7 line up with our expectations, but others do not. The *HML* slopes have a familiar pattern – strongly negative slopes for low *B/M* growth stocks and strongly positive slopes for high *B/M* value stocks. In general, however, the *RMW* and *CMA* slopes for factors from the 2x3 sorts do not confirm the evidence in Fama and French (1995) that high *B/M* value stocks tend to be less profitable and grow more slowly than low *B/M* growth stocks. There is only one negative *RMW* slope – suggesting weak profitability – among the ten portfolios in the two highest *B/M* quintiles. The *CMA* slopes are slightly negative – suggesting high investment – for the portfolios in the lowest *B/M* quintile (extreme growth stocks), but the slopes are also negative for three of five portfolios in the highest *B/M* quintile (extreme value stocks).

We suggest an explanation for these results. With 2x3 sorts, value, profitability, and investment effects are smeared in *HML*, *RMW*, and *CMA* because, for example, the sorts on *Size* and *B/M* that produce *HML* do not control for profitability and investment. The factors from the 2x2x2x2 sorts jointly control for *Size*, *B/M*, *OP*, and *Inv*, and the slopes for these factors, also in Panel B of Table 7, are more consistent with our priors. The *CMA* slopes increase from strongly negative for extreme growth portfolios to strongly positive for extreme value portfolios. As expected given the weak profitability of value stocks, the *RMW* slopes produced by the 2x2x2x2 factors are negative for the ten portfolios in the two highest *B/M* quintiles. The *RMW* slopes are also negative for all microcap portfolios and for the portfolio of extreme growth stocks in the second *Size* quintile, but this is consistent with the evidence in Fama and French (1995) that among small stocks there is a large dose of low profitability firms.

The advantage of the factors from the 2x2x2x2 sorts is also apparent in the tests for other sets of LHS portfolios. The 2x2x2x2 factors always produce exposures to *HML*, *RMW*, and *CMA* that conform better to the characteristics of stocks in the portfolios, so henceforth, we show intercepts and slopes for only these factors. The choice is inconsequential for the intercepts since, especially for the five-factor model, they are quite similar for different definitions of the factors.

As in Fama and French (1993), the portfolio of microcap stocks in the lowest *B/M* quintile is the big embarrassment of the three-factor model. The five-factor slopes from the 2x2x2x2 sorts provide new information about stocks in this portfolio. The portfolio's *HML* slope is not extreme, -0.29 (*t* = -5.11) versus -0.52 or less for other portfolios in the lowest *B/M* quintile. But the portfolio has the most extreme *RMW* and *CMA* slopes, -0.68 (*t* = -8.62) and -0.49 (*t* = -5.53). The *RMW* and *CMA* slopes suggest that the portfolio is dominated by microcaps whose returns behave like those of unprofitable firms that grow rapidly. The hits to the estimate of expected returns implied by the negative five-factor *RMW* and *CMA* slopes for this portfolio absorb 40% of the intercept produced by the three-factor model (-0.50, t = -5.21), but the five-factor model leaves a large unexplained average return (-0.30, t = -3.37). There is a similar negative intercept in the results to come whenever the LHS assets include a portfolio of small stocks with strong negative exposures to *RMW* and *CMA*.

B. 25 *Size-OP* Portfolios

The *GRS* test and other statistics in Table 5 say that the five-factor model and the three-factor model that includes *RMW* provide similar descriptions of average returns on the 25 portfolios formed on *Size* and profitability. The five-factor intercepts for the portfolios (Panel A of Table 8) show no patterns and are mostly close to zero. This is in line with the evidence in Table 5 that the average absolute intercepts are smaller for the *Size-OP* portfolios than for other LHS portfolios. The highest profitability microcap portfolio (upper right corner of the intercept matrix) produces the most extreme five-factor intercept, -0.19 (*t* = -2.44), but it is modest relative to the most extreme intercept in other sorts.

The five-factor *HML* slopes for the 25 *Size-OP* portfolios (Panel B of Table 8) show a clear pattern for megacaps – strongly positive for the least profitable and strongly negative for the most profitable. Thus, among megacaps low profitability is associated with value and high profitability is associated with growth. The negative correlation between *HML* slopes and profitability is weaker among smaller firms. For microcaps, the *HML* slopes for the lowest four *OP* quintiles cluster between 0.30 and 0.39, and even the highest profitability portfolio has a slight tilt toward value.

As expected, the *RMW* slopes for the 25 portfolios formed on *Size* and profitability increase from strongly negative for low profitability portfolios to strongly positive for high profitability portfolios. In contrast, the *CMA* slopes are generally close to zero. Thus, portfolios formed on profitability show little exposure to the investment factor.

The microcap portfolio in the lowest profitability quintile is not a problem for the five-factor model in the *Size*-*OP* sorts. Its five-factor intercept is -0.11% per month (t = -1.29). This portfolio shows strong negative exposure to *RMW* (-1.12, t = -15.42) but modest negative exposure to *CMA* (-0.14, t = -1.74). This is in contrast to the *Size*-*B/M* sorts, in which the big problem is microcaps with extreme negative exposures to *RMW* and *CMA*. In short, the portfolios formed on *Size* and *OP* are less of a challenge for the five-factor model than portfolios formed on *Size* and *B/M* in large part because the *Size*-*OP* portfolios do not isolate microcaps with the strong negative exposures to *RMW* and *CMA* that are typical of firms that invest a lot despite low profitability.

The *Size*-*OP* portfolios are, however, a problem for the three-factor model of Fama and French (1993). Panel A of Table 8 shows that the model produces negative intercepts that are far from zero for the three small stock portfolios in the lowest *OP* quintile. The estimate for the low *OP* microcap portfolio, for example, is -0.32% per month (t = -3.25). Four of the five portfolios in the highest *OP* quintile produce positive three-factor intercepts, and the intercept for the megacap portfolio is the most extreme, 0.21% per month (t = 3.68). The results suggest that the three-factor model will have serious problems in applications when portfolios have strong tilts toward high or low profitability.

C. 25 *Size-Inv* Portfolios

Table 5 says that the five-factor model improves the description of average returns on the 25 *Size*-*Inv* portfolios provided by the original three-factor model. Table 9, which shows the three-factor and five-factor intercepts and the five-factor *HML*, *RMW*, and *CMA* slopes, gives the details.

The *CMA* slopes for the *Size*-*Inv* portfolios show the expected pattern – positive for low investment portfolios and negative for high investment portfolios. There is also a pattern in the *HML* slopes – positive for low investment portfolios and, except for microcaps, negative for high investment portfolios. Thus, low investment tends to be associated with value and high investment is associated with growth.

The story in the *RMW* slopes is more complicated. As expected, the portfolios of stocks in the lowest quintile of *Inv* show negative exposure to *RMW* that is stronger for small stocks. In other words, low investment tends to be associated with low profitability. But except for megacaps, extreme high investment also tends to be associated with negative exposure to *RMW*. Thus, megacaps aside, extreme high investment apparently does not imply high profitability.

The big problems of the three-factor model in the tests on the 25 *Size*-*Inv* portfolios are the strong negative intercepts for the portfolios in the three smallest *Size* quintiles and the highest *Inv* quintile. Switching to the five-factor model moves these intercepts toward zero. The improvements trace to negative slopes for the investment and profitability factors, which lower five-factor estimates of expected returns. For example, the microcap portfolio in the highest *Inv* quintile produces the most extreme three-factor intercept, -0.55% (t = -7.18), but the portfolio's strong negative *RMW* and *Inv* slopes (-0.36, *t* = -5.78, and -0.52, *t* = -7.48) lead to a less extreme five-factor intercept, -0.40%. (t = -5.48). This portfolio's intercept is nevertheless sufficient (on Bonferroni's inequality) for a strong rejection of the five-factor model as a description of expected returns on the 25 *Size*-*Inv* portfolios.

The problem for the five-factor model posed by the microcap portfolio in the highest *Inv* quintile is much the same as the problem posed by the microcap portfolio in the lowest *B/M* quintile in Table 7. Both show strong negative exposures to *RMW* and *CMA*, like those of firms that invest a lot despite low profitability, but their *RMW* and *CMA* slopes do not come close to explaining the low average returns of the portfolios (Table 1).

D. *Size-OP-Inv* Portfolios

Table 10 shows three-factor and five-factor regression intercepts and five-factor *RMW* and *CMA* slopes for the 32 portfolios from the 2x4x4 sorts on *Size*, *OP*, and *Inv*. (To save space the five-factor HML slopes are not shown.) These sorts are interesting because the profitability and investment characteristics of stocks in the portfolios are known, whereas in other sorts, one or both characteristics are suggested by *RMW* and *CMA* slopes. The *RMW* and *CMA* slopes in Table 10 line up as expected. For small and big stocks, the *RMW* slopes are positive for high profitability quartiles and negative for low *OP* quartiles. The *CMA* slopes are positive for low investment quartiles and negative for high *Inv* quartiles.

The *Size*-*OP*-*Inv* sorts provide clear information about the failures of the five-factor model. By far the biggest problem in Table 10 is the portfolio of small stocks in the lowest profitability and highest investment quartiles. Its intercept, -0.54% per month (t = -5.88) easily rejects the model as a description of expected returns on the 32 *Size*-*OP*-*Inv* portfolios. Low profitability per se is not a problem for the five-factor model in the results for small stocks; two of the other three portfolios in the lowest *OP* quartile produce positive intercepts and one is 2.37 standard errors from zero. There is suggestive evidence that for small stocks high investment alone is associated with five-factor problems; the other three small stock portfolios in the highest *Inv* quartile produce negative five-factor intercepts and two are more than two standard errors below zero.

If one looks to big stocks for confirmation of the five-factor problems observed for small stocks, none is found. The intercepts for the four big stock portfolios in the highest investment quartile split evenly between positive and negative, and the one that is more than two standard errors from zero is positive. Most important, the portfolio of big stocks in the lowest *OP* and highest *Inv* quartiles (the lethal combination for small stocks) produces a minor five-factor intercept, 0.06% per month (t = 0.67). Thus, if the market overprices small stocks that invest a lot despite low profitability, the problem does not carry over to big stocks.

The three-factor model of Fama and French (1993) faces more general problems in the tests on the 32 *Size*-*OP*-*Inv* portfolios. Because they are not helped by the negative *RMW* and *CMA* slopes of the five-factor model, the three-factor intercepts for small stock portfolios that combine low profitability and high investment are even more extreme than the five-factor intercepts. For example, the intercept for the portfolio of small stocks in the lowest *OP* and highest *Inv* quartiles is -0.89% per month (t = -8.04) in the three-factor model versus -0.54% (t = -5.88) in the five-factor model. Portfolios of big stocks that combine low *OP* and high *Inv* and portfolios of small or big stocks that combine high *OP* and low *Inv* are not problems for the five-factor model, but they are problems for the three-factor model. In short, the *Size*-*OP*-*Inv* sorts provide the most direct evidence that strong profitability and investment tilts are general problems for the three-factor model.

VIII. Conclusions

There are patterns in average returns related to *Size*, *B/M*, profitability, and investment. The *GRS* test easily rejects a five-factor model directed at capturing these patterns, but we estimate that the model explains between 69% and 93% of the cross-section variation in expected returns for the *Size*, *B/M*, *OP*, and *Inv* portfolios we examine.

Judged on regression intercepts, the three sets of factors we use – (i) separate 2x3 sorts on *Size* and *B/M*, *OP*, or *Inv*, (ii) separate 2x2 sorts, and (iii) 2x2x2x2 sorts that jointly control for *Size*, *B/M*, *OP*, and *Inv* – provide similar descriptions of average returns on the LHS portfolios examined.

Armed with the evidence presented here, which version of the factors would we choose if starting fresh? We might prefer the factors from the 2x2 *Size*-*B/M*, *Size*-*OP*, and *Size*-*Inv* sorts over those from the 2x3 sorts (the original approach). Since the 2x2 versions of *HML*, *RMW*, and *CMA* use all stocks and the 2x3 versions exclude 40%, the 2x2 factors are better diversified. In the tests of the five-factor model, however, the performance of the two sets of factors is similar for the LHS portfolios we examine, so the choice between them seems inconsequential.

The *HML*, *RMW*, and *CMA* slopes produced by the factors from the 2x2x2x2 sorts, which jointly control for *Size*, *B/M*, *OP*, and *Inv*, seem to better identify value, profitability, and investment exposures. This is, for example, an advantage for performance attribution in studies of portfolio performance. Closer to home, it helps us identify the characteristics of the small stocks in the portfolios that produce glaring contradictions of the five-factor model, specifically, strong negative exposures to *RMW* and *CMA*, typical of firms that invest a lot despite low profitability. Unfortunately, four variables may be the most we can control at the same time. If we add momentum, for example, correlations among the variables are likely to result in poor diversification of some of the portfolios used to construct factors.

If parsimony is an issue, however, our results suggest that *HML* is a redundant factor in the sense that its high average return is fully captured by its exposures to R_M - R_F, *SMB*, and especially *RMW* and *CMA*. Thus, in applications where the sole interest is abnormal returns (regression intercepts), our tests suggest that a four-factor model that drops *HML* performs as well as (no better and no worse than) the five-factor model. But if one is also interested in measuring portfolio tilts toward value, profitability, and investment, the five-factor model is the choice.

As noted in the introduction, the paper closest to ours is Hou, Xue, and Zhang (2012). They examine a four-factor model that, in addition to $R_M - R_F$, includes factors that are much like our *SMB, RMW,* and *CMA* and are constructed from 2x3x3 sorts that jointly control for *Size*, profitability, and investment. They do not comment on why *HML* is not in the model, and they only compare the performance of their four-factor model to that of the CAPM, the three-factor model of Fama and French (1993), and Carhart's (1997) four-factor model, which adds a momentum factor. Their investigation of models is more restricted than ours, and they do not consider alternative definitions of the factors. More important, they are primarily concerned with explaining the returns associated with anomaly variables not used to construct their factors, and they focus on VW portfolios from univariate sorts on each variable. Value weight portfolios from univariate sorts on variables other than *Size* are typically dominated by big stocks, however, and one of the main messages here and in Fama and French (1993, 2012) is that the problems of asset pricing models are largely concentrated in small stocks.

Finally, there is an issue with respect to the market factor that is untouched here. Empirical tests of the CAPM, from Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) to Fama and French (1992) and Frazzini and Pedersen (2013), find that the premium for market beta is much smaller than predicted by the version of the CAPM in which there is unrestricted riskfree borrowing and lending. Davis, Fama, and French (2000) document a similar result for the multivariate beta in the Fama-French three-factor model. These results suggest that the predictions of models (like those examined here) that include a standard market factor, $R_{Mt} - R_{Ft}$, are too high for assets with market betas greater than 1.0 and too low for assets with betas less than 1.0.

Fortunately, although the market betas for the LHS assets examined here show some dispersion, they are never far from 1.0. In applications in which betas differ a lot from 1.0, however, misspecification of the market premium may be an important issue. We have examined a simple approximate cure. Specifically, assume all market betas are 1.0, drop the market premium from the RHS of the asset pricing regression, and use asset returns measured net of the market return as LHS variables. Appendix Table A4 compares the *GRS* statistics and average absolute intercepts obtained for the standard excess return five-factor model and the "net-of-market" four-factor version of the same model. At least for the LHS portfolios examined here, the standard and net-of-market versions of the models produce similar results.

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Appendix

A1. Summary Statistics for the Components of the Factors

Table A1 shows the means, standard deviations, and *t*-statistics for the means for the portfolios used to construct *SMB*, *HML*, *RMW*, and *CMA*.

A2. Five-Factor Regressions to Explain the Returns for *Size-B/M-OP* and *Size-B/M-Inv* Portfolios

Table A2 shows the intercepts and *HML*, *RMW*, and *CMA* slopes from five-factor regressions to explain monthly excess returns for the 32 portfolios from 2x4x4 sorts on *Size*, *B/M*, and operating profitability. The portfolios of small and big stocks with the highest *B/M* and *OP* (highly profitable extreme value stocks) produce extreme intercepts, negative for big stocks (-0.28% per month), and positive for small stocks (0.21%), but they are only -1.42 and 1.06 standard errors from zero, suggestive of chance results. The imprecision of these intercepts is due to poor diversification: highly profitable extreme value stocks are rare, especially for big stocks. The regression R^2 for these portfolios (not shown in Table A2) are low, 0.57 for big stocks and 0.67 for small stocks.

For both small and big stocks, the *HML* slopes for the 32 *Size*-*B/M*-*OP* portfolios increase from strongly negative for the low-*B/M* portfolios to strongly positive for high-*B/M* portfolios. The *RMW* slopes increase from strongly negative for the low profitability portfolios to strongly positive for the high *OP* portfolios. None of this is surprising, given that the LHS sorts are on *Size*, *B/M*, and *OP*.

Investment is not a sort variable, and the *CMA* slopes are more interesting. The *CMA* slopes are negative for the small portfolios in the lowest *B/M* quartile and positive for the small portfolios in the highest *B/M* quartile. Thus, for small stocks we have the expected result that growth, i.e., low *B/M*, is associated with high investment and value is associated with low investment. Note, however, that the *CMA* slopes are more negative (investment is apparently stronger) for less profitable small stocks in the lowest *B/M* quartile, an unexpected result. It is tempting to infer that the result is driven by unprofitable startups, but the same pattern in *CMA* slopes is observed for big stocks in the lowest *B/M* quartile.

The big problem for the five-factor model in Table A2 is the negative intercept (-0.36% per month, t = -3.52) for the portfolio of small stocks in the lowest *OP* and *B/M* quartiles (small, low profitability growth stocks). This portfolio has negative *HML*, *RMW*, and *CMA* slopes, but the hits to expected return implied by the slopes don't fully explain the low average excess return on the portfolio, -0.04% per month (Table 2). The problem for the five-factor model posed by this portfolio is much the same as the big problems in the tests on the 25 *Size-B/M*, the 25 *Size-Inv*, and the 32 *Size-OP-Inv* portfolios. In a nutshell, small growth stocks that invest a lot despite low profitability fare much worse than predicted by the five-factor model. The 2x4x4 sorts on *Size*, *B/M*, and *OP* add to the puzzle since the portfolio of big stocks in the lowest *B/M* and *OP* quartiles also has strong negative exposures to *HML*, *RMW*, and *CMA*, but it has a positive five-factor intercept (0.24% per month, t = 1.39). Thus, big growth stocks that invest a lot despite low profitability are not a problem for the five-factor model.

Table A3 shows five-factor intercepts and *HML*, *RMW*, and *CMA* slopes for the 32 portfolios from 2x4x4 sorts on *Size*, *B/M*, and *Inv*. The *HML* and *CMA* slopes behave as expected, given that the LHS sorts are on *Size*, *B/M*, and *Inv*. The *HML* slopes are negative for low *B/M* portfolios and strongly positive for high *B/M* portfolios. The *CMA* slopes fall from strongly positive for low investment portfolios to strongly negative for high *Inv* portfolios. The sorts are not on profitability, but for big stocks there is a clear pattern in the *RMW* slopes – positive for low *B/M* growth stocks and negative for high *B/M* value stocks. Small high *B/M* stocks also have negative exposure to *RMW*, but for small stocks, all portfolios in the lowest and highest *Inv* quartiles show negative exposure to *RMW* that is strong for low *Inv* portfolios and weaker for high *Inv* portfolios. Apparently low profitability is a deterrent to investment for some small firms, but small firms that invest a lot have a slight low profitability tilt.

The average absolute five-factor intercept for the 32 *Size-B/M-Inv* portfolios, 0.087% per month (Table 5), is lower than for the two other 2x4x4 sorts. The portfolios of small and big stocks in the lowest *B/M* quartile and the highest *Inv* quartile (growth stocks that invest a lot) produce intercepts more than 3.5 standard errors from zero but of opposite sign – negative (-0.23% per month, t = -3.95) for small stocks and positive (0.27%, t = 3.53) for big stocks. A distinct difference between the two portfolios is the slightly negative exposure of the small stock portfolio to *RMW* versus the strong positive slope for the portfolio of big stocks with low *B/M* and high *Inv*. The *Size-B/M-Inv* sorts do not produce a portfolio of small stocks that invest a lot despite extreme low profitability, and this probably explains why the biggest five-factor problem in these sorts is less serious than in the other 2x4x4 sorts.

A3. Net-of-market Regressions

Table A4 shows the GRS statistics, average absolute intercepts, and average regression R^2 for the five-factor excess return regression models of Table 5. The table also shows the same statistics for the corresponding four-factor "net-of-market" models in which the LHS return is the portfolio's return in excess of the market return and no market factor is included among the RHS variables. (See the regression equation in the table.)

The average regression R^2, 0.88 to 0.92 in the excess return regression models, drop to 0.49 to 0.62 in the net-of-market regressions. This result implies that the market factor accounts for a large fraction of the variances of the LHS excess returns in the excess return regressions, and the other four factors together account for smaller fractions of net-of-market variances. The residual variances (not shown) are, however, similar for the excess return and net-of-market versions of the models.

More important, comparing the GRS statistics and average absolute intercepts shows that performance is quite similar for excess and net-of-market returns. Sometimes the excess return regression performs a little better and sometimes the advantage goes to the net-of-market regressions, but the differences are minor. These results may, however, be misleading. The LHS portfolios examined here may not have enough spread in market betas to produce a meaningful contrast of the excess return and net-of-market models.

Table 1 – Average monthly excess returns for portfolios formed on *Size* and *B/M*, *Size* and *OP*, *Size* and *Inv*; July 1963 to December 2012, 594 months

At the end of each June stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *B/M* groups (Low to High), again using NYSE breakpoints. The intersections of the two sorts produce 25 value-weight *Size-B/M* portfolios. In the sort for June of year *t*, *B* is book equity at the end of the fiscal year ending in year *t*-1 and *M* is market cap at the end of December of year *t*-1, adjusted for changes in shares outstanding between the measurement of *B* and the end of December. The *Size-OP* and *Size-Inv* portfolios are formed in the same way, except that the second sort variable is operating profitability or investment. Operating profitability, *OP*, in the sort for June of year *t* is measured with accounting data for the fiscal year ending in year *t*-1 and is revenues minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense all divided by book equity. Investment, *Inv*, is the change in total assets from the fiscal year ending in year *t*-2 to the fiscal year ending in *t*-1, divided by *t*-1 total assets. The table shows averages of monthly returns in excess of the one-month Treasury bill rate. Highlighted portfolios cause lethal problems in the asset pricing tests of later tables.

	Low	2	3	4	High
Panel A: *Size-B/M* portfolios					
Small	0.19	0.76	0.80	0.97	1.11
2	0.42	0.68	0.90	0.90	0.97
3	0.45	0.73	0.75	0.85	1.03
4	0.56	0.53	0.68	0.81	0.81
Big	0.42	0.48	0.44	0.52	0.58
Panel B: *Size-OP* portfolios					
Small	0.51	0.89	0.86	0.89	0.82
2	0.54	0.74	0.79	0.77	0.93
3	0.49	0.73	0.67	0.73	0.89
4	0.51	0.61	0.58	0.66	0.78
Big	0.33	0.28	0.39	0.43	0.54
Panel C: *Size-Inv* portfolios					
Small	0.96	0.94	0.92	0.86	0.29
2	0.88	0.86	0.89	0.85	0.43
3	0.86	0.88	0.76	0.79	0.45
4	0.74	0.68	0.68	0.71	0.51
Big	0.67	0.49	0.45	0.44	0.37

Table 2 – Averages of monthly excess returns for portfolios formed on (i) *Size*, *B/M*, and *OP*, (ii) *Size*, *B/M*, and *Inv*, and (iii) *Size*, *OP* and *Inv*; July 1963 to December 2012, 594 months

At the end of June each year *t* stocks are allocated to two *Size* groups (Small and Big) using the NYSE median market cap as breakpoint. Stocks in each *Size* group are allocated independently to four *B/M* groups (Low *B/M* to High *B/M* for fiscal year *t*-1), four *OP* groups (Low *OP* to High *OP* for fiscal year *t*-1) and four *Inv* groups (Low *Inv* to High *Inv* for fiscal year *t*-1) using NYSE breakpoints specific to the *Size* group. The table shows averages of monthly returns in excess of the one-month Treasury bill rate on the 32 portfolios formed from each of three sorts. Highlighted portfolios cause lethal problems in the asset pricing tests of later tables.

	Small				Big			
Panel A: Portfolios formed on *Size*, *B/M*, and *OP*								
	Low *B/M*	2	3	High *B/M*	Low B/M	2	3	High *B/M*
Low *OP*	-0.04	0.69	0.80	0.88	0.18	0.19	0.32	0.55
2	0.61	0.73	0.84	1.05	0.38	0.46	0.43	0.66
3	0.61	0.84	1.03	1.26	0.36	0.55	0.64	0.85
High *OP*	0.75	1.09	1.18	1.56	0.50	0.60	0.76	0.67
Panel B: Portfolios formed on *Size*, *B/M* and *Inv*								
	Low *B/M*	2	3	High *B/M*	Low *B/M*	2	3	High *B/M*
Low *Inv*	0.66	0.94	1.16	1.19	0.53	0.65	0.59	0.73
2	0.81	0.88	0.88	1.04	0.46	0.50	0.48	0.55
3	0.80	0.92	0.97	0.93	0.46	0.51	0.53	0.69
High *Inv*	0.33	0.71	0.82	0.98	0.44	0.38	0.33	0.60
Panel C: Portfolios formed on *Size*, *OP*, and *Inv*								
	Low *OP*	2	3	High *OP*	Low *OP*	2	3	High *OP*
Low *Inv*	0.82	0.97	1.14	1.23	0.59	0.63	0.74	0.65
2	0.89	0.86	0.88	0.99	0.26	0.38	0.59	0.61
3	0.57	0.90	0.90	1.03	0.49	0.54	0.44	0.50
High *Inv*	-0.15	0.53	0.71	0.72	0.23	0.20	0.33	0.61

Table 3 – Construction of *Size*, *B/M*, profitability, and investment factors

We use independent sorts to assign stocks to two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label these portfolios with two or four letters. The first always describes the *Size* group, small (*S*) or big (*B*). In the 2x3 sorts and 2x2 sorts, the second describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (C), neutral (N), or aggressive (A). In the 2x2x2x2 sorts, the second character is *B/M* group, the third is *OP* group, and the fourth is *Inv* group. The factors are *SMB* (small minus big), *HML* (high minus low *B/M*), *RMW* (robust minus weak *OP*), and *CMA* (conservative minus aggressive *Inv*).

Sort	Breakpoints	Factors and their components
2x3 sorts on *Size* and *B/M*, or *Size* and *OP*, or *Size* and *Inv*	*Size*: NYSE median *B/M*: 30^{th} & 70^{th} NYSE percentiles *OP*: 30^{th} & 70^{th} NYSE percentiles *Inv*: 30^{th} & 70^{th} NYSE percentiles	$SMB_{B/M} = (SH + SN + SL) / 3 – (BH + BN + BL) / 3$ $SMB_{OP} = (SR + SN + SW) / 3 – (BR + BN + BW) / 3$ $SMB_{Inv} = (SC + SN + SA) / 3 – (BC + BN + BA) / 3$ $SMB = (SMB_{B/M} + SMB_{OP} + SMB_{Inv}) / 3$ $HML = (SH + BH) / 2 – (SL + BL) / 2 = [(SH – SL) + (BH – BL)] / 2$ $RMW = (SR + BR) / 2 – (SW + BW) / 2 = [(SR – SW) + (BR – BW)] / 2$ $CMA = (SC + BC) / 2 – (SA + BA) / 2 = [(SC – SA) + (BC – BA)] / 2$
2x2 sorts on *Size* and *B/M,* or *Size* and *OP,* or *Size* and *Inv*	*Size*: NYSE median *B/M*: NYSE median *OP*: NYSE median *Inv*: NYSE median	$SMB = (SH + SL + SR + SW + SC + SA) / 6 – (BH + BL + BR + BW + BC + BA) / 6$ $HML = (SH + BH) / 2 – (SL + BL) / 2 = [(SH – SL) + (BH – BL)] / 2$ $RMW = (SR + BR) / 2 – (SW + BW) / 2 = [(SR – SW) + (BR – BW)] / 2$ $CMA = (SC + BC) / 2 – (SA + BA) / 2 = [(SC – SA) + (BC – BA)] / 2$
2x2x2x2 sorts on *Size*, *B/M*, *OP*, and *Inv*	*Size*: NYSE median *B/M*: NYSE median *OP*: NYSE median *Inv*: NYSE median	$SMB = (SHRC + SHRA + SHWC + SHWA + SLRC + SLRA + SLWC + SLWA) / 8$ $- (BHRC + BHRA + BHWC + BHWA + BLRC + BLRA + BLWC + BLWA) / 8$ $HML = (SHRC + SHRA + SHWC + SHWA + BHRC + BHRA + BHWC + BHWA) / 8$ $- (SLRC + SLRA + SLWC + SLWA + BLRC + BLRA + BLWC + BLWA) / 8$ $RMW = (SHRC + SHRA + SLRC + SLRA + BHRC + BHRA + BLRC + BLRA) / 8$ $- (SHWC + SHWA + SLWC + SLWA + BHWC + BHWA + BLWC + BLWA) / 8$ $CMA = (SHRC + SHWC + SLRC + SLWC + BHRC + BHWC + BLRC + BLWC) / 8$ $- (SHRA + SHWA + SLRA + SLWA + BHRA + BHWA + BLRA + BLWA) / 8$

Table 4 – Summary statistics for monthly factor returns; July 1963 to December 2012, 594 months

R_M-R_F is the value-weight return on the market portfolio of all sample stocks minus the one-month Treasury bill rate. At the end of each June, stocks are assigned to two *Size* groups using the NYSE median market cap as breakpoint. Stocks are also assigned independently to two or three book-to-market equity (*B/M*), operating profitability (*OP*), and investment (*Inv*) groups, using NYSE medians of *B/M*, *OP*, and *Inv* or the 30th and 70th NYSE percentiles. In the first two blocks of Panel A, the *B/M* factor, *HML*, uses the VW portfolios formed from the intersection of the *Size* and *B/M* sorts (2x2 = 4 or 2x3 = 6 portfolios), and the profitability and investment factors, *RMW* and *CMA*, use four or six VW portfolios from the intersection of the *Size* and *OP* or *Inv* sorts. In the third block, *HML*, *RMW*, and *CMA* use the intersections of the *Size*, *B/M*, *OP*, and *Inv* sorts (2x2x2x2 = 16 portfolios). HML_B is the average return on the portfolio(s) of big high *B/M* stocks minus the average return on the portfolio(s) of big low *B/M* stocks, HML_S is the same but for portfolios of small stocks, *HML* is the average of HML_S and HML_B, and $HML_{S\text{-}B}$ is the difference between them. RMW_S, RMW_B, *RMW*, and $RMW_{S\text{-}B}$ and CMA_S, CMA_B, *CMA*, and $CMA_{S\text{-}B}$ are defined in the same way, but using high and low *OP* or *Inv* instead of *B/M*. In the 2x2x2x2 sorts, *SMB* is the average return on the eight portfolios of small stocks minus the average return on the eight portfolios of big stocks. In the separate 2x3 *Size*-*B/M*, Size-*OP*, and *Size*-*Inv* sorts, there are three versions of *SMB*, one for each 2x3 sort, and *SMB* is the average of the three. *SMB* in the separate 2x2 sorts is defined similarly. Panel A of the table shows average monthly returns (Mean), the standard deviations of monthly returns (Std Dev) and the *t*-statistics for the average returns. Panel B shows the correlations of the same factor from different sorts and Panel C shows the correlations for each set of factors.

Panel A: Averages, standard deviations, and *t*-statistics for monthly returns

	2x3 factors					2x2 factors					2x2x2x2 factors				
	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*
Mean	0.46	0.29	0.38	0.26	0.33	0.46	0.29	0.29	0.17	0.22	0.46	0.29	0.30	0.26	0.15
Std Dev	4.51	3.10	2.90	2.15	2.02	4.51	3.15	2.18	1.53	1.49	4.51	2.93	2.18	1.53	1.18
t-statistic	2.47	2.24	3.22	2.93	4.03	2.47	2.26	3.23	2.77	3.65	2.47	2.44	3.39	4.10	3.08

	HML_S	HML_B	$HML_{S\text{-}B}$	RMW_S	RMW_B	$RMW_{S\text{-}B}$	CMA_S	CMA_B	$CMA_{S\text{-}B}$
2x3 factors									
Mean	0.55	0.21	0.34	0.33	0.19	0.14	0.45	0.22	0.23
Std Dev	3.27	3.13	2.71	2.71	2.36	2.70	2.01	2.68	2.48
t-statistic	4.10	1.67	3.02	2.97	1.94	1.29	5.43	2.00	2.24
2x2 factors									
Mean	0.41	0.16	0.25	0.21	0.14	0.08	0.33	0.11	0.22
Std Dev	2.41	2.38	1.98	1.94	1.70	2.00	1.54	1.88	1.71
t-statistic	4.20	1.66	3.11	2.65	1.94	0.92	5.30	1.44	3.18
2x2x2x2 factors									
Mean	0.38	0.23	0.15	0.30	0.22	0.08	0.23	0.06	0.17
Std Dev	2.42	2.38	2.02	2.19	1.53	2.23	1.24	1.59	1.60
t-statistic	3.81	2.32	1.84	3.29	3.47	0.84	4.61	0.98	2.58

Table 4 (continued)

Panel B: Correlations between different versions of the same factor

	SMB			***HML***			***RMW***			***CMA***		
	2x3	2x2	2x2x2x2	2x3	2x2	2x2x2x2	2x3	2x2	2x2x2x2	2x3	2x2	2x2x2x2
2x3	1.00	1.00	0.98	1.00	0.97	0.94	1.00	0.96	0.80	1.00	0.95	0.83
2x2	1.00	1.00	0.98	0.97	1.00	0.96	0.96	1.00	0.83	0.95	1.00	0.87
2x2x2x2	0.98	0.98	1.00	0.94	0.96	1.00	0.80	0.83	1.00	0.83	0.87	1.00

Panel C: Correlations between different factors

	2x3 factors					**2x2 factors**					**2x2x2x2 factors**				
	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*
R_M-R_F	1.00	0.28	-0.30	-0.21	-0.40	1.00	0.30	-0.35	-0.12	-0.44	1.00	0.25	-0.34	-0.27	-0.43
SMB	0.28	1.00	-0.12	-0.36	-0.11	0.30	1.00	-0.16	-0.32	-0.13	0.25	1.00	-0.21	-0.33	-0.21
HML	-0.30	-0.12	1.00	0.08	0.70	-0.35	-0.16	1.00	0.04	0.71	-0.34	-0.21	1.00	0.63	0.37
RMW	-0.21	-0.36	0.08	1.00	-0.11	-0.12	-0.32	0.04	1.00	-0.19	-0.27	-0.33	0.63	1.00	0.15
CMA	-0.40	-0.11	0.70	-0.11	1.00	-0.44	-0.13	0.71	-0.19	1.00	-0.43	-0.21	0.37	0.15	1.00

Table 5 – Summary statistics for tests of three-, four-, and five-factor models; July 1963 to December 2012, 594 months

The table tests the ability of three-, four-, and five-factor models to explain monthly excess returns on 25 *Size-B/M* portfolios (Panel A), 25 *Size-OP* portfolios (Panel B) 25 *Size-Inv* portfolios (Panel C), 32 *Size-B/M-OP* portfolios (Panel D), 32 *Size-B/M-Inv* portfolios (Panel E), and 32 *Size-OP-Inv* portfolios (Panel F). For each set of 25 or 32 regressions, the table shows the factors that augment R_M-R_F and *SMB* in the regression model, the *GRS* statistic testing whether the expected values of all 25 or 32 intercept estimates are zero, the average absolute value of the intercepts, $A|a_i|$, $A|a_i|/A|\bar{r}_i|$, the average absolute value of the intercept a_i over the average absolute value of $\bar{r}_i$, which is the average return on portfolio i, minus the average of the portfolio returns, $A(a_i^2)/A(\bar{r}_i^2)$, the average squared intercept over the average squared value of $\bar{r}_i$, and $A(\hat{a}_i^2)/A(\hat{\mu}_i^2)$, which is $A(a_i^2)/A(\bar{r}_i^2)$ corrected for sampling error in the numerator and denominator.

	2x3 factors					2x2 factors					2x2x2x2 factors				
	GRS	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(a_i^2)}{A(\bar{r}_i^2)}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$	*GRS*	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(a_i^2)}{A(\bar{r}_i^2)}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$	*GRS*	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(a_i^2)}{A(\bar{r}_i^2)}$	$\frac{A(\hat{a}_i^2)}{A(\hat{\mu}_i^2)}$
Panel A: 25 *Size-B/M* portfolios															
HML	3.60	0.103	0.54	0.40	0.39	3.54	0.102	0.54	0.38	0.37	3.42	0.097	0.51	0.39	0.37
HML RMW	3.12	0.097	0.51	0.29	0.25	3.12	0.097	0.51	0.30	0.26	3.30	0.090	0.47	0.29	0.24
HML CMA	3.51	0.102	0.54	0.41	0.40	3.46	0.100	0.53	0.39	0.37	3.19	0.096	0.51	0.39	0.36
RMW CMA	2.87	0.101	0.53	0.30	0.22	2.81	0.095	0.50	0.27	0.19	2.80	0.089	0.47	0.25	0.14
HML RMW CMA	2.86	0.097	0.51	0.28	0.24	2.84	0.095	0.50	0.28	0.23	2.83	0.090	0.47	0.25	0.19
Panel B: 25 Size-OP portfolios															
HML	2.30	0.107	0.68	0.55	0.52	2.31	0.108	0.68	0.54	0.52	1.91	0.090	0.56	0.45	0.38
RMW	1.67	0.068	0.43	0.21	0.11	1.78	0.077	0.48	0.25	0.15	1.73	0.061	0.38	0.19	0.06
HML RMW	1.65	0.063	0.40	0.17	0.06	1.76	0.059	0.37	0.15	0.04	1.63	0.066	0.41	0.19	0.07
HML CMA	2.95	0.135	0.85	0.83	0.90	2.79	0.133	0.83	0.80	0.86	2.03	0.102	0.64	0.55	0.51
RMW CMA	1.82	0.076	0.48	0.23	0.13	1.62	0.066	0.42	0.17	0.06	1.59	0.069	0.43	0.19	0.05
HML RMW CMA	1.84	0.074	0.46	0.22	0.12	1.71	0.067	0.42	0.18	0.07	1.59	0.070	0.44	0.20	0.08
Panel C: 25 Size-Inv portfolios															
HML	4.61	0.111	0.64	0.55	0.57	4.47	0.107	0.61	0.52	0.53	4.40	0.099	0.57	0.56	0.57
CMA	4.14	0.106	0.61	0.48	0.48	4.16	0.108	0.62	0.49	0.49	4.32	0.123	0.71	0.62	0.63
HML RMW	4.41	0.106	0.61	0.55	0.56	4.30	0.104	0.60	0.52	0.53	4.49	0.114	0.66	0.63	0.66
HML CMA	4.10	0.100	0.58	0.45	0.45	4.08	0.098	0.57	0.43	0.42	3.81	0.084	0.48	0.40	0.37
RMW CMA	3.42	0.087	0.50	0.34	0.30	3.38	0.084	0.48	0.32	0.27	3.59	0.083	0.48	0.34	0.28
HML RMW CMA	3.41	0.087	0.50	0.34	0.31	3.38	0.084	0.48	0.32	0.28	3.65	0.082	0.47	0.34	0.29

Table 5 (continued)

	2x3 factors					2x2 factors					2x2x2x2 factors				
	GRS	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(a_i^2)}{A(\bar{r}_i^2)}$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$	GRS	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(a_i^2)}{A(\bar{r}_i^2)}$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$	GRS	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{A(a_i^2)}{A(\bar{r}_i^2)}$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$
Panel D: 32 *Size-B/M-OP* portfolios															
HML	2.51	0.152	0.60	0.39	0.34	2.56	0.151	0.60	0.38	0.33	2.32	0.134	0.53	0.32	0.26
HML RMW	1.97	0.112	0.45	0.20	0.13	2.31	0.113	0.45	0.21	0.14	1.93	0.098	0.39	0.19	0.12
HML CMA	2.96	0.169	0.67	0.47	0.44	2.95	0.165	0.66	0.44	0.41	2.29	0.144	0.57	0.32	0.26
RMW CMA	2.02	0.137	0.54	0.24	0.15	2.07	0.129	0.51	0.21	0.12	1.75	0.109	0.43	0.17	0.07
HML RMW CMA	2.02	0.136	0.54	0.23	0.17	2.21	0.129	0.51	0.21	0.14	1.77	0.111	0.44	0.18	0.10
Panel E: 32 Size-B/M-Inv portfolios															
HML	2.70	0.130	0.64	0.42	0.38	2.80	0.135	0.67	0.43	0.40	2.80	0.132	0.65	0.44	0.40
HML RMW	2.32	0.122	0.60	0.42	0.39	2.50	0.130	0.64	0.45	0.43	2.49	0.123	0.61	0.42	0.37
HML CMA	2.41	0.105	0.52	0.31	0.25	2.53	0.111	0.55	0.32	0.27	2.36	0.117	0.58	0.34	0.28
RMW CMA	1.70	0.098	0.48	0.29	0.19	1.73	0.093	0.46	0.26	0.15	1.83	0.083	0.41	0.22	0.08
HML RMW CMA	1.75	0.093	0.46	0.26	0.19	1.91	0.094	0.46	0.26	0.19	1.87	0.087	0.43	0.23	0.14
Panel F: 32 Size-OP-Inv portfolios															
HML	4.36	0.182	0.79	0.66	0.68	4.16	0.179	0.78	0.65	0.66	4.03	0.171	0.74	0.61	0.60
HML RMW	3.79	0.142	0.62	0.39	0.37	3.86	0.142	0.62	0.39	0.37	3.59	0.154	0.67	0.45	0.43
HML CMA	3.91	0.177	0.77	0.65	0.67	3.84	0.177	0.77	0.65	0.66	3.69	0.142	0.62	0.49	0.48
RMW CMA	2.95	0.105	0.46	0.25	0.20	3.13	0.100	0.44	0.24	0.20	3.04	0.105	0.45	0.25	0.19
HML RMW CMA	2.97	0.105	0.46	0.25	0.21	3.13	0.100	0.43	0.24	0.20	3.07	0.103	0.45	0.25	0.20

Table 6 – Using four factors in regressions to explain average returns on the fifth: July 1963 - December 2012, 594 months

R_M-R_F is the value-weight return on the market portfolio of all sample stocks, minus the one month Treasury bill rate; *SMB* (small minus big) is the size factor; *HML* (high minus low *B/M*) is the value factor; *RMW* (robust minus weak *OP*) is the profitability factor; and *CMA* (conservative minus aggressive *Inv*) is the investment factor. The 2x3 factors are constructed using separate sorts of stocks into two *Size* groups and three *B/M* groups (*HML*), three *OP* groups (*RMW*), or three *Inv* groups (*CMA*). The 2x2 factors use the same approach except the second sort for each factor produces two rather than three portfolios. Each of the factors from the 2x3 and 2x2 sorts uses 2x3 = 6 or 2x2 = 4 portfolios to control for *Size* and one other variable (*B/M*, *OP*, or *Inv*). The 2x2x2x2 factors use the 2x2x2x2 = 16 portfolios to jointly control for *Size*, *B/M*, *OP*, and *Inv*.

	Int	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R^2
2x3 factors							
R_M-R_F							
Coef	0.78		0.25	0.03	-0.40	-0.92	0.24
t-statistic	4.67		4.41	0.37	-4.76	-7.86	
SMB							
Coef	0.39	0.13		0.04	-0.48	-0.16	0.17
t-statistic	3.20	4.41		0.78	-8.33	-1.85	
HML							
Coef	-0.03	0.01	0.02		0.23	1.04	0.51
t-statistic	-0.40	0.37	0.78		5.28	22.78	
RMW							
Coef	0.43	-0.09	-0.22	0.20		-0.44	0.21
t-statistic	5.36	-4.76	-8.33	5.28		-7.72	
CMA							
Coef	0.27	-0.10	-0.04	0.45	-0.21		0.57
t-statistic	4.87	-7.86	-1.85	22.78	-7.72		

Table 6 (continued)

	Int	R_M-R_F	*SMB*	*HML*	*RMW*	*CMA*	R^2
2x2 factors							
R_M-R_F							
Coef	0.75		0.27	0.01	-0.43	-1.34	0.26
t-statistic	4.53		5.04	0.08	-3.69	-8.30	
SMB							
Coef	0.38	0.15		-0.03	-0.63	-0.17	0.17
t-statistic	3.07	5.04		-0.34	-7.53	-1.38	
HML							
Coef	0.00	0.00	-0.01		0.25	1.09	0.53
t-statistic	0.07	0.08	-0.34		5.67	22.90	
RMW							
Coef	0.29	-0.05	-0.14	0.21		-0.52	0.21
t-statistic	5.11	-3.69	-7.53	5.67		-9.26	
CMA							
Coef	0.18	-0.08	-0.02	0.43	-0.25		0.60
t-statistic	4.56	-8.30	-1.38	22.90	-9.26		
2x2x2x2 factors							
R_M-R_F							
Coef	0.75		0.19	-0.23	-0.33	-1.34	0.25
t-statistic	4.53		3.17	-2.20	-2.29	-8.95	
SMB							
Coef	0.43	0.09		0.13	-0.65	-0.33	0.15
t-statistic	3.70	3.17		1.87	-6.79	-3.01	
HML							
Coef	0.02	-0.04	0.04		0.85	0.48	0.48
t-statistic	0.24	-2.20	1.87		18.56	7.91	
RMW							
Coef	0.20	-0.03	-0.11	0.43		-0.21	0.46
t-statistic	4.20	-2.29	-6.79	18.56		-4.58	
CMA							
Coef	0.19	-0.09	-0.05	0.20	-0.17		0.27
t-statistic	4.32	-8.95	-3.01	7.91	-4.58		

Table 7 – Regressions for 25 *Size-B/M* portfolios; July 1963 to December 2012, 594 months

At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *B/M* groups (Low *B/M* to High *B/M*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size*-*B/M* portfolios. The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size*-*B/M* portfolios. The RHS variables are the excess market return, $Mkt = R_M - R_F$, the *Size* factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using either independent 2x3 sorts on *Size* and each of *B/M*, *OP*, and *Inv* or 2x2x2x2 sorts that jointly control for the four variables. Panel A of the table shows the three-factor and five-factor intercepts produced by the factors from the 2x3 sorts. Panel B shows five-factor regression slopes for *HML*, *RMW*, and CMA, using the factors from the 2x3 and 2x2x2x2 sorts.

$$R(t)-R_F(t) = a + b[R_M(t)-R_F(t)] + sSMB + hHML(t) + rRMW(t) + cCMA(t) + e(t)$$

Panel A: Three-factor and five-factor regression intercepts using factors from the 2x3 sorts

	a					***t*(*a*)**				
	Low *B/M*	2	3	4	High *B/M*	Low *B/M*	2	3	4	High *B/M*
Three-factor: *Mkt SMB HML*										
Small	-0.50	0.01	0.02	0.15	0.14	-5.21	0.13	0.37	2.74	2.29
2	-0.17	-0.04	0.12	0.07	-0.03	-2.81	-0.65	2.25	1.37	-0.48
3	-0.05	0.05	0.02	0.07	0.12	-0.81	0.77	0.34	1.01	1.59
4	0.15	-0.10	-0.04	0.07	-0.09	2.32	-1.43	-0.49	1.01	-1.04
Big	0.17	0.03	-0.08	-0.12	-0.18	3.49	0.49	-1.06	-1.89	-1.94
Five-factor: *Mkt SMB HML RMW CMA*										
Small	-0.30	0.11	0.01	0.12	0.12	-3.37	1.72	0.15	2.02	1.92
2	-0.12	-0.10	0.05	-0.00	-0.05	-1.82	-1.77	0.89	-0.05	-0.76
3	0.03	-0.02	-0.07	-0.01	0.04	0.54	-0.32	-1.04	-0.20	0.57
4	0.18	-0.23	-0.13	0.05	-0.10	2.78	-3.29	-1.72	0.71	-1.21
Big	0.12	-0.10	-0.11	-0.15	-0.10	2.49	-1.74	-1.49	-2.38	-0.99

Table 7 (continued)

Panel B: Five-factor regression slopes for 25 *Size-B/M* portfolios

	Low *B/M*	2	3	4	High *B/M*	Low *B/M*	2	3	4	High *B/M*
2x3 factors										
			h					***t*(*h*)**		
Small	-0.43	-0.13	0.10	0.27	0.52	-10.04	-4.22	3.85	9.98	17.48
2	-0.46	-0.01	0.29	0.43	0.69	-15.25	-0.34	11.63	16.67	24.42
3	-0.43	0.11	0.38	0.52	0.67	-14.64	3.49	12.23	16.88	18.68
4	-0.46	0.09	0.39	0.52	0.80	-15.17	2.59	11.13	15.81	20.17
Big	-0.31	0.03	0.26	0.63	0.84	-13.92	0.98	7.55	20.87	18.52
			r					***t*(*r*)**		
Small	-0.48	-0.31	-0.02	0.04	0.00	-10.72	-9.47	-0.57	1.56	0.14
2	-0.11	0.15	0.21	0.16	0.05	-3.35	5.15	7.82	6.07	1.73
3	-0.11	0.20	0.25	0.17	0.18	-3.65	5.94	7.53	5.18	4.66
4	-0.08	0.25	0.20	0.02	0.08	-2.58	7.09	5.45	0.57	1.80
Big	0.19	0.25	0.02	0.10	-0.17	8.35	8.37	0.46	3.04	-3.44
			c					***t*(c)**		
Small	-0.13	0.01	0.09	0.11	0.09	-2.01	0.30	2.16	2.62	1.90
2	-0.11	0.07	0.01	0.10	-0.00	-2.39	1.65	0.34	2.59	-0.02
3	-0.22	0.02	0.03	0.10	0.07	-4.92	0.41	0.66	2.13	1.33
4	-0.03	0.22	0.11	0.06	-0.05	-0.56	4.35	2.08	1.15	-0.78
Big	-0.07	0.23	0.14	0.01	-0.15	-2.02	5.52	2.64	0.13	-2.15
2x2x2x2 factors										
			h					***t*(*h*)**		
Small	-0.29	0.11	0.32	0.54	0.85	-5.11	2.62	8.78	15.35	21.32
2	-0.52	0.05	0.42	0.69	0.98	-12.15	1.26	12.12	19.71	23.95
3	-0.59	0.11	0.52	0.81	0.96	-14.10	2.61	12.40	20.01	19.50
4	-0.58	0.15	0.59	0.88	1.05	-14.04	3.21	12.79	20.63	19.34
Big	-0.59	-0.00	0.45	0.92	1.10	-19.44	-0.07	10.32	25.64	17.51
			r					***t*(*r*)**		
Small	-0.68	-0.61	-0.25	-0.23	-0.28	-8.62	-10.11	-5.00	-4.63	-5.07
2	-0.11	0.05	0.05	-0.08	-0.22	-1.84	0.98	1.13	-1.54	-3.82
3	-0.00	0.16	0.07	-0.17	-0.16	-0.05	2.69	1.13	-2.92	-2.36
4	0.03	0.12	-0.04	-0.39	-0.24	0.46	1.88	-0.57	-6.49	-3.21
Big	0.43	0.32	-0.15	-0.17	-0.56	10.24	6.22	-2.43	-3.44	-6.41
			c					***t*(*c*)**		
Small	-0.49	-0.21	0.03	0.14	0.23	-5.53	-3.14	0.59	2.48	3.71
2	-0.39	-0.01	0.06	0.17	0.25	-5.83	-0.20	1.08	3.18	3.95
3	-0.41	0.08	0.10	0.21	0.27	-6.35	1.15	1.57	3.26	3.55
4	-0.23	0.29	0.18	0.09	0.24	-3.54	4.04	2.53	1.40	2.88
Big	-0.09	0.28	0.22	0.09	0.22	-2.01	4.93	3.25	1.60	2.29

Table 8 –Regressions for 25 *Size-OP* portfolios; July 1963 - December 2012, 594 months

At the end of each June, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *OP* (profitability) groups (Low *OP* to High *OP*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size-OP* portfolios. The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 *Size-OP* portfolios. The RHS variables are the excess market return, $Mkt = R_M\text{-}R_F$, the *Size* factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2x2x2x2 sorts on *Size*, *B/M*, *OP*, and *Inv*. Panel A shows three-factor intercepts and their *t*-statistics. Panel B shows five-factor intercepts and slopes for *HML*, *RMW*, and *CMA*, and *t*-statistics for these coefficients.

$$R(t)\text{-}R_F(t) = a + b[R_M(t)\text{-}R_F(t)] + sSMB + hHML(t) + rRMW(t) + cCMA(t) + e(t)$$

	Low *OP*	2	3	4	High *OP*	Low *OP*	2	3	4	High *OP*
Panel A: *Mkt SMB HML*										
			a					***t*(*a*)**		
Small	-0.32	0.02	-0.01	0.00	-0.11	-3.25	0.28	-0.22	0.01	-1.44
2	-0.25	-0.06	-0.01	-0.04	0.08	-2.97	-1.00	-0.09	-0.55	1.00
3	-0.19	0.05	-0.02	-0.01	0.14	-1.93	0.72	-0.40	-0.19	1.70
4	-0.09	-0.01	-0.06	0.04	0.14	-0.84	-0.12	-0.97	0.61	1.92
Big	-0.12	-0.19	-0.01	0.05	0.21	-1.33	-2.73	-0.12	1.08	3.68
Panel B: *Mkt SMB HML RMW CMA*										
			a					***t*(*a*)**		
Small	-0.11	0.02	-0.06	-0.08	-0.19	-1.29	0.33	-0.94	-1.11	-2.44
2	-0.08	-0.08	-0.03	-0.13	-0.01	-1.09	-1.13	-0.46	-2.01	-0.11
3	0.03	0.10	-0.05	-0.07	0.03	0.30	1.39	-0.84	-1.16	0.36
4	0.13	0.07	-0.09	-0.04	0.07	1.44	1.08	-1.36	-0.58	0.99
Big	0.08	-0.07	-0.01	-0.02	0.11	1.03	-1.17	-0.18	-0.38	2.25
			h					***t*(*h*)**		
Small	0.31	0.39	0.36	0.31	0.15	5.89	9.76	9.54	7.31	3.10
2	0.22	0.33	0.33	0.14	0.01	5.01	8.17	9.23	3.71	0.27
3	0.32	0.38	0.27	0.22	-0.11	6.24	9.02	7.17	5.57	-2.34
4	0.41	0.48	0.31	0.11	-0.08	7.58	11.59	7.66	2.64	-1.85
Big	0.45	0.48	0.14	-0.16	-0.35	9.37	12.82	3.93	-6.00	-11.35
			r					***t*(*r*)**		
Small	-1.12	-0.04	0.14	0.40	0.47	-15.42	-0.64	2.63	6.72	7.04
2	-0.94	-0.06	0.13	0.50	0.63	-15.41	-1.02	2.53	9.27	9.88
3	-1.13	-0.32	0.06	0.36	0.73	-15.68	-5.55	1.20	6.64	11.12
4	-1.20	-0.62	0.06	0.36	0.46	-16.00	-10.74	1.04	6.50	7.37
Big	-1.07	-0.70	-0.17	0.38	0.58	-16.02	-13.36	-3.37	10.08	13.36
			c					***t*(*c*)**		
Small	-0.14	0.02	0.14	0.07	-0.01	-1.74	0.25	2.35	1.05	-0.17
2	-0.06	0.13	0.01	0.04	-0.11	-0.93	2.04	0.09	0.71	-1.60
3	-0.16	0.05	0.11	0.01	-0.06	-2.04	0.84	1.88	0.13	-0.79
4	-0.07	0.14	0.11	0.11	-0.04	-0.87	2.16	1.79	1.83	-0.56
Big	-0.15	0.01	0.21	0.02	-0.02	-2.01	0.21	3.61	0.55	-0.34

Table 9 –Regressions for 25 *Size-Inv* portfolios; July 1963 - December 2012, 594 months

At the end of June each year, stocks are allocated to five *Size* groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five *Inv* (investment) groups (Low *Inv* to High *Inv*), again using NYSE breakpoints. The intersections of the two sorts produce 25 *Size-Inv* portfolios. The LHS variables are the monthly excess returns on the 25 *Size-Inv* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the value factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2x2x2x2sorts on *Size*, *B/M*, *OP*, and *Inv*. Panel A shows three-factor intercepts and their *t*-statistics. Panel B shows five-factor intercepts and slopes for *HML*, *RMW*, and *CMA*, and *t*-statistics for these coefficients.

$$R(t)\text{-}R_F(t) = a + b[R_M(t)\text{-}R_F(t)] + sSMB + hHML(t) + rRMW(t) + cCMA(t) + e(t)$$

	Low *Inv*	2	3	4	High *Inv*	Low *Inv*	2	3	4	High *Inv*
Panel A: *Mkt SMB HML*										
			a					***t*(*a*)**		
Small	0.05	0.09	0.09	0.02	-0.55	0.56	1.55	1.38	0.31	-7.18
2	-0.01	0.04	0.11	0.03	-0.30	-0.20	0.70	1.86	0.45	-4.48
3	0.08	0.15	0.06	0.08	-0.20	1.00	2.33	1.05	1.17	-2.63
4	0.02	0.01	0.02	0.12	-0.03	0.24	0.09	0.27	1.83	-0.32
Big	0.17	0.09	0.02	0.07	0.05	1.97	1.60	0.38	1.37	0.70
Panel B: *Mkt SMB HML RMW CMA*										
			a					***t*(*a*)**		
Small	0.19	0.09	0.07	0.05	-0.40	2.22	1.54	1.12	0.76	-5.48
2	-0.02	-0.04	0.08	0.02	-0.16	-0.25	-0.61	1.25	0.29	-2.59
3	0.04	0.12	0.00	0.11	-0.08	0.44	1.97	0.06	1.65	-1.08
4	-0.06	-0.06	-0.00	0.12	0.13	-0.78	-0.82	-0.02	1.81	1.65
Big	0.04	-0.00	-0.02	0.05	0.11	0.51	-0.06	-0.46	0.99	1.63
			h					***t*(*h*)**		
Small	0.41	0.42	0.35	0.29	0.10	7.85	12.13	9.14	7.60	2.13
2	0.42	0.40	0.32	0.30	-0.14	11.04	10.62	8.73	8.03	-3.66
3	0.35	0.49	0.32	0.24	-0.16	7.02	12.69	8.77	5.95	-3.76
4	0.43	0.48	0.42	0.13	-0.25	9.41	11.59	11.09	3.15	-5.51
Big	0.18	0.13	0.15	-0.14	-0.46	3.76	4.12	4.98	-4.45	-11.02
			r					***t*(*r*)**		
Small	-1.02	-0.23	-0.06	-0.12	-0.36	-13.92	-4.81	-1.07	-2.29	-5.78
2	-0.49	0.10	-0.02	0.16	-0.19	-9.17	1.97	-0.42	3.08	-3.61
3	-0.24	-0.24	0.13	0.03	-0.08	-3.47	-4.41	2.46	0.46	-1.25
4	-0.25	-0.07	-0.04	0.02	-0.23	-3.97	-1.15	-0.69	0.39	-3.52
Big	-0.10	-0.07	0.09	0.27	0.30	-1.54	-1.57	2.28	6.14	5.12
			c					***t*(*c*)**		
Small	0.22	0.27	0.16	-0.05	-0.52	2.70	4.91	2.58	-0.90	-7.48
2	0.53	0.41	0.25	-0.11	-0.68	9.01	6.92	4.37	-1.93	-11.59
3	0.57	0.44	0.24	-0.24	-0.69	7.36	7.32	4.22	-3.78	-10.20
4	0.76	0.46	0.15	-0.04	-0.71	10.80	7.20	2.58	-0.57	-9.98
Big	0.95	0.69	0.16	-0.16	-0.69	12.69	14.20	3.46	-3.20	-10.70

Table 10 –Regressions for 32 *Size-OP-Inv* portfolios; July 1963 - December 2012, 594 months

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *OP* groups (Low *OP* to High *OP*) and four *Inv* groups (Low *Inv* to High *Inv*), using NYSE *OP* and *Inv* breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size-OP-Inv* portfolios. The LHS variables in the 32 regressions are the excess returns on the 32 *Size-OP-Inv* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the *B/M* factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2x2x2x2 sorts on *Size*, *B/M*, *OP*, and *Inv*. Panel A shows three-factor intercepts and their *t*-statistics. Panel B shows five-factor intercepts and slopes for *RMW* and *CMA*.

$$R(t)-R_F(t) = a + b[R_M(t)-R_F(t)] + sSMB + hHML(t) + rRMW(t) + cCMA(t) + e(t)$$

	Small								Big							
OP	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
Panel A: *Mkt SMB HML*																
	a				***t*(*a*)**				***a***				***t*(*a*)**			
Low *Inv*	-0.08	0.07	0.26	0.29	-0.76	0.85	2.83	3.00	0.05	0.12	0.19	0.16	0.53	1.33	2.16	1.73
2	0.10	0.05	0.09	0.13	1.12	0.75	1.60	1.83	-0.24	-0.11	0.15	0.21	-2.65	-1.26	2.08	2.53
3	-0.25	0.15	0.10	0.22	-2.68	2.30	1.84	3.21	-0.07	0.02	0.02	0.15	-0.78	0.31	0.27	1.75
High *Inv*	-0.89	-0.28	-0.08	-0.11	-8.04	-3.33	-1.17	-1.52	-0.23	-0.27	-0.08	0.27	-2.30	-2.91	-0.87	2.87
Panel B: *Mkt SMB HML RMW CMA*																
	a				***t*(*a*)**				***a***				***t*(*a*)**			
Low *Inv*	0.06	0.01	0.13	0.09	0.77	0.14	1.45	0.96	0.06	-0.00	0.01	-0.01	0.79	-0.01	0.18	-0.17
2	0.18	-0.00	0.01	-0.03	2.37	-0.07	0.10	-0.50	-0.18	-0.10	0.05	0.08	-2.14	-1.22	0.72	0.98
3	-0.12	0.18	0.08	0.10	-1.41	2.73	1.33	1.59	0.11	0.06	-0.04	0.05	1.28	0.74	-0.55	0.58
High *Inv*	-0.54	-0.21	-0.06	-0.14	-5.88	-2.52	-0.89	-2.18	0.06	-0.16	-0.10	0.27	0.67	-1.76	-1.15	3.05
	r				***t*(*r*)**				***r***				***t*(*r*)**			
Low *Inv*	-1.15	-0.21	0.15	0.47	-16.24	-3.05	2.01	6.11	-0.75	-0.13	0.38	0.45	-11.60	-1.79	5.30	6.02
2	-0.87	-0.11	0.19	0.60	-13.19	-2.06	4.05	10.22	-0.69	-0.41	0.22	0.48	-9.49	-5.96	3.60	6.96
3	-0.73	-0.22	0.13	0.62	-9.91	-3.88	2.67	11.72	-0.88	-0.11	0.38	0.61	-12.13	-1.51	6.20	8.79
High *Inv*	-1.23	-0.02	0.18	0.61	-15.66	-0.25	3.22	10.89	-0.87	-0.17	0.55	0.57	-11.68	-2.24	7.45	7.55
	c				***t*(*c*)**				***c***				***t*(*c*)**			
Low *Inv*	0.33	0.61	0.65	0.76	4.21	7.95	7.77	8.92	0.73	0.93	0.71	0.61	10.08	11.54	8.91	7.38
2	0.40	0.45	0.33	0.44	5.44	7.65	6.31	6.74	0.35	0.40	0.39	0.30	4.36	5.17	5.78	3.99
3	0.00	0.04	0.03	0.08	0.02	0.59	0.58	1.38	-0.22	-0.12	-0.03	-0.01	-2.67	-1.50	-0.45	-0.15
High *Inv*	-0.87	-0.41	-0.31	-0.46	-9.94	-5.07	-5.04	-7.37	-0.90	-0.49	-0.45	-0.59	-10.84	-5.64	-5.58	-7.05

Table A1 – Means, standard deviations (Std Dev) and *t*-statistics for the means for the portfolios used to construct *SMB*, *HML*, *RMW*, and *CMA*; July 1963 - December 2012, 594 months

We use independent sorts to form two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label the portfolios with two or four letters. The first is small (*S*) or big (*B*). In the 2x3 and 2x2 sorts, the second is the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (C), neutral (*N*), or aggressive (A). In the 2x2x2x2 sorts, the second character is the *B/M* group, the third is the *OP* group, and the fourth is the *Inv* group.

	2x3 Sorts						2x2 Sorts			
Size-B/M	*SL*	*SN*	*SH*	*BL*	*BN*	*BH*	*SL*	*SH*	*BL*	*BH*
Mean	0.88	1.28	1.43	0.85	0.91	1.07	0.99	1.40	0.85	1.01
Std Dev	6.92	5.47	5.62	4.68	4.37	4.70	6.45	5.45	4.53	4.40
t-statistic	3.11	5.69	6.21	4.45	5.10	5.54	3.73	6.27	4.57	5.60
Size-OP	*SW*	*SN*	*SR*	*BW*	*BN*	*BR*	*SW*	*SR*	*BW*	*BR*
Mean	0.98	1.23	1.31	0.77	0.84	0.95	1.07	1.28	0.79	0.92
Std Dev	6.70	5.35	6.00	5.00	4.40	4.42	6.20	5.72	4.55	4.42
t-statistic	3.58	5.62	5.34	3.74	4.65	5.26	4.20	5.45	4.21	5.08
Size-Inv	*SC*	*SN*	*SA*	*BC*	*BN*	*BA*	*SC*	*SA*	*BC*	*BA*
Mean	1.37	1.31	0.92	1.03	0.91	0.81	1.36	1.03	0.96	0.85
Std Dev	6.15	5.24	6.63	4.40	4.10	5.22	5.75	6.21	4.10	4.72
t-statistic	5.42	6.11	3.38	5.72	5.44	3.80	5.77	4.04	5.70	4.38

	2x2x2x2 *Size-B/M-OP-Inv* Sorts							
	SLWC	*SLWA*	*SLRC*	*SLRA*	*SHWC*	*SHWA*	*SHRC*	*SHRA*
Mean	1.11	0.66	1.32	1.12	1.40	1.21	1.61	1.51
Std Dev	7.23	7.42	5.40	6.19	5.57	5.66	5.25	5.56
t-statistic	3.75	2.17	5.97	4.40	6.11	5.21	7.46	6.63
	BLWC	*BLWA*	*BLRC*	*BLRA*	*BHWC*	*BHWA*	*BHRC*	*BHRA*
Mean	0.73	0.74	0.99	0.89	0.99	0.90	1.21	1.15
Std Dev	5.19	5.49	4.18	4.77	4.37	4.72	4.81	5.54
t-statistic	3.44	3.27	5.78	4.52	5.53	4.65	6.13	5.05

Table A2 – Five-factor regression results for 32 *Size-B/M-OP* portfolios; July 1963 - December 2012, 594 months

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *B/M* groups (Low *B/M* to High *B/M*) and four *OP* groups (Low *OP* to High *OP*), using NYSE *B/M* and *OP* breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size*-*B/M*-*OP* portfolios. The LHS variables are the excess returns on the 32 *Size*-*B/M*-*OP* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the *B/M* factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2x2x2x2 sorts on *Size*, *B/M*, *OP*, and *Inv*. The table shows five-factor regression intercepts, *HML*, *RMW*, and *CMA* slopes, and *t*-statistics for the intercepts and slopes.

$$R(t)-R_F(t) = a + b[R_M(t)-R_F(t)] + sSMB + hHML(t) + rRMW(t) + cCMA(t) + e(t)$$

	Small								Big							
B/M	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
	a				***t*(*a*)**				***a***				***t*(*a*)**			
Low *OP*	-0.36	0.07	-0.01	-0.16	-3.52	0.68	-0.18	-2.19	0.24	-0.16	-0.09	-0.08	1.39	-1.53	-1.07	-1.26
2	0.04	-0.09	-0.03	-0.03	0.44	-1.13	-0.44	-0.35	0.26	-0.08	-0.14	-0.10	2.28	-0.85	-1.75	-1.18
3	-0.10	-0.05	0.07	0.19	-1.48	-0.89	1.15	1.50	0.04	-0.03	-0.12	0.08	0.56	-0.49	-1.41	0.66
High *OP*	-0.13	0.04	0.05	0.21	-2.14	0.64	0.57	1.06	0.10	-0.08	0.06	-0.28	1.58	-0.81	0.42	-1.42
	h				***t*(*h*)**				***h***				***t*(*h*)**			
Low *OP*	-0.24	0.22	0.57	0.90	-3.89	3.66	11.98	19.74	-0.76	0.00	0.49	0.99	-7.19	0.05	10.05	26.21
2	-0.32	0.29	0.66	1.07	-5.35	6.09	18.66	23.19	-0.68	-0.06	0.48	1.10	-9.95	-1.13	9.95	22.09
3	-0.21	0.38	0.76	1.00	-5.12	10.16	21.86	13.17	-0.68	-0.01	0.60	0.77	-16.12	-0.29	11.30	9.89
High *OP*	-0.20	0.55	0.79	0.92	-5.30	14.31	13.99	7.73	-0.56	0.00	0.44	1.08	-14.30	0.08	5.27	8.93
	r				***t*(*r*)**				***r***				***t*(*r*)**			
Low *OP*	-1.38	-0.96	-0.69	-0.41	-16.02	-11.59	-10.40	-6.44	-0.88	-0.55	-0.67	-0.64	-5.95	-6.13	-9.80	-12.08
2	-0.23	0.04	-0.12	-0.10	-2.77	0.66	-2.45	-1.48	-0.27	0.18	-0.25	-0.19	-2.84	2.32	-3.66	-2.76
3	0.19	0.29	0.18	-0.03	3.26	5.58	3.78	-0.29	0.56	0.38	0.32	0.31	9.60	6.65	4.37	2.84
High *OP*	0.72	0.54	0.47	0.34	13.61	9.95	5.92	2.04	0.64	0.68	0.33	0.39	11.86	8.30	2.80	2.28
	c				***t*(*c*)**				***c***				***t*(*c*)**			
Low *OP*	-0.57	-0.05	0.20	0.35	-5.99	-0.56	2.69	4.88	-0.82	0.05	0.17	0.23	-4.98	0.50	2.31	3.96
2	-0.34	0.18	0.28	0.25	-3.62	2.36	4.99	3.52	-0.25	0.43	0.50	0.08	-2.34	5.07	6.64	1.01
3	-0.14	0.28	0.14	0.13	-2.14	4.82	2.54	1.08	-0.14	0.31	0.27	-0.30	-2.14	4.89	3.22	-2.48
High *OP*	-0.07	0.05	0.18	0.58	-1.24	0.76	2.07	3.15	-0.01	0.29	-0.10	-0.45	-0.24	3.13	-0.74	-2.39

Table A3 – Five-factor regression results for 32 *Size-B/M-Inv* portfolios; July 1963 - December 2012, 594 months

At the end of June each year, stocks are allocated to two *Size* groups (Small and Big) using the NYSE median as the market cap breakpoint. Small and big stocks are allocated independently to four *B/M* groups (Low *B/M* to High *B/M*) and four *Inv* groups (Low *Inv* to High *Inv*), using NYSE breakpoints for the small or big *Size* group. The intersections of the three sorts produce 32 *Size-B/M-Inv* portfolios. The LHS variables in the 32 regressions are the excess returns on the 32 *Size-B/M-Inv* portfolios. The RHS variables are the excess market return, R_M-R_F, the *Size* factor, *SMB*, the *B/M* factor, *HML*, the profitability factor, *RMW*, and the investment factor, *CMA*, constructed using 2x2x2x2 sorts on *Size*, *B/M*, *OP*, and *Inv*. The table shows five-factor regression intercepts, *HML*, *RMW*, and *CMA* slopes, and *t*-statistics for the intercepts and slopes.

$$R(t)\text{-}R_F(t) = a + b[R_M(t)\text{-}R_F(t)] + sSMB + hHML(t) + rRMW(t) + cCMA(t) + e(t)$$

	Small								Big							
B/M	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
	a				***t*(*a*)**				***a***				***t*(*a*)**			
Low *Inv*	-0.02	0.08	0.23	0.01	-0.24	0.95	2.90	0.10	0.06	0.01	-0.12	-0.07	0.64	0.12	-1.44	-0.94
2	0.10	-0.00	-0.05	-0.01	1.41	-0.01	-0.93	-0.19	0.05	-0.07	-0.08	-0.12	0.63	-0.81	-0.95	-1.56
3	0.11	0.05	0.10	-0.12	1.95	0.76	1.70	-1.35	0.10	-0.02	-0.02	-0.00	1.34	-0.25	-0.24	-0.05
High *Inv*	-0.23	-0.09	-0.04	-0.06	-3.95	-1.34	-0.61	-0.53	0.27	-0.17	-0.26	-0.02	3.53	-1.89	-2.57	-0.20
	h				***t*(*h*)**				***h***				***t*(*h*)**			
Low *Inv*	-0.10	0.26	0.69	0.98	-1.90	5.30	14.60	18.92	-0.43	-0.08	0.41	0.84	-7.14	-1.45	8.12	18.34
2	-0.16	0.39	0.61	1.04	-3.50	10.19	18.04	22.28	-0.42	-0.11	0.54	0.99	-7.81	-2.32	10.81	20.66
3	-0.16	0.38	0.70	0.94	-4.63	9.72	20.29	16.78	-0.47	-0.06	0.62	1.30	-10.08	-1.17	11.38	22.49
High *Inv*	-0.35	0.40	0.75	1.01	-9.78	10.27	17.04	14.27	-0.90	0.18	0.55	1.01	-19.37	3.32	9.10	15.49
	r				***t*(*r*)**				***r***				***t*(*r*)**			
Low *Inv*	-0.75	-0.43	-0.45	-0.30	-10.01	-6.32	-6.79	-4.20	0.30	0.27	-0.04	-0.20	3.47	3.75	-0.60	-3.14
2	-0.16	0.15	0.03	-0.22	-2.48	2.75	0.66	-3.44	0.44	0.28	-0.26	-0.35	5.98	4.01	-3.77	-5.16
3	0.10	0.21	0.00	-0.14	2.05	3.78	0.10	-1.79	0.49	0.42	-0.16	-0.44	7.65	6.33	-2.08	-5.40
High *Inv*	-0.08	-0.06	-0.29	-0.17	-1.54	-1.18	-4.67	-1.72	0.36	0.20	0.00	-0.40	5.55	2.62	0.03	-4.36
	c				***t*(*c*)**				***c***				***t*(*c*)**			
Low *Inv*	0.37	0.69	0.47	0.64	4.47	9.19	6.45	7.90	0.57	0.96	0.86	0.67	5.99	11.88	10.99	9.39
2	0.38	0.45	0.47	0.38	5.49	7.47	8.96	5.26	0.27	0.61	0.47	0.14	3.27	7.88	6.07	1.93
3	-0.02	0.14	0.05	0.24	-0.42	2.24	0.85	2.74	-0.00	0.05	-0.06	-0.48	-0.06	0.68	-0.68	-5.28
High *Inv*	-0.70	-0.36	-0.27	-0.37	-12.42	-5.89	-3.93	-3.33	-0.70	-0.35	-0.50	-0.47	-9.70	-4.03	-5.24	-4.62

Table A4 – Summary statistics for asset pricing tests on excess and net-of-market returns; July 1963 to December 2012, 594 months

The table uses four- and five-factor models to explain returns on 25 *Size-B/M* portfolios, 25 *Size-OP* portfolios 25 *Size-Inv* portfolios, 32 *Size-B/M-OP* portfolios, 32 *Size-B/M-Inv* portfolios, and 32 *Size-OP-Inv* portfolios. For each set of 25 or 32 regressions, the table shows the *GRS* statistic testing whether the expected values of all 25 or 32 intercepts are zero, the average absolute value of the intercepts, $A|a|$, and the average of the regression R^2, $A(R^2)$. The regression models are,

Excess: $R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$

Net-of-market: $R_{it} - R_{Mt} = a_i + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$

	2x3 Factors			2x2 Factors			2x2x2x2 Factors		
	GRS	$A\|a\|$	$A(R^2)$	*GRS*	$A\|a\|$	$A(R^2)$	*GRS*	$A\|a\|$	$A(R^2)$
25 *Size-B/M* portfolios									
Excess	2.86	0.097	0.92	2.84	0.095	0.92	2.83	0.090	0.91
Net-of-market	2.09	0.081	0.65	2.15	0.078	0.65	2.06	0.074	0.62
25 *Size-OP* portfolios									
Excess	1.84	0.074	0.93	1.71	0.067	0.93	1.59	0.070	0.92
Net-of-market	2.01	0.085	0.60	1.87	0.081	0.60	1.64	0.078	0.55
25 *Size-Inv* portfolios									
Excess	3.41	0.087	0.93	3.38	0.084	0.93	3.65	0.082	0.92
Net-of-market	3.39	0.083	0.63	3.37	0.082	0.63	3.61	0.084	0.59
32 *Size-B/M-OP* portfolios									
Excess	2.02	0.136	0.86	2.21	0.129	0.86	1.77	0.111	0.85
Net-of-market	2.16	0.138	0.52	2.29	0.132	0.52	1.89	0.116	0.50
32 *Size-B/M-Inv* portfolios									
Excess	1.75	0.093	0.88	1.91	0.094	0.88	1.87	0.087	0.88
Net-of-market	1.63	0.085	0.55	1.68	0.086	0.55	1.67	0.082	0.53
32 *Size-OP-Inv* portfolios									
Excess	2.97	0.105	0.89	3.13	0.100	0.89	3.07	0.103	0.88
Net-of-market	3.06	0.105	0.52	3.19	0.102	0.52	3.19	0.102	0.49