

Not Only What But also When – A Theory of Dynamic Voluntary Disclosure

PRELIMINARY AND INCOMPLETE

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1 Introduction

In this paper, we study a dynamic model of voluntary disclosure of multiple news. Corporate voluntary disclosure is one of the major sources of information in capital markets. The extant theoretical literature on voluntary disclosure focuses on static models in which an interested party (e.g., a firm) may privately observe a single piece of private information (e.g., Dye 1985 and Jung and Kwon 1988). Corporate disclosure environments, however, are characterized by multi-period and multi-dimensional flow of information from the firm to the market. The interaction between these two dimensions plays a critical role. When deciding whether to disclose one piece of information a manager must also consider the possibility of learning and potentially disclosing a new piece of information in the future.

To better understand the dynamic interaction between firms and the capital market, we extend Dye's (1985) voluntary disclosure model with uncertainty about information endowment to a two-period and two-signal setting. In particular, we study a two-period setting in which a manager who cares about both periods' stock price may receive up to two private signals about the value of the firm. In each period, the manager may voluntarily disclose any subset of the signals he has received but not yet disclosed. Our model demonstrates how dynamic considerations shape the disclosure strategy of a privately informed agent and the market reactions to what he releases and when. Our setting is such that absent information asymmetry, the firm's price at the end of the second period is independent of the arrival and disclosure times of the firm's private information. Nevertheless, our model shows that in equilibrium, the market price depends not only on what information has been disclosed so far, but also on when it was disclosed. In particular, we show that the price at the end of the second period given disclosure of one signal is higher if the signal is disclosed later in the game. This result might be counter intuitive, as one might expect the market to reward the manager for early disclosure of information, since then he seems less likely to be "hiding something."

The intuition for our finding that the price at the end of the second period given disclosure of one signal is higher if the signal is disclosed later in the game is as follows. Let time be $t \in \{0, 1\}$ and suppose it is now $t = 1$. Consider the following two histories on the equilibrium-path in which the manager disclosed a single signal, x . In history 1, the manager disclosed x at $t = 0$ while in history 2 he disclosed x at $t = 1$. The market price depends on x and on what the market believes

about the second signal given the history. Let y denote that second signal. The market considers three possibilities: (i) the manager did not learn y , (ii) the manager learned y at $t = 1$, (iii) the manager learned y at $t = 0$. Obviously, if the manager did not learn y the market's inference is independent of the observed history. In case the agent learned y at $t = 1$, the market's inference is also independent of the observed history because in both cases the manager would reveal y if and only if it would increase the market price at $t = 1$ (relative to non-disclosure of y). But what if the agent knew y at $t = 0$? If x is disclosed at $t = 0$ then the market can infer that y is less than x and is small enough that revealing it at $t = 1$ would not increase the price at $t = 1$ (relative to non-disclosure of y). If x is disclosed at $t = 1$, the market additionally infers that in case the manager knew y already at $t = 0$ but learned x only at $t = 1$, then y is lower than the threshold for disclosure of a single signal at $t = 0$.¹

On the face of it, one might expect that this additional negative inference about y if x is disclosed later, should lead to more negative beliefs about y . But the opposite is true in equilibrium! Why? There are two effects that impact investors' beliefs in opposite directions. On one hand, the lower threshold implies that the expected value of y conditional on the manager knowing y is lower. On the other hand, the lower threshold implies that it is less likely that the manager learned y at $t = 0$ but did not disclose it, which increases the expected value of y . This second effect always dominates! The reason is that the manager still discloses at $t = 1$ all signals y that exceed the market beliefs about y at this time. Hence, in case of the history with late disclosure, investors additionally rule out any y that is above the disclosure threshold at $t = 0$ but below the threshold for disclosure at $t = 1$. Since the disclosure threshold at $t = 1$ equals the average y for all managers' types who do not disclose y at $t = 1$ (including the informed and uninformed), ruling out these types, which are lower than the expected value of y according to investors' beliefs, increases the expectation of y and hence increase the market price.

To further characterize the strategic behavior and market inferences in our model, we characterize threshold equilibria in Section 4. We show that a threshold equilibrium exists under suitable conditions.² We then characterize the equilibrium disclosure strategy and the properties of the

¹To simplify the demonstration of the intuition, we assume that the agent follows a threshold strategy at $t = 0$. However, our proof does not make this assumption and applies to any equilibrium.

²In most of the existing voluntary disclosure literature (e.g., Verrecchia 1983, Dye 1985, Acharya et al. 2011), the equilibrium always exists, is unique and is characterized by a threshold strategy. In our model, due to multiple periods and signals, existence of a threshold equilibrium is not guaranteed, and therefore we provide sufficient conditions for existence (similar to Pae 2005).

corresponding equilibrium prices. We find that managers that assign higher weight to the first period's stock price compared to second period's stock price tend to issue voluntary disclosure more frequently in the first period, i.e., their first period's disclosure threshold is lower. Assigning higher weight to the short term price compared to the long term price may be due to many reasons that are outside the scope of our model. Several such examples are: managers that face higher short term incentives, managers of firms that are about to issue new debt or equity, managers of firms with higher probability to be taken-over, managers with shorter expected horizon with the firm. The manager's disclosure threshold is also affected by the probability of obtaining private information. In particular, similar to the single period models, the disclosure threshold is decreasing in the likelihood of the manager to obtain private information.

1.1 Related Literature

The voluntary disclosure literature goes back to Grossman and Hart (1980), Grossman (1981) and Milgrom (1981), who established the “unraveling result,” stating that under certain assumptions (including: common knowledge that an agent is privately informed, disclosing is costless and information is verifiable), in equilibrium all types disclose their information. In light of companies propensity to withhold some private information, the literature on voluntary disclosure evolved around settings in which the unraveling result does not prevail. The two major streams of this literature are: (i) assuming that disclosure is costly (pioneered by Jovanovic 1982 and Verrecchia 1983) and (ii) investors' uncertainty about information endowment (pioneered by Dye 1985 and Jung and Kwon 1988). Our model follows Dye (1985) and Jung and Kwon (1988) and extends it to a multi-signal and multi-period setting.

In spite of the vast literature that models voluntary disclosure, very little has been done on multi-period settings and on multi-signals settings. Corporate disclosure environments however, are characterized by multi-period and multi-dimensional flow of information from the firm to the market.³

To the best of our knowledge the only papers that study multi-period voluntary disclosures are Shin (2003 and 2006), who discusses how his single period disclosure setting can be extended to multiple disclosure periods, Einhorn and Ziv (2008) and Beyer and Dye (2011). The settings studied in these papers as well as the dynamic considerations of the agents are very different from

³For example, this gap in the literature is pointed out in a survey by Hirst, Koonce and Venkataraman (2008), who write “much of the prior research ignores the iterative nature of management earnings forecasts.”

ours. Shin (2003, 2006) studies a setting in which a firm may learn a binary signal for each of its independent projects, where each project may either fail or succeed. In this binary setting, Shin (2003, 2006) studies the “sanitization” strategy, under which the agent discloses only the good (success) news. Einhorn and Ziv (2008) study a setting in which in each period the manager may obtain a single signal about the period’s cash flows, where at the end of each period the realized cash flows are publicly revealed. If the agent chooses to disclose his private signal, he incurs some disclosure costs. Finally, Beyer and Dye (2011) study a reputation model in which the manager may learn a single private signal in each of the two periods. The manager can be either “forthcoming” and disclose any information he learns or he may be “strategic.” At the end of each period, the firm’s signal/cash flow for the period becomes public and the market updates beliefs about the value of the firm and the type of the agent. Importantly, the option to “wait for a better signal” that is behind our main result is not present in any of these papers.

Our paper also adds to the understanding of management’s decision to selectively disclose information. Most voluntary disclosure models assume a single signal setting, in which the manager can either disclose all of his information or not disclose at all. In practice, managers sometimes voluntarily disclose part of their private information while concealing another part of their private information. To the best of our knowledge, the only exceptions in which agents may learn multiple-signals are Shin (2003, 2006), which we discussed above, and Pae (2005). The latter considers a single period setting in which the agent can learn up to two signals. We add to Pae’s (2005) model dynamic considerations, which are again crucial for creating the option value of waiting for a better signal.

2 Model setup

We study a two period setting, $t \in \{0, 1\}$, in which an agent (the manager) may receive private signals about his firm’s value (his type). The value of the firm, V , is the realization of a normally distributed random variable. Let’s assume, without loss of generality, that $\tilde{V} \sim N(0, \sigma^2)$. The manager might obtain up to two private signals, each of the form $\tilde{S}_i = \tilde{V} + \tilde{\varepsilon}_i$ where $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2 \sim N(0, \sigma_\varepsilon^2)$ and are independent of \tilde{V} and of each other. The manager’s probability of learning the signal \tilde{S}_i at a given period (given that the manager hasn’t yet learned this signal) is independent of whether the other signal has been observed and of the realizations of the signals. We denote this probability by p . In each period, the manager can publicly disclose all or part of the signals he has obtained

(and hasn't yet disclosed). We follow Dye (1985) and assume that an uninformed manager can not credibly convey the fact that he did not obtain a signal. Any disclosure is assumed to be truthful and does not impose direct cost on the manager or the firm. The manager's objective is to maximize a weighted average of the firm's price over the two periods. For simplicity and without loss of generality we assume that the manager weighs the prices equally across the two periods. In each period, based on the publicly available information, investors set the firm's price to equal its expected value. The publicly available information at time t includes the signals that were disclosed by the manager (or the lack of disclosure) and the time in which the disclosure was made. We denote by x the first signal that is disclosed, we denote by t_x the time at which x is being disclosed and the time at which the signal x was observed by the manager we denote by τ_x . Similarly, we denote the other signal that the manager might have received by y and the time at which the manager learned y we denote by τ_y . Finally, we let

$$h(x, t_x, t)$$

denote investors' expectation at time t of the signal y conditional on the fact that only x was disclosed until period t and it was disclosed at time t_x . The structure of the game and all parameters of the model are common knowledge.⁴

From the properties of the joint normal distribution of the signals it follows that the conditional expectation of the firm value given that the manager obtained a single signal is given by:

$$E(\tilde{V}|S_1 = s_1) = \beta_1 s_1,$$

where $\beta_1 = \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}$. Note that also $E(\tilde{S}_2|S_1 = s_1) = \beta_1 s_1$.

Following disclosure of two signals the conditional expectation of the firm value is given by

$$E(\tilde{V}|S_1 = s_1, S_2 = s_2) = \beta_2(s_1 + s_2),$$

where $\beta_2 = \frac{\sigma^2}{2\sigma^2 + \sigma_\varepsilon^2}$. Finally, the expectation of the firm value given disclosure of a single signal, x , at $t = t_x$ as calculated at the end of period t is given by

$$\beta_2(x + h(x, t_x, t)).$$

Note that $\beta_2 < \beta_1 < 2\beta_2 < 1$ and $\beta_2(1 + \beta_1) = \beta_1$.

Figure 1 summarizes the sequence of events in the model.

⁴All the model's analysis and results are robust to the introduction of a third period in which the private signals learned by the manager are publicly revealed.

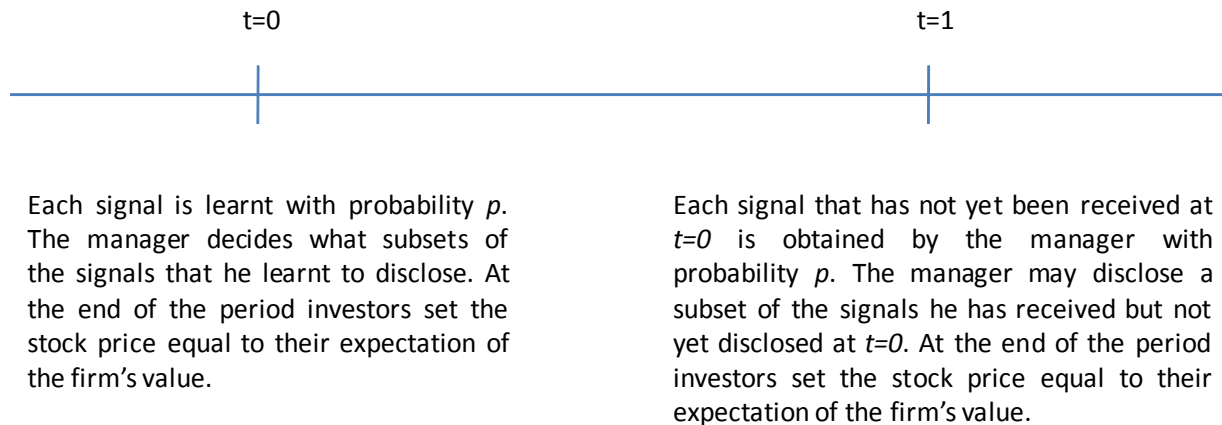


Figure 1: Timeline

3 Properties of any Equilibrium

Multiple equilibria are common in signaling models. In section 4, we identify and analyze a specific class of equilibria in which the manager follows a threshold strategy. In the current section, we show that our main result, that the inference about the firm's value at a given point in time depends not only on what information has been disclosed so far but also on when it was disclosed, holds in any equilibrium. In particular, we show that the price at the end of the second period is higher when the manager disclosed a single signal, x , at $t = 1$ and none at $t = 0$ than when the manager disclosed the same single signal, x , but at $t = 0$ and non at $t = 1$.

At $t = 1$ the number of signals that were disclosed can be zero, one or two. To demonstrate our main result, the only relevant case is when exactly one signal is being disclosed. If both signals were disclosed there is no information asymmetry, so the price is independent of when the signals were disclosed. If no signal was disclosed we cannot condition on the time of disclosure.

We consider two possible equilibrium scenarios. In the first scenario, a signal x is disclosed at $t = 0$ and in the other it is disclosed at $t = 1$. In both cases, this is the only signal that the manager discloses. At $t = 1$, the market sets a price of $\beta_2(x + h(x, 0, 1))$ in the first scenario and $\beta_2(x + h(x, 1, 1))$ in the second one. We argue that $h(x, 1, 1) \geq h(x, 0, 1)$ so that later disclosure receives a better interpretation. That is, investors' valuation of the firm is higher if the manager discloses x at $t = 1$ rather than at $t = 0$.

Note that in equilibrium, under both scenarios (given disclosure of only x at either $t = 0$ or

$t = 1$), the market cannot perfectly tell whether the manager learned a second signal, y , or not.⁵

We refer to managers who by the end of $t = 1$ know also the second signal y as ‘*informed*’ and those who have not learned y as ‘*uninformed*’ (note that in this section we are analyzing managers that disclose a single signal, x , so the only uncertainty is whether the manager did learn the other signal, y , and if so, what is the realization of y). Formally, the set of informed is given by $\{\tau_x \leq 1, \tau_y \leq 1\}$ and those who are *uninformed* by $\{\tau_x \leq 1, \tau_y > 1\}$, where $\tau_y > 1$ indicates that the manager did not learn y in either periods. We denote the set of uninformed managers by A . It is useful to define a subset of *informed* managers which we refer to as ‘*potential disclosers*’. These are informed managers who have not disclosed y at $t = 0$ and therefore may potentially disclose y at $t = 1$. In some cases we need also to exclude types that would have disclosed y at $t = 0$. This occurs for example, if x is disclosed at $t = 0$, when we can rule out the possibility that the manager knew both signals at $t = 0$ and $y \in D_0(y|x)$, where $D_0(y|x)$ is the set of signals y that would have been disclosed in $t = 0$ if the manager learned both y and x at $t = 0$ (this includes y that would have been disclosed at $t = 0$ either with or without disclosure of x). We denote the set of *potential disclosers* at $t = 1$ by B . The set of potential disclosers at $t = 1$ depends on the history, i.e., on whether x was disclosed at $t = 0$ or at $t = 1$. We let B_0 denote the set of potential disclosers at $t = 1$ when x is disclosed at $t = 0$ and B_1 the set of potential disclosers at $t = 1$ for the scenario in which x is disclosed at $t = 1$.

All the types that belong to the set B_0 are managers that disclosed the signal x at $t = 0$ and learned the signal y by $t = 1$.⁶ But, we can say more about the types that belong to B_0 . At $t = 1$ investors know that at $t = 0$ the manager either did not yet learn y or else did learn y and chose not to disclose it. Therefore, at $t = 1$, given that the agent disclosed only x at $t = 0$, investors know that they are not facing a manager that learned a signal y at $t = 0$ which would have been disclosed at $t = 0$, i.e., investors know that they are not facing a manager with $\{y \in D_0(y|x), \tau_y = 0, \tau_x = 0\}$.

⁵To see why, suppose instead that a manager who learns a single signal that equals x never discloses it. Following the disclosure of only x it would become commonly known that the manager did learn the other signal he does not disclose it. The usual unraveling argument then implies that in equilibrium the manager would disclose also the second signal, y . This precludes a disclosure of a single signal being part of the equilibrium. If instead, only a manager who learned a single signal that equals x discloses it, following disclosure of only x it becomes commonly known that the manager does not know the other signal. In this case, managers that learned both x and a low signal y would be better off revealing x but not y - contrary to the assumption.

⁶A manager’s type is determined by whether and when he received each signal, whether and when he disclosed each of the signals he received and what are the realizations of the signals he received.

Therefore, the set of potential disclosers at $t = 1$ when x is disclosed at $t = 0$ is given by

$$B_0 = \{\text{Informed agents who have disclosed only } x \text{ at } t = 0\} \\ \setminus \{y \in D_0(y|x), \tau_y = 0, \tau_x = 0\}.$$

Following similar logic, all the types that belong to the set B_1 are managers that disclosed the signal x at $t = 1$ and learned the signal y by $t = 1$. Here, at $t = 1$ investors can infer that the manager they are facing does not belong to more subsets of informed managers than in the case of B_0 . As in the case of B_0 , investors know that had the manager learned both x and $y \in D_0(y|x)$ at $t = 0$ he would have disclosed y at $t = 0$. In addition, had the manager learned at $t = 1$ a signal $y > x$ he would have disclosed y at $t = 1$ (either instead of or in addition to x).⁷ Finally, let us denote by $D_0(y)$ the set of managers that learn only the signal y at $t = 0$, i.e., $\tau_y = 0, \tau_x = 1$, that would have disclosed their signal y at $t = 0$. Had the manager learned only $y \in D_0(y)$ at $t = 0$ (and x at $t = 1$) he would have disclosed it at $t = 0$. In summary, the set of potential disclosers at $t = 1$ when x is disclosed at $t = 1$ is given by

$$B_1 = \{\text{Informed agents who have disclosed only } x \text{ at } t = 1\} \\ \setminus \left\{ \{y \in D_0(y|x), \tau_y = 0, \tau_x = 0\} \cup \{y > x, \tau_y = 1\} \cup \{y \in D_0(y), \tau_y = 0, \tau_x = 1\} \right\}.$$

At $t = 1$, given disclosure of only x (either at $t = 0$ or at $t = 1$), potential disclosers are myopic in deciding whether to disclose their second signal, y . That is, potential disclosers will disclose y if and only if it is higher than the market perception about y when they do not disclose y , which is given by $h(x, \cdot, 1)$. The myopic disclosure policy can be characterized by a set and a distribution over that set, which we denote by $S_{A,B}^f$. Formally, for arbitrary sets A and B , with distributions f^A and f^B respectively, we define $S_{A,B}^f$ as:

$$S_{A,B}^f = A \cup \{B \cap \{(y, \tau_y) : y \leq E_y(S_{A,B}^f)\}\}. \quad (1)$$

Or equivalently:

$$S_{A,B}^f = A \cup B \setminus \{(y, \tau_y) \in B : y > E_y(S_{A,B}^f)\}.$$

where $E_y(S_{A,B}^f)$ is the expectation of y when calculated over the union of the set A and the set $\{B \cap \{(y, \tau_y) : y \leq E_y(S_{A,B}^f)\}\}$ such that the distribution f assigns a weight to each of the sets and

⁷This follows from the fact that at $t = 1$ the price is increasing in the reported signal and the manager is myopic at $t = 1$, i.e., at $t = 1$ the manager makes the disclosure decision that maximizes the price. We later confirm that the price at $t = 1$ is indeed increasing in the disclosed signal.

its corresponding distribution according to the relative likelihood of y belonging to each set. $S_{A,B}^f$ captures strategic behavior by the potential disclosers in the sense that they disclose the signal y if and only if the signal y is greater than investors beliefs about y absent disclosure of y . The definition of $S_{A,B}^f$ is an implicit definition that relies on the existence and uniqueness of a fixed point. We verify this in the following Lemma.

Lemma 1 *Equation 1 has a unique solution $S_{A,B}^f$.*

Proof. See appendix ■

To gain better intuition for the definition of $S_{A,B}^f$ consider a Dye (1985) setting in which an agent, whose type y is distributed according to a standard normal distribution, may learn his type with probability p . When learnings his type, the agent needs to decide whether or not to disclose it. The agent's objective is to maximize investors' beliefs about his type. Both the set of uninformed agents, A^{Dye} , and the set of potential disclosers (informed agents), B^{Dye} , consists of all the real numbers where the distribution over both sets is standard normal. The set $\{B^{Dye} \cap \{(y, \tau_y) : y \leq E_y(S_{A,B}^f)\}\}$ is all values of y such that $y < E_y(S_{A,B}^f)$. Denoting by $\phi(\cdot)$ and $\Phi(\cdot)$ respectively the pdf and cdf of the standard normal distribution, we have $E_y(S_{A,B}^f) = \frac{(1-p) \int_{-\infty}^{\infty} z\phi(z)dz + p \int_{-\infty}^{E_y(S_{A,B}^f)} z\phi(z)dz}{(1-p) + p\Phi(E_y(S_{A,B}^f))}$. In equilibrium of such a Dye (1985) setting, investors' beliefs given no disclosure, $E_y(S_{A,B}^f)$, equal the disclosure threshold of an informed agent.

We now return to our two-period and two-signal setting. The definitions of $S_{A,B}^f$ and the sets B_0 and B_1 can be used to express $h(x, 0, 1)$ and $h(x, 1, 1)$ in terms of S_{A,B_0}^f and S_{A,B_1}^f . To see that, note that when a manager who disclosed x does not disclose y by $t = 1$ investors know that either they are facing an uninformed manager, or that they are facing a potential discloser who chose not to disclose y , i.e., a manager with y lower than the price given no disclosure of y ($y \leq E(y|y \in S_{A,B}^f)$). Therefore, in equilibrium, investors' beliefs about y at $t = 1$ are given by $E(y|y \in S_{A,B}^f)$. In particular, we can express $h(x, 0, 1)$ and $h(x, 1, 1)$ in terms of S_{A,B_0}^f and S_{A,B_1}^f as follows:

$$\begin{aligned} h(x, 0, 1) &= E(y|y \in S_{A,B_0}^f) \equiv E_y(S_{A,B_0}^f) \\ h(x, 1, 1) &= E(y|y \in S_{A,B_1}^f) \equiv E_y(S_{A,B_1}^f) \end{aligned} \tag{2}$$

In the following, to simplify notation and readability, we will abuse the notation and omit the reference to the distribution of the sets A , B and $S_{A,B}^f$. That is, for any two sets A , B and their

respective distributions we will denote $S_{A,B}^f$ by $S_{A,B}$ and the expectation over the union of these sets given the distributions over the sets by $E_y(A \cup B)$.

A key argument that we will use is the following extension of the minimum principle that appeared first in Acharya, DeMarzo and Kremer (2011):⁸

Lemma 2 *Generalized Minimum Principle*

For any two sets A and B (and their respective distributions) we have:

- (i) $E_y(A \cup B) \geq E_y(S_{A,B})$.⁹
- (ii) Suppose that $B' \supseteq B''$. Then $E_y(S_A, B'') \geq E_y(S_A, B')$.
- (iii) Suppose that $B' \supset B''$. Then $S_{A, B''} = S_{A, B'}$ if and only if every $y \in B' \setminus B''$ satisfies $y > E_y(S_A, B'')$.¹⁰

While we use Lemma 2 in a specific context, note that it holds for arbitrary sets A , B and distributions over A and B . We provide a formal proof of the Lemma in the appendix but the logic and intuition can be demonstrated through the following three simple examples.

Examples: In all three examples, we consider two disjoint sets. Each element in a set is multi-dimensional, but we are interested only in one dimension of the element - call it the value of y . Suppose that for set A y is uniformly distributed on $[0, 1]$, i.e., $y_A \sim U [0, 1]$.

1. Suppose that B is such that $y_B \sim U [0, 1]$. In this case $E_y(A \cup B) = 0.5$ while $E_y(S_{A,B}) < 0.5$ because $S_{A,B}$ is defined as $A \cup B$ excluding some high types in B .
2. Suppose that $B'' = \emptyset$ and B is as defined in example 1 above. In this case we have $S_{A, B''} = A$ and $E_y(S_{A, B''}) = 0.5 > E_y(S_{A, B})$
3. Suppose that $B'' = \emptyset$ but B' is a unit mass distributed uniformly on $[0.5, 1]$. In this case we have $S_{A, B''} = S_{A, B'} = A = 0.5$.

Based on Lemma 2, we argue that the interpretation of a disclosed signal when there was no new disclosure following it deteriorates over time. The following Lemma formalizes it.

Lemma 3 $h(x, 0, 0) \geq h(x, 0, 1)$.

⁸ Acharya et al. (2011) established point (i) of the Lemma below.

⁹ $E_y(A \cup B) = E_y(S_{A,B})$ if and only if $E_y(S_{A,B})$ is greater than the highest element in B .

¹⁰ Note that (ii) and (iii) imply that if there are elements $y \in B' \setminus B''$ such that $y < E_y(S_{A, B''})$ then $E_y(S_{A, B''}) > E_y(S_{A, B'})$.

Proof Consistent with our notation earlier, let the set A denote the set of uniformed managers who disclosed x at $t = 0$ and did not learn y by the end of $t = 1$. Let B_0 denote the set of managers who disclosed x at $t = 0$ and become informed of y by the end of $t = 1$. The claim follows from Lemma 2 as $h(x, 0, 0) = E_y(A \cup B_0)$ and $h(x, 0, 1) = E_y(S_{A, B_0})$. **QED**

The above Lemma implies the following Corollary.

Corollary 1 *A manager that has disclosed x at $t = 0$ is myopic with respect to the decision to release y . That is, conditional on disclosing x at $t = 0$ an informed manager reveals also y at $t = 0$ if and only if $y > h(x, 0, 0)$.*

We now turn to our main result.

Theorem 1 $h(x, 1, 1) \geq h(x, 0, 1)$.

The proof of the Theorem is quite complex and therefore, the formal proof is deferred to the Appendix. Below we describe our strategy to prove the Theorem, which is by way of contradiction. In the particular case in which the manager follows a threshold strategy, the proof of Theorem 1 can be easily obtained using Lemma 2. In particular, for a manager that follows a threshold strategy it is easy to see that $h(x, 0, 1) > h(x, 1, 1)$ implies that $B_0 \supset B_1$, which based on equation (2) and part (ii) of Lemma 2 leads to a contradiction. We want to show that Theorem 1 holds for any equilibrium and therefore we do not rely on the equilibrium structure (e.g., a threshold strategy). instead, we use a slightly more involved argument. We assume by contradiction that $h(x, 0, 1) > h(x, 1, 1)$ and find a set $\widehat{B}_0 \supset B_1$ such that also $\widehat{B}_0 \supseteq B_0$. We then show that for all $y \in \widehat{B}_0 \setminus B_0$ we have $y > E_y(S_{A, B_0}) = h(x, 0, 1)$. Finally, based on part (iii) of Lemma 2 we obtain a contradiction.

4 A Threshold Equilibrium

The objective of this section is to demonstrate the existence of a threshold equilibrium under suitable conditions. In a static model with a single signal, existence of a threshold equilibrium is trivial since the payoff upon disclosure is increasing in the manager's type while the payoff upon non disclosure is fixed. Hence, if a given type chooses to disclose his type so would a higher type. This simple argument is not applicable in our dynamic setting. The reason is that an agent's

expected payoff upon non-disclosure in the first period also increases in his type. Moreover, the relation between the expected payoff of an agent that discloses a signal in the first period and his type is not straight forward. This complicates the analysis and requires few interim steps before establishing existence of a threshold equilibrium. Our proof strategy is to first derive prices that would occur if the market believes that the agent follows a threshold strategy. For these prices, we then show that under suitable conditions the agent's expected payoff upon disclosure in $t = 0$ is increasing faster in his type, x , as compared to his expected payoff upon non-disclosure in $t = 0$. Therefore, given these prices the agent's best response would indeed be to follow a threshold strategy.

We define a threshold strategy in our dynamic setting with two signals in the following way.

Definition 1 *Denote the information set of an agent by $\{s'_1, s_2\}$ where $s_i \in \{\mathcal{R}, \emptyset\}$ and $s_i = \emptyset$ implies that the agent has not learned this signal yet. We say that the equilibrium is a threshold equilibrium if an agent with information set $\{s_1, s_2\}$ who discloses s_1 at time $t_i \in \{0, 1\}$ also discloses any $s'_1 > s_1$ by time t_i when his information set is $\{s'_1, s_2\}$.¹¹*

Since the equilibrium reporting strategy in $t = 1$ is always a threshold strategy as defined above, we will focus on the manager's disclosure decision at $t = 0$. We first assume (and later confirm) that there exists a threshold equilibrium in which a manager that learns a single signal, x , at $t = 0$ discloses it at $t = 0$ if and only if $x > x^*$. Note that a manager that does not disclose at $t = 0$ is always better off disclosing any signal greater than x^* (and even lower signals) at $t = 1$ over not disclosing also at $t = 1$. It proves convenient to partition the set of managers that learn at $t = 0$ a signal $x \geq x^*$ into the following three subsets: (i) managers that learn only x at $t = 0$, (ii) managers that learn both signals at $t = 0$ but the signal y (where $y < x$) is sufficiently high such that if the manager doesn't disclose any signal at $t = 0$ he will disclose y at $t = 1$ (as well as x) and (iii) managers that learn both signals at $t = 0$ but the signal y is sufficiently low such that if the managers doesn't disclose any signal at $t = 0$ he will not disclose y at $t = 1$ (and will disclose x at $t = 1$).

We start by discussing managers in subset (i). If such a manager discloses x at $t = 0$ he will disclose y at $t = 1$ only if $y \geq h(x, 0, 1)$ (in case he learns y at $t = 1$). On the other hand, if the manager does not disclose x at $t = 0$ he benefits from two "real options." The first option value

¹¹While we do not know whether a non-threshold equilibrium exists, one can show that it is always the case that the equilibrium reporting strategy of the second period is a threshold strategy as defined above.

will be realized if he learns at $t = 1$ a sufficiently high value of y such that at $t = 1$ he will disclose only y and conceal x (for $y > y^H(x)$). This increases his payoff at $t = 1$ relative to the case in which he discloses x at $t = 0$. The second option value will be realized if the manager does not learn y at $t = 1$ or if he learns a sufficiently low y ($y < h(x, 1, 1)$) such that he does not disclose it. In this case, since $h(x, 1, 1) > h(x, 0, 1)$ (see Theorem 1) the manager's payoff at $t = 1$ is higher than his payoff would have been had he disclosed x at $t = 0$. In order for a partially informed agent (an agent that learned a single signal at $t = 0$) to disclose x at $t = 0$ the expected value of his two real options should be (weakly) lower than the decrease in the price at $t = 0$ when he does not disclose at $t = 0$ relative to the price when he does disclose x at $t = 0$. This implies that $h(0) < \beta_2(x + h(x, 0, 0))$.

Formally, if the manager decides to disclose x at $t = 0$ his expected payoff is:

$$\begin{aligned} E_{t=0}(U|\tau_x = 0, \tau_y \neq 0, t_x = 0, x) &= \beta_2(x + h(x, 0, 0)) + E_y[\max\{\beta_2(x + h(x, 0, 1)), \beta_2(x + y)\}|x] \\ &= \beta_2(x + h(x, 0, 0)) + (1 - p)\beta_2(x + h(x, 0, 1)) \\ &+ p\beta_2\left[\left(x + \int_{-\infty}^{h(x, 0, 1)} h(x, 0, 1) f(y|x) dy\right) + \int_{h(x, 0, 1)}^{\infty} (x + y) f(y|x) dy\right]. \end{aligned}$$

If he withholds information at $t = 0$ his expected payoff is:¹²

$$\begin{aligned} E_{t=0}(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x) &= h(0) + E_y[\max\{\beta_2(x + h(x, 1, 1)), \beta_2(x + y), \beta_2(y + h(y, 1, 1))\}|x] \\ &= h(0) + (1 - p)\beta_2(x + h(x, 1, 1)) \\ &+ p\beta_2\left[\int_{-\infty}^{h(x, 1, 1)} (x + h(x, 1, 1)) f(y|x) dy + \int_{h(x, 1, 1)}^{y^H(x)} (x + y) f(y|x) dy + \int_{y^H(x)}^{\infty} (y + h(y, 1, 1)) f(y|x) dy\right]. \end{aligned}$$

Such a manager prefers to disclose x at $t = 0$ over not disclosing it if

$$E_{t=0}(U|\tau_x = 0, \tau_y \neq 0, t_x = 0, x) \geq E_{t=0}(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x). \quad (3)$$

Next, we consider managers in subset (ii). If such managers, whose second signal $y > h(x, 1, 1)$, do not disclose x at $t = 0$ they will disclose y (and x) at $t = 1$. Therefore, such managers will not benefit from any of the real options that managers in subset (i) might benefit when they do not disclose x at $t = 0$.

Finally, managers in subset (iii), whose second signal $y < h(x, 1, 1)$, will not disclose y at $t = 1$. Such managers will always benefit at $t = 1$ from the fact that $h(x, 1, 1) > h(x, 0, 1)$. So they trade

¹²The agent considers $E_y[\max\{\beta_2(x + h(x, 1, 1)), \beta_2(x + y), \beta_2(y + h(y, 1, 1)), h(1)\}|x]$ where $h(1)$ is the price at $t = 1$ when the agent hasn't made any disclosure. However, for any $x > x^*$ an agent that did not disclose at $t = 0$ is better off disclosing x at $t = 1$ over not disclosing at all. Therefore, we omit $h(1)$ in the agent's expected utility.

off higher price at $t = 0$ against lower price at $t = 1$. Such a manager will disclose x at $t = 0$ if

$$\beta_2(x + h(x, 0, 0)) + \beta_2(x + h(x, 0, 1)) \geq h(0) + \beta_2(x + h(x, 1, 1)). \quad (4)$$

Later in the paper, we demonstrate some characteristics of prices that always hold under a threshold disclosure strategy. As mentioned at the beginning of this section, the major challenge in proving existence of a threshold equilibrium is to show that for an agent that learns only x at $t = 0$ the expected payoff from disclosing x at $t = 0$ is increasing in x faster than his expected payoff from not disclosing x at $t = 0$. That is, showing that $LHS - RHS$ of inequality 3 is increasing in x . It turns out that given the above mentioned characteristics of prices, that always hold, a sufficient condition for $LHS - RHS$ of inequality 3 to increase in x is that $LHS - RHS$ of inequality 4 is increasing in x , i.e., that

$$\frac{\partial}{\partial x}h(x, 0, 0) + \frac{\partial}{\partial x}h(x, 0, 1) \geq \frac{\partial}{\partial x}h(x, 1, 1) - 1.$$

Since there are few steps we need to take prior to proving existence of a threshold equilibrium, we outline the structure of the remainder of this section. In order to characterize the prevailing prices in our dynamic setting when the market believes that the agent follows a threshold strategy it is useful to study a variant of a Dye (1985) setting. In Section 4.1, we study a variant of Dye (1985) in which the disclosure threshold of the agent is determined exogenously and is stochastic. Equipped with the insights from the variant static model, we characterize in section 4.2 the prices that prevail when the market believes that the agent follows a threshold strategy. The characteristics of the prices derived in section 4.2 set the ground for Section 4.3, in which we establish the existence of a threshold equilibrium under suitable conditions. Finally, in Section 4.4 we further characterize the equilibrium and offer some empirical predictions.

4.1 A Variant of a Static Model

We briefly present and discuss few properties of a static voluntary disclosure setting similar to Dye (1985) and Jung and Kwon (1988). These properties will be later used in characterizing prices in the dynamic setting and in proving existence of a threshold equilibrium. Assume that an agent's type (his firm's value) is the realization of $\tilde{s} \sim N(\mu, \sigma^2)$ and with probability p the agent learns this value. If the agent learns the realization of \tilde{s} he may choose to disclose it. We are interested in investors' beliefs about the firm value given no disclosure by the agent. In particular, we assume that the agent's strategy is to disclose s if and only if $s \geq z$, where z is some exogenously determined

disclosure threshold. Note that unlike Dye (1985) and Jung and Kwon (1988), we are not looking at an equilibrium strategy, but rather on some exogenously determined disclosure threshold strategy. We will refer to this setting as “*Dye setting with exogenous disclosure threshold.*” Let us denote investors’ expectation of the firm value given no disclosure and given the agent’s disclosure threshold is z by $h^{stat}(\mu, z)$.

Figure 2 plots $h^{stat}(\mu, z)$ for standard normal distribution with $p = 0.5$.

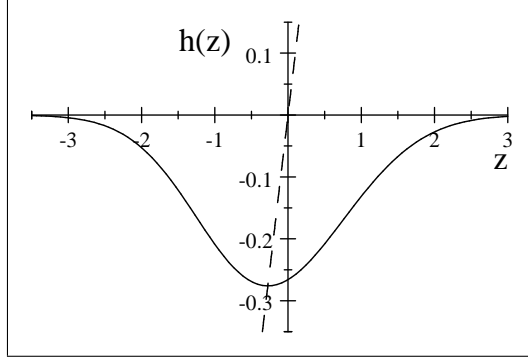


Figure 2: Price Given No-Disclosure in a Dye Setting with Exogenous Disclosure Threshold z

For $z \rightarrow \infty$ none of the agents discloses, and hence, following no disclosure investors do not revise their beliefs relative to the prior. For $z \rightarrow -\infty$ all agents that obtain a signal disclose it, and therefore, following no disclosure investors infer that the agent is uninformed. This also results in investors’ beliefs that equal the prior distribution. As the exogenous disclosure threshold, z , increases from $-\infty$, following no disclosure investors know that the agent is either uninformed or that the agent is informed and his type is lower than z . This results in investors’ expectation about s being lower than the prior mean (zero) for any finite disclosure threshold. The following Lemma provides further characterization of investors’ expectation about s given no disclosure, $h^{stat}(\mu, z)$.

Corollary 2 *Consider a Dye setting with exogenous disclosure threshold. Then:*

1. $h^{stat}(\mu + \Delta, z + \Delta) = h^{stat}(\mu, z) + \Delta$ for any constant Δ ; this implies that $h_1^{stat}(\mu, z) + h_2^{stat}(\mu, z) = 1$.
2. The Generalized Minimum Principle (Lemma 2) implies that $h^{stat}(\mu, z)$ satisfies $z^* = \arg \min_z h^{stat}(\mu, z)$ if and only if $z^* = h^{stat}(\mu, z^*)$. This implies that the equilibrium disclosure threshold in standard Dye (1985) and Jung and Kwon (1988) minimizes $h^{stat}(\mu, z)$.

Note that for all $z < h^{stat}(\mu, z)$ ($z > h^{stat}(\mu, z)$) the price given no disclosure, $h^{stat}(\mu, z)$, is decreasing (increasing) in z . Further analysis shows that for $p < 0.95$ the slope of $h^{stat}(\mu, z)$ with respect to z is always higher than -1 . We will later use this lower bound of the slope.

For the analysis of our dynamic model it will prove useful to consider a variant of this model. The variant is still a static model but the threshold for disclosure depends on μ and the agent follows a random disclosure policy. In particular, with probability λ_i , $i \in \{1, \dots, K\}$, where $\sum_{i=1}^K \lambda_i = p$, the agent discloses only if his type is above $z_i(\mu)$. The reasons we consider a disclosure threshold that depends on μ is that in our dynamic setting investors update their beliefs about the undisclosed signal, y , based on the value of the disclosed signal, x . The reason we consider a random disclosure policy is as follows. In our dynamic setting, when by $t = 1$ the agent disclosed a single signal investors do not know whether the agent learned a second signal and if so, whether he learned it at $t = 0$ or at $t = 1$. Since the agent follows different disclosure thresholds at the two possible dates investors' beliefs about the agent's disclosure threshold for the signal y are stochastic.

We next analyze some properties of the static setting with random threshold disclosure policy. Let us denote by $h^{stat}(\mu, \{z_i(\cdot)\})$ the conditional expectation of the type given no disclosure and given the disclosure thresholds, $z_i(\cdot)$.

Lemma 4 *Suppose that: (i) for all i $z_i(\mu) \leq h^{stat}(\mu, \{z_i(\cdot)\})$, (ii) $z'_i(\mu) \in [0, c]$ and (iii) $p \leq 0.95$. Then $\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\cdot)\}) \in (1 - c, 2)$*

The intuition for the random case, in which $K > 1$, is somehow complicated, and therefore, we defer it to the Appendix, where we formally prove the Lemma. In order to provide the basic intuition for the result, we analyze the particular case in which the disclosure strategy is non-random, i.e., $K = 1$. We start by providing the two simplest examples, for the cases where $z'(\mu) = 1$ and for the case $z'_i(\mu) = 0$. These examples are useful in demonstrating the basic logic and how it can be analyzed using Figure 2. These two examples also provide most of the intuition for the case with no restriction on $z'_i(\mu)$, which is presented in example 3.

Examples:

1. Assume that $z'(\mu) = 1$ (and $K = 1$). Using point 1 in Corollary 2 we have $\frac{d}{d\mu} h^{stat}(\mu, z(\mu)) = \frac{\partial}{\partial \mu} h^{stat}(\mu, \{z(\mu)\}) + z'(\mu) * \frac{\partial}{\partial z} h^{stat}(\mu, \{z(\mu)\}) = 1$. The intuition can be demonstrated using figure 2. A unit increase in μ (keeping $z_i(\cdot)$ constant) shifts the entire graph both upwards

and right by one unit. However, since also $z_i(\cdot)$ increases by a unit, the overall effect is an increase in $h^{stat}(\mu, \{z(\mu)\})$ by one unit.

2. Assume that $z'_i(\mu) = 0$ and the agent discloses his signal if it is higher than z^* , i.e., $z_i(\mu) = z^*$. From Corollary 2 we know that $\frac{\partial}{\partial \mu} h^{stat}(\mu, z^*) + \frac{\partial}{\partial z^*} h^{stat}(\mu, z^*) = 1$ and therefore $\frac{\partial}{\partial \mu} h^{stat}(\mu, z^*) = 1 - \frac{\partial}{\partial z^*} h^{stat}(\mu, z^*)$. We also know that since $z^* < h^{stat}(\mu, \{z^*\})$ we have $\frac{\partial}{\partial z^*} h^{stat}(\mu, z^*) \in (-1, 0)$. Hence, we conclude that $\frac{\partial}{\partial \mu} h^{stat}(\mu, z^*) \in (1, 2)$. The intuition can be demonstrated using figure 2. The effect of a unit increase in μ can be presented as a sum of two effects: (i) a unit increase in the disclosure threshold z as well as a shift of the entire graph both to the right and upwards by one unit and (ii) a unit decrease in the disclosure threshold z (as $z'_i(\mu) = 0$). The first effect is similar to example 1 above and therefore increases $h^{stat}(\mu, \{z_i(\cdot)\})$ by one. The second effect increases $h^{stat}(\mu, \{z_i(\cdot)\})$ by the absolute value of the slope of $h^{stat}(\mu, \{z_i(\cdot)\})$. So, in summary we have $\frac{\partial}{\partial \mu} h^{stat}(\mu, z^*) = 1 - \frac{\partial}{\partial z^*} h^{stat}(\mu, z^*)$. Moreover, $\frac{\partial}{\partial \mu} h^{stat}(\mu, z^*) \in [1, 2]$.

3. The general case for $K = 1$. Assume that $z'(\mu) = c$. This is a more general case and both of the examples above are a particular case of it. Following a similar logic, we conclude that $\frac{d}{d\mu} h^{stat}(\mu, \{z(\cdot)\}) = \frac{\partial}{\partial \mu} h^{stat}(\mu, z(\cdot)) + c * \frac{\partial}{\partial z} h^{stat}(\mu, z(\cdot)) = 1 + (c - 1) \frac{\partial}{\partial z} h^{stat}(\mu, z(\cdot))$.¹³

4.2 Prices Given a Threshold Disclosure Strategy

In this section, we assume the existence of a threshold equilibrium in which a manager who learns only x at $t = 0$ discloses it if and only if $x \geq x^*$. We will derive some characteristics of prices that are consistent with such disclosure strategy.

Recall two observations that we discussed earlier. First, at $t = 1$, the manager behaves myopically he follows a threshold strategy at $t = 1$. Second, for any $x \geq x^*$ the price at $t = 0$ given no disclosure, $h(0)$, is lower than the price given disclosure of x . That is $h(0) < \beta_2(x + h(x, 0, 0))$.

Another intuitive observation is that an agent who learns both signals at $t = 0$ ($\tau_x = \tau_y = 0$), where $y < x$, and discloses x at $t = 0$, behaves myopically with respect to the disclosure of his signal y . In particular, such a type discloses also y at $t = 0$ if and only if $y \geq h(x, 0, 0)$. This has been established in section 3, where we have also shown that for $y \in (h(x, 0, 1), h(x, 0, 0))$ the agent

¹³Recall that $\frac{\partial}{\partial \mu} h^{stat}(\mu, z(\cdot)) = 1 - \frac{\partial}{\partial z} h^{stat}(\mu, z(\cdot))$.

discloses y at $t = 1$ and if $y < h(x, 0, 1)$ he never discloses y . That is, conditional on disclosing x at $t = 0$ the agent is myopic in both periods with respect to the disclosure of y .

Next, we characterize the slopes of $h(x, 0, 0)$, $h(x, 1, 1)$ and $h(x, 0, 1)$. This will be useful later, when we show that the agent's expected payoff from disclosing his first signal at $t = 0$ is increasing in his signal faster than his expected payoff from concealing this signal at $t = 0$.

Claim 1 *Suppose there exists a threshold equilibrium in which an agent that learns only x at $t = 0$ discloses it if and only if $x \geq x^*$. Then, the following are upper and lower bounds for the slopes of $h(x, 0, 0)$, $h(x, 1, 1)$ and $h(x, 0, 1)$.*

$$\begin{aligned} \frac{\partial}{\partial x} h(x, 0, 0) & \begin{cases} = \beta_1 & \text{if } h(x, 0, 0) < x \\ \in (2\beta_1 - 1, \beta_1) & \text{if } h(x, 0, 0) > x \end{cases} \\ \frac{\partial}{\partial x} h(x, 1, 1) & \begin{cases} = \beta_1 & \text{if } h(x, 1, 1) < x^* \\ \in (2\beta_1 - 1, 2\beta_1) & \text{if } h(x, 1, 1) > x^* \end{cases} \\ \frac{\partial}{\partial x} h(x, 0, 1) & \begin{cases} = \beta_1 & \text{if } h(x, 0, 1) < x \\ \in (2\beta_1 - 1, \beta_1) & \text{if } h(x, 0, 1) > x \end{cases} \end{aligned}$$

Proof of Claim 1

We start by analyzing $h(x, 0, 0)$.

As we showed in Section 3, for any x that is disclosed at $t = 0$ such that $h(x, 0, 0) < x$ (the non-binding case)¹⁴, if $\tau_y = 0$ the agent is myopic with respect to the disclosure of y and discloses it whenever $y \geq h(x, 0, 0)$. This makes the analysis of the effect of an increase in x on $h(x, 0, 0)$ qualitatively similar to the analysis of an increase in the mean of the distribution in a standard Dye (1985) and Jung and Kwon (1988) equilibrium. In Dye (1985) and Jung and Kwon (1988) an increase in the mean of the distribution results in an identical increase in both the equilibrium beliefs and the equilibrium disclosure threshold. This case is captured by example 1 of Section 4.1. The quantitative difference in our dynamic setting is that a unit increase in x increases investors' beliefs about y by β_1 (rather than by 1) and therefore also increases both the beliefs about y and the disclosure threshold by β_1 . As a result, in our dynamic setting for $h(x, 0, 0) < x$ we have $h'(x, 0, 0) = \beta_1$.¹⁵

In the binding case, i.e., for all x such that $h(x, 0, 0) > x$ (if such $x > x^*$ exists) we know that if $\tau_y = 0$ then $y < x$. This case is captured by example 3 of Section 4.1. In particular, an increase

¹⁴We use the term non-binding to indicate that the constraint $y < x$ is not binding. The reason is that since $h(x, 0, 0) < x$ the constraint $y < h(x, 0, 0)$ also implies that $y < x$.

¹⁵Since both the beliefs about y and the disclosure threshold increase at the same rate, the probability that the agent learned y at $t = 0$ but did not disclose it is independent of x .

in x increases the beliefs about y at a rate of β_1 while the increase in the constraint/disclosure threshold ($y < x$) is at a rate of 1. Therefore, this is a particular case of example 3 of Section 4.1 in which we increase the mean by β_1 and $z'(\mu) = c = \frac{1}{\beta_1}$. From example 3 we know that an increase in the beliefs about y given a unit increase in x (which is equivalent to an increase of β_1 in μ in example 3) is given by $\beta_1 (1 + (c - 1) \frac{\partial}{\partial z} h^{stat}(\mu, z(\cdot)))$. Substituting $c = \frac{1}{\beta_1}$ and rearranging terms yields

$$h'(x, 0, 0) = \beta_1 + (1 - \beta_1) \frac{\partial}{\partial z} h^{stat}(\mu, z(\cdot)).$$

Since $\frac{\partial}{\partial z} h^{stat}(\mu, z(\cdot)) \in (-1, 0)$ we have $\frac{\partial}{\partial x} h(x, 0, 0) \in [2\beta_1 - 1, \beta_1]$.

Since the analysis of how $h(x, 1, 1)$ and $h(x, 0, 1)$ vary with x is more involved and more technical we defer it to the Appendix, where we prove the remainder of Claim 1. The reason these cases are more complicated is that when pricing the firm at $t = 1$ investors do not know whether the agent learned y at $t = 0$ or at $t = 1$ (in case the agent did in fact learn y). Investors' inference about y depends on when the agent learned it, and therefore the analysis of $h(x, 1, 1)$ and $h(x, 0, 1)$ requires analysis of a stochastic disclosure threshold. Lemma 4 will be useful in conducting this analysis.

Given the characterization of a threshold equilibrium that we have developed so far, we are now ready to establish the existence of a threshold equilibrium.

4.3 Existence of a threshold equilibrium

Proposition 1 *For $\beta_1 > 0.5$ and $p < 0.95$ there exists a threshold equilibrium in which an agent that learns at $t = 0$ only one signal discloses it at $t = 0$ if and only if it is greater than x^* . If the agent learns two signals at $t = 0$ and one of them is greater than x^* he makes a disclosure at $t = 0$. In particular, he may choose to disclose at $t = 0$ both signals or just the higher one. Disclosing a single signal $x < x^*$ at $t = 0$ is not part of the equilibrium disclosure strategy.¹⁶*

Proof. The sketch of the proof is as follows. We show that if the highest signal learned by an agent at $t = 0$ is sufficiently high he will disclose it at $t = 0$ (and if he learned a second signal he sometimes discloses it as well). If his highest signal is sufficiently low the agent will not make a disclosure at $t = 0$. Finally, using the properties of the slopes of the various prices that we derived in Claim (1), we show that the difference between the agent's expected payoff at $t = 0$ from disclosing a signal

¹⁶We believe that the threshold equilibrium exists for a wider set of (all) parameters, however, for tractability reasons we restrict the set of parameters for which we show the existence of a threshold equilibrium.

and his expected payoff at $t = 0$ from not disclosing the signal is increasing in the signal. This guarantees the existence of a threshold equilibrium. The following Lemma formalizes these three conditions, which taken together they are sufficient for for proving Proposition 1.

Lemma 5 *In any threshold equilibrium*

- (a) *For sufficiently high (low) realizations of x , an agent that learns a single signal, x , at $t = 0$ ($\tau_x = 0, \tau_y \neq 0$) discloses (does not disclose) x at $t = 0$.*
- (b) *For sufficiently high (low) realizations of x , an agent that learns both signals at $t = 0$ ($\tau_x = \tau_y = 0$) and does not disclose y at $t = 0$ discloses (does not disclose) x at $t = 0$.*
- (c) *On the equilibrium path, the difference between the agent's expected payoff if he discloses x at $t = 0$ and if he does not disclose at $t = 0$ is increasing in x .*

The proof of the Lemma, which is sufficient condition for Proposition 1 to hold, is in the appendix.

■

4.4 Further characterization of the equilibrium

While we have already established the existence of a threshold equilibrium under suitable conditions, we have not yet discussed how x^* , the disclosure threshold at $t = 0$ for an agent that learned a single signal, is determined. We complete this analysis below.

In most signaling models, and in particular voluntary disclosure models, the agent's private information consists of a single signal. In such settings, the disclosure threshold equals the signal for which the manager is indifferent between disclosing and not disclosing his signal. In our richer setting with two signals and two periods, the difference between types is multidimensional, and therefore the simple indifference condition used in the standard models does not apply. We next discuss how the disclosure threshold is determined.

At the beginning of Section 4, when discussing the manager's trade-offs we partitioned the set of agents that make a disclosure at $t = 0$ into subsets (i) – (iii). We will use the same partition in order to describe how the threshold for disclosure of a single signal at $t = 0$ is determined.

For a given x , if an agent in subset (iii) prefers to disclose x at $t = 0$ then it is easy to see that every agent in subset (ii) strictly prefers to disclose x at $t = 0$. It is not easy, however, to determine whether the fact that an agent in subset (iii) prefers to disclose x at $t = 0$ over not disclosing it implies that also a type in subset (i) prefers disclosure of x at $t = 0$ over non-disclosure at $t = 0$.

The reason is that a type in subset (i) that does not disclose x at $t = 0$ may benefit from either one of the real options, or none of them, while a type in subset (iii) benefits for sure from just one of the real options (the increased price at $t = 1$).

To obtain an equilibrium with a threshold for disclosure of a single signal at $t = 0$, we set x^* to equal the lowest value of x for which all agents with $x = x^*$ from all subsets (i) – (iii) weakly prefer to disclose x^* at $t = 0$ over not disclosing at $t = 0$. Note that the binding constraint might be either equation (3) or equation (4). Since there are agents that strictly prefer to disclose x^* at $t = 0$ over not disclosing at $t = 0$ (these are agents in subset (ii) and agents in either subset (i) or (iii)) the price given disclosure of $x < x^*$ at $t = 0$, which is off the equilibrium path, must be sufficiently low to prevent the above types from deviating from the equilibrium strategy and disclosing $x < x^*$ at $t = 0$. This implies that a necessary condition for our equilibrium is that prices exhibit a discontinuity at x^* .

Our model demonstrates how strategic considerations affect an agent’s voluntary disclosure decisions in a dynamic setting. In light of the focus of the current literature on static models, an interesting question is how the dynamic/multi-period nature of our setting affects the agent’s disclosure decisions. One way to demonstrate this effect is to compare the disclosure strategy in a single period setting (as in Pae 2005) to the disclosure strategy in our setting. In particular, suppose that the probability of obtaining each signal in the single period setting equals the probability of obtaining each signal in the first period of our two-period setting. That is, the first period of our two-period setting differs from the single period setting only in that there is a future in which the agent may learn additional information and may voluntarily disclose information. Our model indicates that the existence of a continuation to the first period decreases the amount of disclosure in the first period. The reason is that in the two-period setting, withholding information in the first period generates a real option to the agent and therefore increasing the disclosure threshold relative to a setting where such real options for non disclosure do not exist.

The disclosure strategy and investors’ beliefs in the single period setting are identical to the ones in the first period of the limit case of the two period setting in which the manager assigns an infinite weight to the first period’s price relative to the second period’s price, or equivalently, when the discount rate used by the manager goes to infinity. This result can be generalized and demonstrate that the higher the weight the manager assigns to the first period’s price relative to the second period’s price, the higher the expected probability of disclosure in the first period. In other

words, increasing the weight assigned to the first period's price decreases the first period's disclosure threshold. Higher weight assigned by the manager to the first period's price can reflect for example: managers that face higher short term incentives, managers of firms that are about to issue new debt or equity, higher probability for the firm to be taken over, shorter expected horizon for the manager with the firm etc. This gives rise to the testable empirical prediction of higher likelihood of early voluntary disclosure by managers of firms that care more about short term price due to various reasons, as the ones mentioned above. Our model also provides some predictions regarding the extent to which managers' voluntary disclosures tend to cluster. For example, conditional on disclosure of two signals the disclosed values are on average closer to each other than in the original distribution of the signals, i.e., values of disclosures are clustered. Another type of clustering on which our model can generate predictions is time clustering of the disclosures.

5 Conclusion

The vast literature on voluntary disclosure models focuses on static models in which an interested party (e.g., a firm's manager) may privately observe a single piece of private information (e.g., Dye 1985 and Jung and Kwon 1988). Many real life voluntary disclosure environments, in particular corporate disclosure environments, are multi-period in their nature and the informed party often obtains more than a single piece of private information. In such settings, the decisions whether to disclose one piece of information must take into account the possibility of learning and potentially disclosing a new piece of information in the future. To the best of our knowledge, such dynamic considerations that affect voluntary disclosure have not been studied in the literature. In this paper, we show that the interaction between these two dimensions plays a critical role in disclosure decisions. We study a dynamic model of voluntary disclosure of multiple news which extends Dye's (1985) and Jung and Kwon's (1988) voluntary disclosure model with uncertainty about information endowment to a two-period and two-signal setting. Our model demonstrates how dynamic considerations shape the strategy of a privately informed agent and the market reactions to the disclosure (or lack of disclosure). Our setting is such that absent information asymmetry, the firm's price at the end of the second period is independent of the disclosure time of the firm's private information. Nevertheless, our model shows that in equilibrium, the market price depends not only on what information has been disclosed so far, but also on when it was disclosed. In particular, we show that the price at the end of the second period given disclosure of one signal

is higher if the signal is disclosed later in the game. As such, the paper illustrates the importance of considering dynamic aspects of voluntary disclosure. The model generates several interesting empirical predictions that can shed some light on, and provide directions for, empirical research. For example: market reaction to later voluntary disclosure is more positive and the amount of voluntary disclosure (the disclosure threshold) in the short term increases (decreases) in the relative weight assigned by the manager to the short term price.

Appendix

Proof Lemma 1

For a constant c let $S_{A,B}^c = A \cup \{B \cap \{(y, \tau_y) : y \leq c\}\}$. For $c \rightarrow -\infty$ we have that $E_y(S_{A,B}^c) = E_y(A) > c$ and for $c \rightarrow \infty$ we have that $E_y(S_{A,B}^c) = E_y(A \cup B) < c$. From continuity we can find c^* for which $E_y(S_{A,B}^{c^*}) = c^*$. This establishes existence.

Now suppose by way of contradiction that there are multiple solutions. Specifically, assume there are $c' < c''$ so that $E_y(S_{A,B}^{c'}) = c'$ and $E_y(S_{A,B}^{c''}) = c''$. When we compare $S_{A,B}^{c'}$ to $S_{A,B}^{c''}$ we note that $S_{A,B}^{c''} \supset S_{A,B}^{c'}$ and that for $(y, \tau_y) \in S_{A,B}^{c''} \setminus S_{A,B}^{c'}$ we have $y < E_y(S_{A,B}^{c''})$. This implies that $S_{A,B}^{c''}$ can be represented as a union of $S_{A,B}^{c'}$ where the average $c' (< c'')$ and a set of types that are lower than c'' . This however, implies that $E_y(S_{A,B}^{c''}) < c''$ and we get a contradiction. QED

Proof of Lemma 2

1. When comparing $S_{A,B}$ to $A \cup B$ we note that we have excluded above average types for which $y > E_y(S_{A,B})$. This results in lower average type.
2. Suppose first that there exists $(y, \tau_y) \in S_{A,B''} \setminus S_{A,B'}$. Since $B' \supseteq B''$ it must be that these $(y, \tau_y) \in B' \cap B''$. From the definition of $S_{A,B}$ since $(y, \tau_y) \in S_{A,B''}$ we conclude that $E_y(S_{A,B''}) > y$. Since $(y, \tau_y) \notin S_{A,B'}$, we conclude that $E_y(S_{A,B'}) < y$ which implies the claim. Hence, we will assume that $S_{A,B'} \supseteq S_{A,B''}$ and we consider $(y, \tau_y) \in S_{A,B'} \setminus S_{A,B''}$; this implies $y < E_y(S_{A,B'})$. Hence, all the elements $(y, \tau_y) \in S_{A,B'} \setminus S_{A,B''}$ have y that is below the average in $S_{A,B'}$ which implies that $E_y(S_{A,B''}) \geq E_y(S_{A,B'})$.
3. Consider the set $S_{A,B''}$, and note that it satisfy the definition for $S_{A,B'}$ given in (1) Hence, the claim follows from uniqueness that was proven in Lemma 1. QED

Proof of Theorem 1

Step 1 Suppose that $h(x, 0, 1) > h(x, 1, 1)$. Then, if x is disclosed at time $t = 1$ then the agent could not have known both signals at $t = 0$.

Proof of Step 1: We know that x is being disclosed with positive probability if it is the only signal known at $t = 0$. Let I denote the payoff for such an agent, who learned only x at $t = 0$, from disclosing x at $t = 0$ and II his payoff from not disclosing at $t = 0$

$$\begin{aligned} I &= \beta(x + h(x, 0, 0)) + E_y[\max\{\beta(x + h(x, 0, 1)), \beta(x + y)\}] \\ II &= h(0) + E_y[\max\{\beta(x + h(x, 1, 1)), \beta(y + h(y, 1, 1)), \beta(x + y), h(1)\}] \end{aligned}$$

where $h(0)$ and $h(1)$ are the prices at the end of $t = 0$ and $t = 1$ respectively, given that no disclosure was made until time t .

We know that for some x we have that $I - II \geq 0$. Consider an agent who knows both signals at $t = 0$ and prefers to disclose just x at $t = 1$ (an agent with $x \in (h(1), h(0))$ and a sufficiently low y). Such an agent knows at time $t = 0$ that he will disclose x and not disclose y at $t = 1$. So, for this to happen it must be that $II' - I' \geq 0$ where:

$$\begin{aligned} I' &= \beta(x + h(x, 0, 0)) + \beta(x + h(x, 0, 1)), \\ II' &= h(0) + \beta(x + h(x, 1, 1)). \end{aligned}$$

This leads to contradiction as $h(x, 0, 1) > h(x, 1, 1), \Rightarrow I' - II' > I - II > 0$.

As we show in Step 2 below, the above provide us with a simple characterization of B_1 . Recall that $D_0(y)$ is the set of signals y that are disclosed at $t = 0$ if this is the only signal the agent knows, i.e., if $\tau_y = 0, \tau_x > 0$.

Step 2 If $h(x, 0, 1) > h(x, 1, 1)$ then $B_1 = \{\tau_y = 0, 1\} \cap \{y < x\} \setminus \{(y, \tau_y) : \tau_y = 0, y \in D_0(y)\}$

Proof: If the agent learned y at $t = 0$ then based on step 1 we conclude that this was the only signal he knew at $t = 0$, so it must be that $y \notin D_0(y)$. Since $x + h(x, 1, 1)$ is increasing in x , the agent is better off disclosing his highest signal at $t = 1$ given that he did not make a disclosure at $t = 0$. Therefore, we conclude that $y \leq x$.

in Step 3 below we will consider the set:

$$\widehat{B}_0 = B_0 \cup \{y < x\} \cap \{\tau_y = 0, 1\}.$$

Clearly $\widehat{B}_0 \supset B_1, \widehat{B}_0 \supseteq B_0$.

Step 3 $\forall y \in \widehat{B}_0 \setminus B_0, y > h(x, 0, 1)$

Proof: We need to consider only agents that become informed. The claim follows from **Corollary 1** showing that an informed agent reveals y if and only if $y > h(x, 0, 0) \geq h(x, 0, 1)$.

From Lemma 2 (iii) we know that $S_{A\widehat{B}_0} = S_{AB_0}$ and from Lemma 2 (ii) we know that $E_y(S_{AB_1}) \geq E_y(S_{A\widehat{B}_0}) = E_y(S_{AB_0})$. This contradicts the assumption that $h(x, 0, 1) > h(x, 1, 1)$.

QED

Proof of Lemma 4

By applying Bayes rule, $h^{stat}(\mu, \{z_i(\cdot)\})$ is given by:

$$h^{stat}(\mu, \{z_i(\cdot)\}) = \frac{(1-p)\mu + \sum_{i=0}^K \lambda_i \int_{-\infty}^{z_i(\mu)} y \phi(y|\mu) dy}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(\mu)|\mu)}.$$

Taking the derivative of $h^{stat}(\mu, \{z_i(\cdot)\})$ with respect to μ and applying some algebraic manipulation yields:

$$\frac{d}{d\mu} h^{stat}(\mu, \{z_i(\cdot)\}) = 1 + \frac{\sum_{i=0}^K \lambda_i (z'_i(\mu) - 1) \phi(z_i(\mu) | \mu) (z_i(\mu) - h^{stat}(\mu, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(\mu) | \mu)}.$$

We start by proving the supremum of this derivative

Given that $z'_i(\mu) \geq 0$ and $z_i(\mu) \leq h^{stat}(\mu, \{z_i(\cdot)\})$ for all $i \in \{1, \dots, K\}$ we have

$$\begin{aligned} \frac{d}{d\mu} h^{stat}(\mu, \{z_i(\cdot)\}) &\leq 1 + \frac{\sum_{i=0}^K \lambda_i \phi(z_i(\mu) | \mu) (z_i(\mu) - h^{stat}(\mu, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(\mu) | \mu)} \\ &\leq 1 + \max_{\substack{z_i \leq h(x) \\ i \in \{1, \dots, K\}}} \frac{\sum_{i=0}^K \lambda_i \phi(z_i(\mu) | \mu) (z_i(\mu) - h^{stat}(\mu, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(\mu) | \mu)} \end{aligned}$$

Due to symmetry, for all $i \in \{1, \dots, K\}$ the maximum is achieved at $z_i(\mu) = z^*(\mu)$. To see this, note that the FOC of the maximization with respect to $z_i(\mu)$ is

$$\begin{aligned} 0 &= (\phi'(z_i(\mu) | \mu) (h^{stat}(\mu, \{z_i(\cdot)\}) - z_i(\mu)) - \phi(z_i(\mu) | \mu)) \left((1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(\mu) | \mu) \right) \\ &\quad - \left(\sum_{i=1}^K \lambda_i \phi(z_i(\mu) | \mu) (h^{stat}(\mu, \{z_i(\cdot)\}) - z_i(\mu)) \right) \phi(z_i(\mu) | \mu) \end{aligned}$$

Since $\phi'(z_i(\mu) | \mu) = -\alpha (z_i(\mu) - \mu) \phi(z_i(\mu) | \mu)$ (for some constant $\alpha > 0$), this simplifies to

$$-\alpha (z_i(\mu) - \mu) (h^{stat}(\mu, \{z_i(\cdot)\}) - z_i(\mu)) = \frac{\sum_{i=0}^K \lambda_i \phi(z_i(\mu) | \mu) (z_i(\mu) - h^{stat}(\mu, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(\mu) | \mu)} + 1$$

In the range $z_i(\mu) \leq h^{stat}(\mu, \{z_i(\cdot)\}) \leq \mu$, the LHS is decreasing in $z_i(\mu)$.¹⁷ The RHS is the same for all i . Therefore, the unique solution to this system of FOC is for all $z_i(\mu)$ to be equal (and note that the maximum is achieved at an interior point since at $z_i(\mu) = h^{stat}(\mu, \{z_i(\cdot)\})$ the LHS is zero and the RHS is positive; and as $z_i(\mu)$ goes to $-\infty$ the LHS goes to $+\infty$ while the RHS is bounded). This implies that the example we discussed following the statement of the Lemma also provides an upper bound. The lower bound can be concluded in a similar way by observing that if we want to minimize the slope we will again choose the same $z_i(\mu)$ for all i and therefore our example provides also a lower bound.

QED

¹⁷Since $z_i(x) \leq h(x, \{z_i(\cdot)\})$ also $h(x, \{z_i(\cdot)\}) \leq E[x|y] = \beta_1 x$.

Proof of Claim 1

The case of $h(x, 0, 0)$ has been proved right bellow the Claim. We analyze the cases of $h(x, 1, 1)$ and $h(x, 0, 1)$ bellow.

Next we analyze $h(x, 1, 1)$.

When an agent discloses $x > x^*$ at $t = 1$ investors know that $\tau_x = 1$ (otherwise the agent would have disclosed x at $t = 0$). Investors' beliefs about the manager's other signal at $t = 1$ is set as a weighted average of three scenarios: $\tau_y = 0$, $\tau_y = 1$ and $\tau_y > 1$. We start by describing the disclosure thresholds conditional on each of the three scenarios.

(i) If $\tau_y > 1$ the agent cannot disclose y and therefore the disclosure threshold is not relevant. In the pricing of the firm conditional on $\tau_y > 1$ investors use $E(y|x)$ which equals $\beta_1 x$.

(ii) If $\tau_y = 1$ investors know that $y < h(x, 1, 1)$ and also that $y < x$. We need to distinguish between the binding case and the non-binding case. In the non-binding case, where $h(x, 1, 1) \leq x$, investors know that $y < h(x, 1, 1)$, so conditional on $\tau_y = 1$ investors set their beliefs as if the manager follows a disclosure threshold of $h(x, 1, 1)$. In the binding case, where $h(x, 1, 1) > x$, investors know that $y < x$, so it is equivalent to a disclosure threshold of x .

(iii) If $\tau_y = 0$ investors know that $y < x^*$ (where $x^* \leq x$) and also $y < h(x, 1, 1)$. Here again we should distinguish between a non-binding case in which $h(x, 1, 1) < x^*$ (if such case exists) and a binding case in which $h(x, 1, 1) > x^*$. In the non-binding case the disclosure threshold is $h(x, 1, 1)$. In the binding case the disclosure threshold is x^* , which is independent of x .

The next Lemma provides an upper and lower bound for $\frac{\partial}{\partial x} h(x, 1, 1)$. The proof of the Lemma, uses the disclosure thresholds for each of the three scenarios above. This Lemma holds also for $h(x, 0, 1)$.

Lemma 6 For $\beta_1 > 0.5$ and $p < 0.95$

$$\frac{\partial}{\partial x} h(x, 1, 1) \in (2\beta_1 - 1, 2\beta_1).$$

Proof of Lemma 6

We next show that for the particular case in which $h(x, 1, 1) < x^*$ (if such case exists) $h'(x, 1, 1) = \beta_1$.

$h(x, 1, 1)$ is a weighted average of the beliefs about y over the three scenarios $\tau_y = 0$, $\tau_y = 1$ and $\tau_y > 1$. That is, we can write

$$h(x, 1, 1) = \lambda_0 h_0 + \lambda_1 h_1 + (1 - \lambda_0 - \lambda_1) h_2,$$

where $\lambda_i = \Pr(\tau_y = i | ND_y)$ and $h_i = E(y | \tau_y = i, ND_y)$ for $i = 0, 1$ and $i = 2$ represents the case of $\tau_y > 1$. ND_y stands for No-Disclosure of y (where x was disclosed at $t = 1$). Assume that $h'(x, 1, 1) = \beta_1$ then for both $\tau_y = 0$ and for $\tau_y = 1$ an increase in x increases both the disclosure threshold $h(x, 1, 1)$ and the expectation of y given no-disclosure, $E(y | \tau_y = i, ND_y)$, at a rate of β_1 . Therefore the probabilities λ_i are independent of x . In addition, for $i = 0, 1, 2$ we have $h'_i = \beta_1$. Computing $h'(x, 1, 1)$ and incorporating the fact that $\lambda'_i = 0$ for all i yields

$$h'(x, 1, 1) = \lambda_0 h'_0 + \lambda_1 h'_1 + (1 - \lambda_0 - \lambda_1) h'_2 = \beta_1.$$

The above only showed consistency of $h'(x, 1, 1) = \beta_1$. Following the same line of logic one can preclude any other value of $h'(x, 1, 1)$. For brevity, we omit this part of the proof.

QED Lemma 6

Finally, we analyze $h(x, 0, 1)$

Recall that Lemma 6 applies also to $h(x, 0, 1)$. However, for $h(x, 0, 1)$ we can show tighter bounds.

We first show that for the case where $h(x, 0, 1) < x$ we have $h'(x, 0, 1) = \beta_1$.

If $h(x, 0, 1) < x$ (the non-binding case) then when pricing the firm at $t = 1$ investors know that if the agent learned y (at either $t = 0$ or $t = 1$) then $y < h(x, 0, 1)$. If the agent did not learn y then investors use in their pricing $E(y|x) = \beta_1 x$. So, the beliefs about y are a weighted average of $E(y|y < h(x, 0, 1))$ and $E(y|x) = \beta_1 x$. This is similar to a Dye (1985) and Jung and Kwon (1988) setting and therefore, in equilibrium we have $h'(x, 0, 1) = \beta_1$.

Next we show that for x such that $h(x, 0, 1) > x$ (if such case exists) $h'(x, 0, 1) \in (2\beta_1 - 1, \beta_1)$.

The argument is similar to the one we made in the proof that $h'(x, 0, 0) \in (2\beta_1 - 1, \beta_1)$ for x such that $h(x, 0, 0) > x$. First note that for $h(x, 0, 1) > x$ investors' beliefs about y conditional on that the agent learned y are independent on whether he learned y at $t = 0$ or at $t = 1$. Moreover, given that $\tau_y \leq 1$ investors know that $y < x$. So from investors' perspective, it doesn't matter if the agent learned y at $t = 0$ or at $t = 1$. Their pricing, $h(x, 0, 1)$, will reflect a weighted average between $E(y|y < x)$ and $E(y|\tau_y > 1, x) = \beta_1 x$. From here on the proof is qualitatively the same as in the proof for $h'(x, 0, 0) \in (2\beta_1 - 1, \beta_1)$, where the only quantitative difference is the probability that the agent learned y .

QED Claim 1

Proof of Lemma 6

In this proof we use a slightly different notation, as part of the proof is more general than our setting.

Suppose that x and y have joint normal distribution and the agent is informed about y with probability p and uninformed with probability $1 - p$. Conditional on being informed the agent's disclosure strategy is assumed to be as follows: with probability λ_i , $i \in \{1, \dots, K\}$, he discloses if his type is above $z_i(x)$, where the various $z_i(x)$ are determined exogenously such that $z_i(x) \leq h(x, \{z_i(\cdot)\})$ for all i (which always holds in our setting). Note that $\sum_{i=1}^K \lambda_i = p$. Let's denote the conditional expectation of y given x and given the disclosure thresholds, $z_i(x)$, by $h(x, \{z_i(\cdot)\})$.

By applying Bayes rule, $h(x)$ is given by:

$$h(x, \{z_i(\cdot)\}) = \frac{(1-p) E[y|x] + \sum_{i=0}^K \lambda_i \int_{-\infty}^{z_i(x)} y \phi(y|x) dy}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x)}.$$

Taking the derivative of $h(x, \{z_i(\cdot)\})$ with respect to x and applying some algebraic manipulation (recall that $\frac{\partial E[y|x]}{\partial x} = \beta_1$) yields:

$$h'(x, \{z_i(\cdot)\}) = \beta_1 + \frac{\sum_{i=0}^K \lambda_i (z'_i(x) - \beta_1) \phi(z_i(x)|x) (z_i(x) - h(x, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x)}. \quad (5)$$

We start by proving the supremum of $h'(x)$.

Given that $z'_i(x) \geq 0$ and $(z_i(x) - h(x, \{z_i(\cdot)\})) \leq 0$ for all $i \in \{1, \dots, K\}$ we have

$$\begin{aligned} h'(x, \{z_i(\cdot)\}) &\leq \beta_1 + \frac{\beta_1 \sum_{i=0}^K \lambda_i \phi(z_i(x)|x) (h(x, \{z_i(\cdot)\}) - z_i(x))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x)} \\ &\leq \beta_1 + \max_{\substack{z_i \leq h(x) \\ i \in \{1, \dots, K\}}} \frac{\beta_1 \sum_{i=0}^K \lambda_i \phi(z_i|x) (h(x, \{z_i(\cdot)\}) - z_i)}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i|x)} \end{aligned}$$

Due to symmetry, for all $i \in \{1, \dots, K\}$ the maximum is achieved at $z_i(x) = z^*(x)$. To see this, note that the FOC of the maximization with respect to $z_i(x)$ is

$$\begin{aligned} 0 &= (\phi'(z_i(x)|x) (h(x, \{z_i(\cdot)\}) - z_i(x)) - \phi(z_i(x)|x)) \left((1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x) \right) \\ &\quad - \left(\sum_{i=1}^K \lambda_i \phi(z_i(x)|x) (h(x, \{z_i(\cdot)\}) - z_i(x)) \right) \phi(z_i(x)|x) \end{aligned}$$

Since $\phi'(z_i(x)|x) = -\alpha (z_i(x) - \beta_1 x) \phi(z_i(x)|x)$ (for some constant $\alpha > 0$), this simplifies to

$$-\alpha (z_i(x) - \beta_1 x) (h(x, \{z_i(\cdot)\}) - z_i(x)) = \frac{\sum_{i=0}^K \lambda_i \phi(z_i(x)|x) (h(x, \{z_i(\cdot)\}) - z_i(x))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x)} + 1$$

In the range $z_i(x) \leq h(x, \{z_i(\cdot)\}) \leq \beta_1 x$, the LHS is decreasing in $z_i(x)$.¹⁸ The RHS is the same for all i . Therefore, the unique solution to this system of FOC is for all $z_i(x)$ to be equal (and note that the maximum is achieved at an interior point since at $z_i(x) = h(x)$ the LHS is zero and the RHS is positive; and as $z_i(x)$ goes to $-\infty$ the LHS goes to $+\infty$ while the RHS is bounded).

Let $z^*(x)$ be the maximizing value. Then

$$\begin{aligned} h'(x, \{z_i(\cdot)\}) &\leq \beta_1 + \frac{\beta_1 \sum_{i=0}^K \lambda_i \phi(z^*(x) | x) (h(x, \{z_i(\cdot)\}) - z^*(x))}{(1-p) + p\Phi(z^*(x) | x)} \\ &= \beta_1 + \frac{p\beta_1 \phi(z^*(x) | x) (h(x, \{z_i(\cdot)\}) - z^*(x))}{(1-p) + p\Phi(z^*(x) | x)}. \end{aligned}$$

The right hand side of the above inequality is identical to the slope in a Dye setting with exogenous disclosure threshold with probability of being uninformed $(1-p)$ and an exogenously determined disclosure threshold of $z^*(x)$, where the disclosure threshold does not change in x . In such a setting, we can think of the effect of a marginal increase in x as the sum of two effects. The first effect is a shift by β_1 in both the distribution and the disclosure threshold. This will increase $h(x)$ by β_1 . The second effect is a decrease in the disclosure threshold by β_1 (as the disclosure threshold does not change in x). Since $z^*(x) < \beta_1 x$ we are in the decreasing part of the beliefs about y given no disclosure (to the left of the minimum beliefs). Therefore, the decrease in the disclosure threshold increases the beliefs about y by the change in the disclosure threshold times the slope of the beliefs about y given no disclosure. Since for $p < 0.95$ the slope of the beliefs about y given no disclosure is greater than -1 , the latter effect increases the beliefs about y by less than β_1 . The overall effect is therefore smaller than $2\beta_1$.

Next we prove the infimum of $h'(x)$.

Equation (5) capture a general case with any number of potential disclosure strategies. In our particular case $K = 1$ where $i = 0$ represents the case of $\tau_y = 0$ and $i = 1$ represents the case of $\tau_y = 1$. So, in our setting equation (5) can be written as

$$\begin{aligned} h'(x, \{z_i(\cdot)\}) &= \beta_1 + \frac{\lambda_0 (z'_0(x) - \beta_1) \phi(z_0(x) | x) (z_0(x) - h(x, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^1 \lambda_i \Phi(z_i(x) | x)} \\ &\quad + \frac{\lambda_1 (z'_1(x) - \beta_1) \phi(z_1(x) | x) (z_1(x) - h(x, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x) | x)} \end{aligned}$$

When calculating $h(x, 1, 1)$ and $h(x, 0, 0)$ in our setting, the disclosure threshold, $z_i(x)$, in any possible scenario (the binding and non-binding case for both $\tau_y = 0$ and $\tau_y = 1$) takes one of

¹⁸Since $z_i(x) \leq h(x, \{z_i(\cdot)\})$ also $h(x, \{z_i(\cdot)\}) \leq E[x|y] = \beta_1 x$.

the following three values: $h(x, \cdot, \cdot)$, x or x^* . Note that whenever $z_i(x) = h(x, \{z_i(\cdot)\})$ we have $\frac{(z'_i(x) - \beta_1) \phi(z_i(x)|x)(z_i(x) - h(x, \{z_i(\cdot)\}))}{(1-p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x)} = 0$.

For the remaining two cases ($z_i(x) = x$ and $z_i(x) = x^*$), for all $i \in \{0, 1\}$ we have $z'_i(x) \leq 1$ and $(z_i(x) - h(x, \{z_i(\cdot)\})) \leq 0$. This implies

$$h'(x) \geq \beta_1 - \frac{(1 - \beta_1) \sum_{i=0}^K \lambda_i \phi(z_i(x)|x) (h(x, \{z_i(\cdot)\}) - z_i(x))}{(1 - p) + \sum_{i=0}^K \lambda_i \Phi(z_i(x)|x)}.$$

Using the same symmetry argument for the first order condition as before, $h'(x, \{z_i(\cdot)\})$ is minimized for some $z^{\min}(x)$ and hence

$$h'(x, \{z_i(\cdot)\}) \geq \beta_1 + \frac{p(1 - \beta_1) \phi(z^{\min}(x)|x) (h(x, \{z_i(\cdot)\}) - z^{\min}(x))}{(1 - p) + p\Phi(z^{\min}(x)|x)}.$$

The right hand side of the above inequality is identical to the slope in a Dye setting with exogenous disclosure threshold in which: the probability of being uninformed is $(1 - p)$, the exogenously determined disclosure threshold is $z^{\min}(x)$ and $\frac{\partial}{\partial x} z^{\min}(x) = 1$. In such a setting, we can think of the effect of a marginal increase in x as the sum of two effects. The first is a shift by β_1 in both the distribution and the disclosure threshold. This will increase $h(x)$ by β_1 . The second effect is an increase in the disclosure threshold by $(1 - \beta_1)$ (as the disclosure threshold increases by 1). Since $z^{\min}(x) < \beta_1 x$ we are in the decreasing part of the beliefs about y given no disclosure (to the left of the minimum beliefs). Therefore, the increase in the disclosure threshold decreases the beliefs about y by the change in the disclosure threshold, $(1 - \beta_1)$, times the slope of the beliefs about y given no disclosure. Since for $p < 0.95$ the slope of the beliefs about y given no disclosure is greater than -1 the latter effect decreases the beliefs about y by less than $(1 - \beta_1)$. The overall effect is therefore greater than $\beta_1 - (1 - \beta_1) = 2\beta_1 - 1$.

QED Lemma 6

Proof of Lemma 5

We start by analyzing the partially informed agent, i.e., $(\tau_x = 0, \tau_y \neq 0)$ and then move to the fully informed agent.

Partially informed agent $(\tau_x = 0, \tau_y \neq 0)$

First note that for sufficiently low realizations of x the agent is always better off not disclosing it at $t = 0$, as they can “hide” behind uninformed agents. Next we establish that for sufficiently high realizations of the only private signal that the agent obtains at $t = 0$ he will disclose it at $t = 0$.

Lemma 7 Consider an agent that obtains a single signal x at $t = 0$. In the threshold equilibrium, the difference between the agent's expected payoff (as calculated at $t = 0$) from disclosing his signal at $t = 0$ and from not disclosing it at $t = 0$ is increasing in x . That is,

$$\frac{\partial}{\partial x} (E(U|\tau_x = 0, \tau_y \neq 0, t_x = 0) - E(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x \geq x_D)) > 0$$

and

$$\frac{\partial}{\partial x} (E(U|\tau_x = 0, \tau_y \neq 0, t_x = 0) - E(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x < x_D)) > 0.$$

Proof. We start by showing that the Proposition holds for the case where $x \geq x_D$ (i.e., $\beta_2(x + h(x, 1, 1)) \geq h(1)$). For simplicity of disposition, we partition the support of x into two cases: realizations of x for which $\beta_2(x + h(x, 1, 1)) \geq h(1)$ and for which $\beta_2(x + h(x, 1, 1)) < h(1)$.¹⁹

Case I - $\beta_2(x + h(x, 1, 1)) \geq h(1)$

Rewriting $E(U|\tau_x = 0, \tau_y \neq 0, t_x = 0, x) - E(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x \geq x_D)$ yields

$$\begin{aligned} & \beta_2 [x + h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)] - h(0) \\ & + p\beta_2 \left[\int_{y^*(x)}^{\infty} (y - h(x, 0, 1)) f(y|x) dy - \int_{y^1(x)}^{\infty} (y - h(x, 1, 1)) f(y|x) dy - \int_{y^H(x)}^{\infty} (h(y, 1, 1) - x) f(y|x) dy \right] \end{aligned}$$

The derivative of this expression with respect to x has the same sign as

$$D = 1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) + p[A + B + C]$$

where

$$\begin{aligned} A &= \frac{\partial}{\partial x} \int_{y^*(x)}^{\infty} (y - h(x, 0, 1)) f(y|x) dy \\ B &= -\frac{\partial}{\partial x} \int_{y^1(x)}^{\infty} (y - h(x, 1, 1)) f(y|x) dy \\ C &= -\frac{\partial}{\partial x} \int_{y^H(x)}^{\infty} (h(y, 1, 1) - x) f(y|x) dy. \end{aligned}$$

To evaluate this derivative we will use the following equations which are easy to obtain:

$$\begin{aligned} \frac{\partial}{\partial x} y^*(x) &= \frac{\partial}{\partial x} h(x, 0, 1), \\ \frac{\partial}{\partial x} f(y|x) &= -\beta_1 \frac{\partial}{\partial y} f(y|x), \\ \frac{\partial}{\partial x} (F(y(x)|x)) &= f(y(x)|x) \left(\frac{\partial}{\partial x} y(x) - \beta_1 \right). \end{aligned}$$

¹⁹Note that on the equilibrium path we are always in case I, i.e., $\beta_2(x + h(x, 1, 1)) \geq h(1)$.

Next we analyze the three terms A, B , and C . Note that the derivative with respect to the limits of integrals is zero for all cases because of the definition of the three cutoffs. Hence we get:

$$A = -\frac{\partial h(x, 0, 1)}{\partial x} (1 - F(y^*(x)|x)) - \beta_1 \int_{y^*(x)}^{\infty} (y - h(x, 0, 1)) \frac{\partial}{\partial y} f(y|x) dy.$$

Note that by integrating $\int_{y^*(x)}^{\infty} (y - h(x, 0, 1)) \frac{\partial}{\partial y} f(y|x) dy$ by parts w.r.t. y we got:

$$\begin{aligned} & \int_{y^*(x)}^{\infty} (y - h(x, 0, 1)) \frac{\partial}{\partial y} f(y|x) dy \\ &= -(y^*(x) - h(x, 0, 1)) f(y^*(x)|x) - \int_{y^*(x)}^{\infty} f(y|x) dy = -(1 - F(y^*(x)|x)). \end{aligned}$$

Plugging back to A we get

$$A = -\left(\frac{\partial h(x, 0, 1)}{\partial x} - \beta_1\right) (1 - F(y^*(x)|x)).$$

Next, we calculate B :

$$\begin{aligned} B &= \int_{y^1(x)}^{\infty} \frac{\partial h(x, 1, 1)}{\partial x} f(y|x) dy + \beta_1 \int_{y^1(x)}^{\infty} (y - h(x, 1, 1)) \frac{\partial}{\partial y} f(y|x) dy \\ &= \frac{\partial h(x, 1, 1)}{\partial x} (1 - F(y^1(x)|x)) - \beta_1 (1 - F(y^1(x)|x)) \\ &= \left(\frac{\partial h(x, 1, 1)}{\partial x} - \beta_1\right) (1 - F(y^1(x)|x)) \end{aligned}$$

Finally, we calculate C :

$$\begin{aligned} C &= (1 - F(y^H(x)|x)) + \beta_1 \int_{y^H(x)}^{\infty} (h(y, 1, 1) - x) \frac{\partial}{\partial y} f(y|x) dy \\ &= (1 - F(y^H(x)|x)) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \end{aligned}$$

Substituting A, B and C back to the whole derivative and re-arranging terms yields:

$$\begin{aligned} D &= 1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \\ &\quad - p \left[\begin{aligned} & \left(\frac{\partial h(x, 0, 1)}{\partial x} - \beta_1\right) (1 - F(y^*(x)|x)) + \left(\frac{\partial h(x, 1, 1)}{\partial x} - \beta_1\right) (1 - F(y^1(x)|x)) + \\ & (1 - F(y^H(x)|x)) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \end{aligned} \right] \\ &= (1 - p) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1))\right) \\ &\quad + p \left[\begin{aligned} & 1 + \frac{\partial h(x, 0, 0)}{\partial x} + \frac{\partial h(x, 0, 1)}{\partial x} F(y^*(x)|x) + \beta_1 (1 - F(y^*(x)|x)) - \frac{\partial h(x, 1, 1)}{\partial x} F(y^1(x)|x) \\ & - \beta_1 (1 - F(y^1(x)|x)) + 1 - F(y^H(x)|x) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \end{aligned} \right] \\ &= (1 - p) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1))\right) \\ &\quad + p \left[\begin{aligned} & 1 + \frac{\partial h(x, 0, 0)}{\partial x} + \frac{\partial h(x, 0, 1)}{\partial x} F(y^*(x)|x) - F(y^*(x)|x) \beta_1 - \frac{\partial h(x, 1, 1)}{\partial x} F(y^1(x)|x) \\ & + F(y^1(x)|x) \beta_1 + (1 - F(y^H(x)|x)) - \beta_1 \int_{y^H(x)}^{\infty} \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \end{aligned} \right] \end{aligned}$$

Rearranging yields:

$$\begin{aligned}
D &= (1-p) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \right) \\
&\quad + p \left[1 + \frac{\partial h(x, 0, 0)}{\partial x} + \left(\frac{\partial h(x, 0, 1)}{\partial x} - \beta_1 \right) F(y^*(x)|x) - \left(\frac{\partial h(x, 1, 1)}{\partial x} - \beta_1 \right) F(y^1(x)|x) \right] \\
&\quad + p\beta_1 \int_{y^H(x)}^{\infty} \frac{1}{\beta_1} - \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy
\end{aligned}$$

Since $\frac{\partial h(x, 0, 1)}{\partial x} \leq \beta_1$ (see Claim 1) and $F(y^1(x)|x) \geq F(y^*(x)|x)$ we have

$$\begin{aligned}
D &\geq (1-p) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \right) \\
&\quad + p \left[1 + \frac{\partial h(x, 0, 0)}{\partial x} + \left(\frac{\partial h(x, 0, 1)}{\partial x} - \frac{\partial h(x, 1, 1)}{\partial x} \right) F(y^1(x)|x) \right] \\
&\quad + p\beta_1 \int_{y^H(x)}^{\infty} \frac{1}{\beta_1} - \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \\
&= (1-p(1-F(y^1(x)|x))) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \right) + \\
&\quad + p(1-F(y^1(x)|x)) \left(1 + \frac{\partial h(x, 0, 0)}{\partial x} \right) + p\beta_1 \int_{y^H(x)}^{\infty} \frac{1}{\beta_1} - \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \\
&= (1-p(1-F(y^1(x)|x))) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \right) \\
&\quad + p \int_{y^1(x)}^{\infty} \left(1 + \frac{\partial h(x, 0, 0)}{\partial x} \right) f(y|x) dy + p \int_{y^H(x)}^{\infty} 1 - \beta_1 \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \\
&= (1-p(1-F(y^1(x)|x))) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \right) + \\
&\quad + p \int_{y^1(x)}^{y^H(x)} \left(1 + \frac{\partial h(x, 0, 0)}{\partial x} \right) f(y|x) dy + p \int_{y^H(x)}^{\infty} 2 + \frac{\partial h(x, 0, 0)}{\partial x} - \beta_1 \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy \\
&\geq (1-p(1-F(y^1(x)|x))) \left(1 + \frac{\partial}{\partial x} (h(x, 0, 0) + h(x, 0, 1) - h(x, 1, 1)) \right) \\
&\quad + p \int_{y^H(x)}^{\infty} 2 + \frac{\partial h(x, 0, 0)}{\partial x} - \beta_1 \frac{\partial h(y, 1, 1)}{\partial y} f(y|x) dy
\end{aligned}$$

So, the following are sufficient conditions to prove the Lemma for case I. For all x :

1. $\frac{\partial}{\partial x} h(x, 0, 0) + \frac{\partial}{\partial x} h(x, 0, 1) \geq \frac{\partial}{\partial x} h(x, 1, 1) - 1$
2. $\frac{\partial h(y, 1, 1)}{\partial y} \leq \left(2 + \frac{\partial h(x, 0, 0)}{\partial x} \right) \frac{1}{\beta_1}$ for any $y > x$

Case II - $x < x_D$ (i.e., $\beta_2(x + h(x, 1, 1)) < h(1)$)

The analysis of Case I was for generic bounds of the integrals $h(x, 0, 1)$ and $y^H(x)$. The difference between Case I and Case II is that the price given no disclosure of y (if the agent does

not obtain a signal y or obtains a low realization of y) is $h(1)$ in Case II and $\beta_2(x + h(x, 1, 1))$ in Case I. This causes the expected payoff of the agent in Case II to be less sensitive to x than in Case I, and therefore $\frac{\partial}{\partial x}(E(U|\tau_x = 0, \tau_y \neq 0, t_x = 0) - E(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x \geq x_D)) > 0$ implies that also $\frac{\partial}{\partial x}(E(U|\tau_x = 0, \tau_y \neq 0, t_x = 0) - E(U|\tau_x = 0, \tau_y \neq 0, t_x \neq 0, x < x_D)) > 0$.

To summarize, conditions 1 and 2 above are sufficient for both cases and therefore for the Proposition as a whole.

Claim 1 shows that condition 2 above holds.

So, it is only left to show that condition 1 holds. Since $\beta_1 > \frac{1}{2}$, the LHS of condition 1 is greater than $2(2\beta_1 - 1) > 0$ and the RHS is less than $2\beta_1 - 1$. Therefore the condition holds.²⁰

■

Fully informed agent ($\tau_x = \tau_y = 0$)

The only case we still haven't analyzed is the case of a fully informed agent that learns both signals at $t = 0$ ($\tau_x = \tau_y = 0$) and y is sufficiently low such that it will not be disclosed.

The analysis bellow shows that such an agent whose signal x is sufficiently high will disclose at least one signal at $t = 0$. In particular, for low realizations of y (such that y will not be disclosed also at $t = 1$) if x is sufficiently high the agent will disclose it at $t = 0$.

Claim 2 *Assume an agent that learned both signals at $t = 0$ and the realization of y is such that he does not disclose y . For sufficiently high realizations of x the agent prefers to disclose x at $t = 0$ over not disclosing x at $t = 0$.*

Proof. We need to show that

$$\beta_2[x + h(x, 0, 0)] + \beta_2[x + h(x, 0, 1)] > h(0) + \beta_2[x + h(x, 1, 1)].$$

Rearranging yields

$$\beta_2[x + h(x, 0, 1)] - h(0) > \beta_2[h(x, 1, 1) - h(x, 0, 0)].$$

Since $h(x, 0, 1)$ is not decreasing for sufficiently high x the LHS of the above inequality, $\beta_2[x + h(x, 0, 1)] - h(0)$, goes to infinity as x goes to infinity. Therefore, it is sufficient to show that $h(x, 1, 1) - h(x, 0, 0)$ is bounded. Both $h(x, 1, 1)$ and $h(x, 0, 0)$ are lower than $\beta_2 x$. From the minimum principle we know that $h(x, 0, 0)$ is higher than the price given no disclosure in a Dye (1985), Jung

²⁰We conjecture that the condition holds also for $\beta_1 < 0.5$, however, we have not yet been able to prove that.

and Kwon (1988) setting where $y \sim N(\beta_1 x, \text{Var}(y|x))$. The price given no disclosure in such a setting is $\beta_1 x - \text{Cons}$, so $h(x, 0, 0) > \beta_1 x - \text{Cons}$. Hence, given that $h(x, 1, 1) < \beta_1 x$ we have $h(x, 1, 1) - h(x, 0, 0) < \text{Cons}$. QED ■

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