On the Optimality of Participatory Budgeting

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Abstract

This paper complements the ongoing empirical discussion surrounding participative budgeting by comparing its economic merits relative to a top-down budgeting alternative. We analyze a screening model of participative budgeting where the subordinate uses the budgeting mechanism to relay private information to his superior, in addition to a signaling model of top-down budgeting where the superior similarly relays private information to the subordinate. In the presence of sufficient ex-ante environmental uncertainty, private interim information availability or both, we find that participative budgeting dominates the more centralized, top-down budgeting counterpart. Contrary to common belief, we find that the agency costs associated with participative budgeting largely persist under top-down budgeting. Consequently, the superior's payoff peaks at boundary levels of informational asymmetry whereas the subordinate's payoff displays unimodality over information asymmetry, regardless of the budgeting mechanism in use. Lastly, we provide empirical interpretations of our results relating the firm's choice of budgeting mechanism to budget-emphasis, budgetary slack and incentive strength.

Keywords: *Participative budgeting theory, Top-down budgeting, Signaling versus Screening.*

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1   Introduction

A key element of any firm's organizational design is the choice of budgeting system with which information is communicated. Consistent with the decentralization trend over the past two decades (Rajan and Wulf (2006) and Roberts (2004)), recent surveys suggest that firms are increasingly engaged in participative, or so-called "bottom-up" budgeting, wherein senior-managers solicit information from lower levels of the firm (Stout and Shastri (2008)). However, transitioning to such systems can be costly, since as Baiman and Evans III (1983) warn, soliciting subordinates' private information requires the provision of informational rents. In this paper we propose a stylized model of the firm's budgeting decision, providing conditions under which participative budgeting dominates traditional, "top-down" budgeting where superiors communicate private information to subordinates. Budgeting research to date has primarily studied participative budgeting empirically, having largely focused on how firm attributes such as performance (Brownell (1982), Frucot and Shearon (1991)), budgetary slack (Dunk (1993)) and budget emphasis (Young (1985)) vary with bottom-up adoption. Accounting literature surveys including Shields and Shields (1998), Shields and Young (1993) and Brown et al. (2009) have concluded that the evidence to date is mixed. As Shields and Shields (1998) point out: "Studies have reported, for example, that participative budgeting has linear positive, linear negative, ordinal and disordinal interaction (with other independent or moderating variables), and no effect on motivation and performance." By analyzing a formal model of the firm's budgeting process, we provide an economic rationale for the firm's choice of budgeting mode and provide a possible explanation for the mixed empirical results to date.

We model a firm who jointly optimizes all organizational design decisions, including the choice of budgeting paradigm, in response to the underlying information structure: ex-ante environmental uncertainty and the availability of private, interim information. Private interim information can be procured by either a superior (the "principal") *or* her subordinate (the "agent"), as we assume that requiring both to acquire the same information would

prove excessively costly. Costly information gathering may include client visits, working with marketing consultants, or inspecting off-site facilities to acquire interim information such as local or future demand estimates, or the likelihood of supply-chain delays. In each of these instances, the acquired information is privately observed and unverifiable; i.e., the information is soft. Interim information serves two purposes: the agent uses it to choose an appropriate level of costly effort, and the principal uses it to better structure the agent's compensation. Consistent with practice (Parker and Kyj (2006)), budgeting enables private information to flow vertically within the firm. Our primary contribution characterizes the optimal information flow: from principal to agent (top-down budgeting) or from agent to principal (bottom-up budgeting).

We find that top-down budgeting outperforms bottom-up budgeting when either there is insufficient interim information available or the firm's range of ex-ante environmental uncertainty is sufficiently small. In such settings, the principal optimally retains all information gathering responsibilities and later signals her findings to the agent. Absent these conditions, the principal optimally delegates information gathering and reporting duties to the agent. Because the agent can misreport private information under bottom-up budgeting, the principal must pay the agent informational rents to solicit truthful reporting. The agent earns rents under top-down budgeting as well, because the principal herself may choose to misreport private information which she communicates to the agent.

To motivate credible communication the budgeting mechanism must undermine the informed party's payoff to misreporting. Under bottom-up budgeting, the optimal contract provides the agent rents to prevent him from unanimously reporting unfavorable news– his preferred information state–while the optimal top-down contract pays the agent rents to prevent the principal from unanimously reporting favorable news–her preferred state. This distinction leads to production inefficiencies in unproductive (productive) states under bottom-up (top-down) budgeting. Because the agent's productivity is unknown when he chooses effort, additional ex-ante environmental uncertainty or additional interim informa-

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tion both drive up (down) the cost of inefficient production in favorable (unfavorable) states. While inefficiencies are always less costly in unfavorable states,[1] bottom-up budgeting may still incur greater agency costs than top-down paradigms due to the presence of additional "control losses." The principal incurs additional control losses under bottom-up budgeting because the agent can coordinate his misreporting with his effort, whereas the principal encounters no such opportunity under top-down budgeting. When there is relatively little environmental uncertainty or available interim information, the principal favors top-down budgeting, because the resulting inefficient production in favorable states proves less costly than the control loss associated with participative budgeting and the associated inefficiencies in the unfavorable state. However, as the cost of inefficient production increases with either additional environmental uncertainty or interim information availability, the difference in cost of inefficient production between the favorable and unfavorable states begins to outweigh the costs associated with control loss, eventually causing the principal to favor bottom-up budgeting.

In spite of the inherent control loss associated with bottom-up budgeting, the principal's and agent's payoffs over the level of interim information and environmental uncertainty remain *qualitatively unchanged* over the two budgeting alternatives. These results suggest that many of the concerns surrounding bottom-up budgeting are equally valid in a top-down construct. More generally, our results demonstrate that the tensions encountered when a privately informed party selects hidden effort persist in certain settings featuring asymmetric information and moral hazard separately. These same two paradigms have both been thoroughly studied in the Economics literature. Bottom-up budgeting shares many common features with the adverse-selection model introduced in Baron and Myerson (1982). In particular, the principal contractually screens the agent's private information. In contrast, under top-down budgeting, the principal herself signals information to the agent, much like

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[1]A price discriminating monopolist faces a similar decision in choosing whether to impose inefficiencies on high- versus low-willingness to pay consumers. As in the classic monopoly model (Maskin and Riley (1984)), the monopolist will generally screen customers by distorting the bundle sold to the low-value customers.

the informed principal model from Maskin and Tirole (1990). An excellent survey of both the signaling and screening literatures can be found in Riley (2001). Surprisingly little research has explicitly compared the two communication mediums or more generally, the firm's internal allocation of private information. One notable exception is Eso and Szentes (2003), where the principal cannot decipher the favorableness of information, but nonetheless must decide how much information to share with the agent at the onset of the game and how much to reveal based on the agent's interim report. Our work differs because the principal in our model can interpret the meaning of interim information. Therefore our top-down setting features an additional tension; namely the principal's incentive to strategically distort her reporting.

To characterize the firm's optimal budgeting decision, we compare the principal's payoff under each regime while holding fixed the level of available interim information and ex-ante environmental uncertainty. The Information Systems literature, beginning with Antle and Fellingham (1995), has already analyzed information preferences within an adverse-selection paradigm; though no work to date has studied such preferences as the recipient of private information varies. To parameterize information quantity, we borrow from Rajan and Saouma (2006) who find that the principal's and agent's preferences vary non-monotonically over (exclusively) the agent's allocation of private information. Similar informational preferences arise in Arya et al. (1997), where the principal can contribute to output, though she cannot commit to her contribution level. Therein, the authors find that information system improvements can aggravate the principal's commitment problem which ultimately reduce her expected profits. In our top-down model, the principal also holds private information, though she cannot commit to truthfully disclosing it. Although the setups are different, our findings coincide with Arya et al. (1997) in that introducing additional information may lower the principal's payoff.

Most importantly, our model addresses the extant empirical budgeting research. Brown et al. (2009) provide a thorough survey of the budgeting literature, both analytical and em-

pirical. The majority of the empirical studies have focused on the consequences of bottom-up budgeting adoptions, including slack (Dunk (1993), Fisher et al. (2002)), incentive contracts (Young (1985)) and firm performance (Brownell (1982), Frucot and Shearon (1991)). A smaller subset of the empirical literature has centered on the identification of antecedents to participative budgeting. Indeed, both Shields and Shields (1998) and Shields and Young (1993) argue that the mixed empirical evidence may have resulted from researchers commingling the antecedents and consequences of participative budgeting. In response, our model treats interim information availability and environmental uncertainty as the sole determinants of the firm's budgeting choice, therefore our discussion on the consequences of participative budgeting centers exclusively on endogenous firm attributes.

We present the model below in Section 2. Section 3 studies the optimal contracts under both top-down and bottom-up budgeting, whereas Section 4 characterizes the relative attributes of each mode. Section 5 discusses the results and concludes.

2 Model

We model a risk-neutral principal (she) and an agent (he) who will incur either high-, θ_H, or low-, θ_L, productivity in the upcoming period with probability $1 > p > 0$ and $1 - p$ respectively, where $\theta_H > \theta_L > 0$ and $\mathrm{E}[\theta] = \bar{\theta}$. We denote the ex-ante range in the firm's profitability, or so-called environmental uncertainty, with $j = \frac{\theta_H}{\theta_L}$. The agent's productivity, θ, denotes the efficiency of his private effort, $e \geq 0$ in generating profits, $\pi = \theta e + \varepsilon$ where ε is mean zero, idiosyncratic noise with finite variance. The agent's private cost of effort is given by $T\frac{e^2}{2}$, with $T > 0$. The agent's cost of effort, T, the likelihood of high productivity, p, and the underlying productivity support, $\{\theta_H, \theta_L\}$, are all common knowledge, albeit the true productivity, θ, is unknown to everyone.

Both the principal and agent are equally capable of collecting interim information,[2]

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[2]We use the terms "collecting information" and "research" synonymously, as well as "informed party" and "researcher".

$\sigma \in \{\sigma_H, \sigma_L\}$, to estimate the agent's forthcoming productivity, θ. To capture the cost of gathering information, such as meeting with clients or visiting production facilities, we assume that the firm can only justify having either the principal *or* agent gather information. The decision as to who ought to collect information is formally made at the outset of the game when the agent is hired at time $t = 0$. The *act* of collecting information, while costly to the researcher, is contractible and thus not subject to moral hazard. For example, visiting a client or touring a factory are both costly though contractible actions; i.e., the researcher can seek reimbursement for any associated costs. The researcher communicates their signal via the budgeting mechanism at the budgeting stage: when the principal reports her findings, we label the process top-down budgeting, whereas when the agent reports, the process is labeled bottom-up or participative budgeting. Thus, the budgeting mode is effectively chosen at the outset, $t = 0$, when the principal outlines information gathering responsibilities. Interim information availability, and the accuracy with which it predicts future productivity are exogenously determined by the nature of the firm's business. To capture both facets, we define, $a \in [0, 1]$, as the level of interim information available, though one may also interpret a as the quality of available interim information. Without loss of generality, the acquired signal σ fully reflects the totality of available information, a. Given a signal, σ_i with $i = H, L$, we label the expected productivity, $\hat{\theta}_i(a) = \mathrm{E}[\theta|\sigma_i; a]$, though for compactness we refer to the expected productivity simply as $\hat{\theta}_i$. To operationalize the level of information available, let:

$$Pr[\theta_i|\sigma_i] = a + (1 - a) \cdot Pr[\theta_i] \quad i = H, L \quad a \in [0, 1]$$

$$Pr[\theta_{-i}|\sigma_i] = (1 - a) \cdot Pr[\theta_{-i}] \quad i = H, L \quad a \in [0, 1].$$

In accordance with the conditional probabilities above, the technology is said to be "productive" or "less productive" upon uncovering a signal $\sigma = \sigma_H$ ("favorable news") or $\sigma = \sigma_L$ ("unfavorable news") respectively. Note that the level of information, a, formally measures the correlation between the signal, σ, and the agent's true productivity, θ. As $a \to 1$, our set-

ting approaches a perfectly-informed paradigm, such as that found in the adverse selection and informed principal literature, while as $a \to 0$, all members of the firm become symmetrically (un)informed, regardless of who collects information. Using the parameterization above, the informed party's signal-contingent expectation of $\hat{\theta}_i$, can be seen in figure 1 as a function of a.

Whereas the level of information collected, a, measures the fraction of uncertainty resolved, it does not capture the nominal uncertainty surrounding the agent's productivity, or equivalently, the firm's ex-ante environmental uncertainty, $j = \frac{\theta_H}{\theta_L}$. For the purpose of our analysis, we only consider variations in environmental uncertainty, j, resulting from linear-mean preserving spreads of the supports, θ_H and θ_L.[3] We do so both for reasons of tractability (linear) and to avoid commingling information effects with productive effects (mean preserving). To measure the information asymmetry, or equivalently, the informativeness of the acquired signal, σ, a metric must consider both the environmental uncertainty, j *and* the level of interim information, a. Since the agent bases his effort on the conditional productivity, $\hat{\theta}_i$, the ratio of high- to low-conditional productivity provides a natural proxy for signal informativeness. From figure 1, the ratio $\frac{\hat{\theta}_H}{\hat{\theta}_L}$ (signal informativeness) increases both in the level of interim information, a, and the underlying ex-ante uncertainty, j.[4] Importantly, we ignore changes in signal informativeness resulting from changes in the likelihood of the productive state, p, because, by construction, varying p will simultaneously affect the unconditional mean-productivity, $\overline{\theta}$, preventing us from delineating production effects from information effects.

Following the prior agency literature, the principal holds the bargaining power: at time $t = 0$, she offers the agent a take-it-or-leave-it contract menu which specifies the budgeting mode and timing thereafter. Under top-down (bottom-up) budgeting, the principal (agent) engages in research and obtains σ at time $t = 1$. At time $t = 2$, a budgeting meeting takes

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[3]In our setting, any generalized mean preserving spread can be attained via a linear mapping.

[4]For the duration of our analysis, references to increases (decreases) in signal informativeness or equivalently, the level of information asymmetry, can be interpreted as either increases (decreases) in the level of interim information, a, the level of ex-ante environmental uncertainty, j, or both.
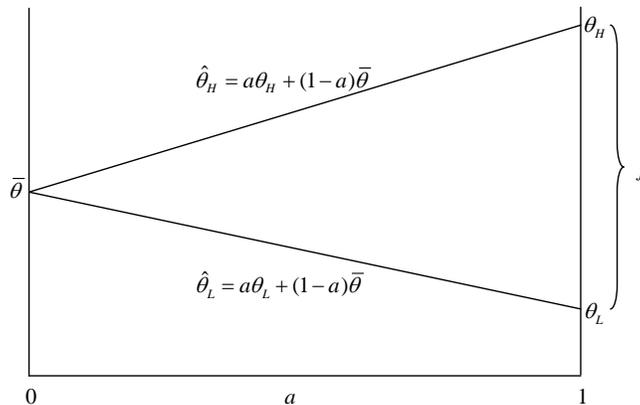
Figure 1: Following the receipt of a signal, σ_i, the conditional expectation, $\hat{\theta}_i$, becomes increasingly distinct from the unconditional expectation, $\bar{\theta}$, as signal informativeness increases due to either increases in the level of interim information, a, the ex-ante environmental uncertainty, j, or both.

place where the informed party publicizes their findings, σ. To credibly communicate the interim signal, σ, the budgeting mechanism mandates that, at $t = 2$, the party charged with collecting information selects a contract from the menu proposed at $t = 0$. The individual contracts take the form of (α_i, β_i) with $i \in \{H, L\}$ and $0 \leq \beta_i \leq 1$, where α_i denotes a fixed salary concession from the agent in exchange for a $(1 - \beta_i)$ share of the $t = 4$ realized profits, π. The principal's residual profits are thus $\beta_i \pi + \alpha_i$.[5]

Without loss of generality, we set the contractible cost of information acquisition to zero.[6] The Revelation Principle allows us to restrict attention to menus with only two individual contracts: contract i is selected if and only if $\sigma = \sigma_i$, though the $t = 0$ menu must satisfy incentive compatibility constraints to prevent misreporting. By virtue of proposing the contract, we assume that the principal must honor the terms she offered at $t = 0$ throughout, though the agent's bargaining position entitles him to leave the firm at any time.[7] In

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[5]While we only consider linear profit sharing contracts, if both the principal and agent could agree on the exact distribution of the noise, ε, or if $\varepsilon \equiv 0$, then the firm could augment profits via a non-linear contract. We thank Ron Dye for raising the possibility of perfect ex-post measures ($\varepsilon \equiv 0$). For a discussion of perfect versus imperfect contracting measures with private information, see Rajan and Saouma (2006).

[6]For a discussion on costly research subject to moral hazard, see Lewis and Sappington (1997).

[7]In equilibrium, both the principal and agent always favor the original contract over their reservation payoffs upon learning or acquiring information. Below, we show that the principal's interim individual rationality constraint is always non-binding, though the agent will always have at least one rationality constraint bind.

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particular, the game ends should the agent leave the firm upon learning σ either directly at $t = 1$ under bottom-up budgeting, or indirectly at $t = 2$ under top-down budgeting. We assume the agent faces sufficiently limited liability to prevent him from buying a full claim to profits from the principal at time $t = 0$; specifically, the principal cannot "sell the firm" prior to information acquisition. However, such a sale is assumed to be feasible once the interim information, σ, has surfaced, as the resulting endogenous liability is significantly smaller.

If the agent stays with the firm beyond $t = 2$, at $t = 3$ he chooses his effort, denoted e_i, to maximize his utility given the contract $\{\alpha_i, \beta_i\}$ specified in the budgeting process. At time $t = 4$, profits, π, are realized, and both the principal and agent receive their payoffs in accordance with the time-line shown in figure 2.

Top-Down Budgeting

Principal acquires information σ_i — Principal selects (α_i, β_i) agent accepts or rejects

$t = 0$ — Principal proposes $\{(\alpha_L, \beta_L), (\alpha_H, \beta_H)\}$ agent accepts or rejects

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3 — Agent selects hidden effort e_i

4 — π realized agent paid $(1 - \beta_i)\pi - \alpha_i$

Agent acquires information σ_i — Agent selects (α_i, β_i) or rejects
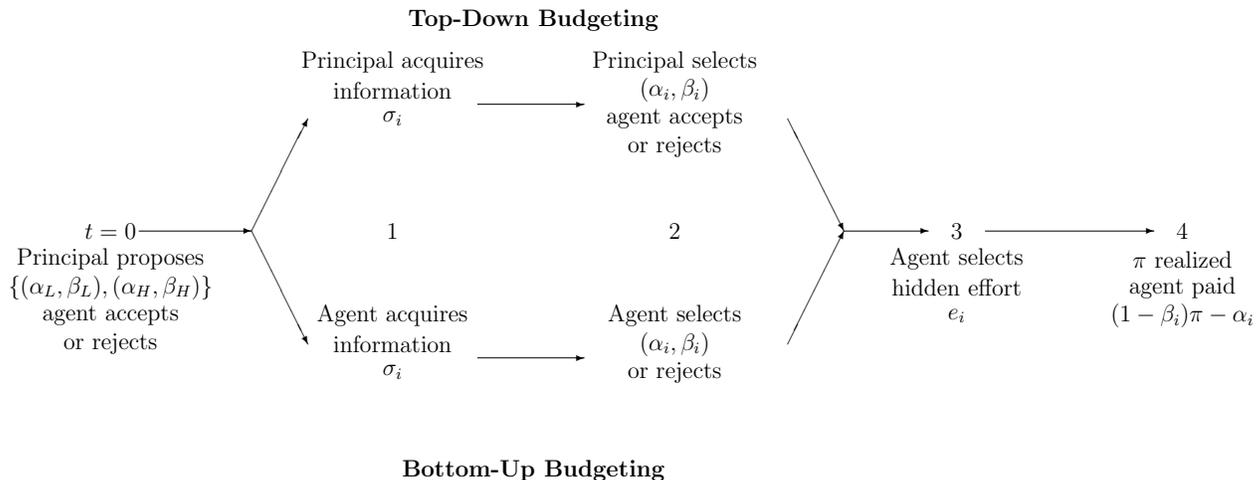
Bottom-Up Budgeting

Figure 2: The top path features the events specific to top-down budgeting, the bottom path features the events specific to bottom-up budgeting, and the middle path contains all the steps shared by both paradigms.

In addition to publicizing private information, the budgeting mechanism effectively imposes a renegotiation of the agent's compensation at $t = 2$, though the terms of renegotiation are limited to the individual contracts agreed upon at $t = 0$. In practice, Fisher et al. (2000) show that budgeting typically involves contract renegotiation as the agent's targets are reestablished. Therefore one may interpret the principal's choice of budgeting mode as

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both a task assignment–who is to collect information–and a commitment device–how to limit renegotiation. However, as the next proposition shows, the choice of budgeting mode has no impact on the principal's payoffs when the principal and agent can contract on the latter's choice of effort.

Lemma 1. *When the agent's effort is contractible, the principal is indifferent between top-down and bottom-up budgeting. Under both modes, the principal instructs the agent to exert first-best effort $e_i^{FB} = \frac{\hat{\theta}_i}{T} \in e^{FB} = \{\frac{\hat{\theta}_H}{T}, \frac{\hat{\theta}_L}{T}\}$ and pays him an effort-contingent wage of $\frac{\hat{\theta}_i^2}{2}T$ following a report $\sigma_i \in \{\sigma_H, \sigma_L\}$. The principal's expected profits are increasing and convex in signal informativeness, whereas the agent always earns his reservation utility.*

The benchmark setting of Lemma 1 highlights the fact that, absent hidden effort, both the principal and agent are entirely indifferent over the choice of budgeting mode. In this setting with a fixed level of ex-ante environmental uncertainty, increases in interim information allow the agent to more efficiently select his level of effort, e_i, resulting in greater profits. Similarly, holding fixed the level of interim information, increasing the level of ex-ante environmental uncertainty raises and lowers the expected profits following a signal σ_H and σ_L respectively. Expected profits rise nonetheless, because the agent exerts greater effort in favorable states than in unfavorable states; i.e, $e_H > e_L$, implying that the marginal profits resulting from an increase in $\hat{\theta}_H$ overpower the negative marginal profits associated with a decrease in $\hat{\theta}_L$, or equivalently, $\left|\frac{d\pi}{d\hat{\theta}_H}\right|_{e=e^{FB}} > \left|\frac{d\pi}{d\hat{\theta}_L}\right|_{e=e^{FB}}$. Profits are convex, because the difference between the two effort levels also widens as $\hat{\theta}_H$ and $\hat{\theta}_L$ diverge from one another with additional signal informativeness.

3   Hidden Effort

As per Lemma 1, if the choice of budgeting mode is to carry any substantive consequences, the agent's effort e must be non-contractible, which we assume hereafter. We define a budgeting equilibrium with moral-hazard as an outcome where:

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i. The informed party maximizes its payoffs by truthfully communicating their interim signal.

ii. The uninformed party correctly infers the signal from their opponent's contract choice.[8]

iii. The agent always selects his payoff maximizing effort.

iv. The agent's interim $(t = 3)$ expected payoffs are non-negative.

v. The uninformed party is free to choose a contract from the original $(t = 0)$ menu if the informed party fails to do so at $t = 2$.

The Revelation Principle guarantees that the first equilibrium requirement is without loss of generality. Conditions (ii)-(iv) imply that both the principal and agent are rational, while condition (v) specifies the necessary off-equilibrium beliefs required to uphold *any* equilibrium with communication.

3.1   Top-Down Budgeting

We first consider the case where the principal collects interim information in accordance with the top-down budgeting paradigm featured in figure 2. The principal thus solves:

$$\max_{\alpha_i,\beta_i} \quad p(\beta_H e_H \hat{\theta}_H + \alpha_H) + (1-p)(\beta_L e_L \hat{\theta}_L + \alpha_L)$$

$$\text{s.t.} \quad \beta_H e_H \hat{\theta}_H + \alpha_H \geq \beta_L e_L \hat{\theta}_H + \alpha_L \tag{1}$$

$$\beta_L e_L \hat{\theta}_L + \alpha_L \geq \beta_H e_H \hat{\theta}_L + \alpha_H \tag{2}$$

$$\beta_i e_i \hat{\theta}_i + \alpha_i \geq 0 \qquad i = H, L \tag{3}$$

$$(1-\beta_i)e_i\hat{\theta}_i - \frac{e_i^2}{2}T - \alpha_i \geq 0 \qquad i = H, L \tag{4}$$

$$e_i \in \arg\max_e \quad (1-\beta_i)\hat{\theta}_i e - \alpha_i - \frac{e^2}{2}T \quad i = H, L \tag{5}$$

$$0 \leq \beta_i \leq 1 \qquad i \in \{H, L\}. \tag{6}$$

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[8]If the signal is completely uninformative $(a = 0)$ then the contract menu consists of two identical contracts and no information is revealed.

Constraints (1) and (2) ensure that the principal truthfully reveals her observed signal to the agent. The individual rationality constraints (3) and (4) guarantee that the principal prefers the contract menu to her reservation payoff at $t = 0$, and that the agent favors the contract over his reservation utility at time $t = 2$ (both set to zero). Finally, (5) characterizes the agent's optimal effort, and (6) prevents the principal from leveraging future profits. Solving (5), the agent puts forth effort $e_i = \frac{(1-\beta_i)}{T}\hat{\theta}_i$ in response to a report of σ_i. Compared with the benchmark setting of Lemma 1, top-down budgeting with moral-hazard causes the agent to distort his effort as his profit-share, $(1 - \beta_i)$, tends away from 1. In other words, any profit share, $1 - \beta_i < 1$, induces effort distortions which lower the total expected output. The principal can avoid effort distortions should she sell the agent the firm's entire stream of profits $(1 - \beta_i = 1)$ at time $t = 2$, however the agent will never accept the principal's proposed price. To see why, notice that the agent's valuation of the firm at time $t = 2$ is entirely contingent on the principal's *reported* signal. If the principal's price is independent of her *observed* signal–as is the case when she holds no stake in the firm's (later) realized profits–then she will always claim to have observed favorable information and the agent will not believe her. As the next proposition shows, the optimal contract sells the agent a claim to all firm profits when the principal announces an unfavorable signal and sells the agent a fraction of the firm's profits when the principal reports favorable news. In the latter case, the rate at which the principal exchanges profit shares for (the agent's) salary concessions is based on the principal's payoff to misreporting, which itself can be traced to signal informativeness.

Proposition 1. *The optimal menu of contracts under top-down budgeting is given by:*

$$\alpha_L = \frac{\hat{\theta}_L^2}{2T} \qquad \alpha_H = \begin{cases} \frac{\hat{\theta}_L\left(2\hat{\theta}_L - \hat{\theta}_H\right)}{4T} & if\ \frac{\hat{\theta}_H}{\hat{\theta}_L} \geq \sqrt{5} - 1 \\ \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} & if\ \frac{\hat{\theta}_H}{\hat{\theta}_L} \in (1, \sqrt{5} - 1) \end{cases}$$

$$\beta_L = 0 \qquad \beta_H = \begin{cases} \frac{1}{2} & if\ \frac{\hat{\theta}_H}{\hat{\theta}_L} \geq \sqrt{5} - 1 \\ \frac{\hat{\theta}_H^2 - \hat{\theta}_H\hat{\theta}_L - \sqrt{2\hat{\theta}_H\hat{\theta}_L^2\left(\hat{\theta}_H - \hat{\theta}_L\right)}}{\hat{\theta}_H^2 - 2\hat{\theta}_H\hat{\theta}_L} & if\ \frac{\hat{\theta}_H}{\hat{\theta}_L} \in (1, \sqrt{5} - 1) \end{cases}.$$

When the principal reports unfavorable news, σ_L, she concedes that the firm's value cannot be lower. Accordingly, the agent accepts to purchase the firm's entire stream of profits and therefore carries out the surplus-maximizing (benchmark) level of effort, e_L^{FB}. The agent's utility will then be given by the benchmark surplus conditional on unfavorable information $\left(e_L^{FB}\hat{\theta}_L - c\left(e_L^{FB}\right)\right)$, net of the price paid to the principal. The proposition above shows that the principal optimally charges the agent the entire benchmark surplus, leaving the agent with his reservation utility of zero.

Unfortunately, the same mechanism is no longer credible when the principal reports favorable news, because the agent will refuse to pay more than the surplus obtained in the unfavorable state, $\left(e_L^{FB}\hat{\theta}_L - c\left(e_L^{FB}\right)\right)$, when the principal's payoff is independent of the firm's realized profits. Instead, Proposition 1 shows that upon observing favorable news, the principal optimally sells the agent a portion of the firm's profits in exchange for salary concessions.

To understand the tradeoffs involved, note that misreporting $\sigma = \sigma_L$ as σ_H misleads the agent into believing that the firm's profits will be higher than conditionally warranted. Therefore, if the agent erroneously believes that the principal obtained a signal σ_H, the agent will exert excessive effort and accept a compensation package worth less than advertised. While the principal benefits from selling the agent an over-valued share of the firm's profits, her misreporting payoff is not without cost, because she must reimburse the agent for exerting excessive effort to satisfy his individual rationality constraint (4). The net payoff to the

principal's rouse traces back to to signal informativeness, $\frac{\hat{\theta}_H}{\hat{\theta}_L}$, which we examine next.

When the principal's signal is relatively uninformative, the difference between correctly- and over-valued firm profits is relatively low. As such, the agent has relatively little to lose from accepting an over-valued share of profits and consequently, the principal has little to gain from misreporting. To mitigate her misreporting payoff, the principal limits the total surplus generated upon announcing a favorable signal by decreasing the agent's share of realized profits, $1 - \beta_H$. In this setting, the principal optimally demands a salary concession from the agent which leaves him with zero rents; i.e. the principal extracts the entire surplus generated. As the informativeness of the principal's signal increases, so does the difference between correctly- and over-valued firm profits, therefore the principal must use increasingly large effort distortions (decreasing $1 - \beta_H$) to credibly communicate her signal. Consequently, the level of surplus destroyed increases with signal informativeness herein. For more elevated levels of signal informativeness, combating additional informativeness with additional effort distortions leads to excessive surplus destruction. Therefore, when the principal's signal is sufficiently informative, the optimal contract menu sustains the principal's truth-telling constraints in the face of additional informativeness by paying the agent rents following favorable news rather than inducing further distortions and extracting all the surplus from the agent. In other words, the principal's and agent's profit shares (β_i) remain constant and the agent begins to earn rents once the signal becomes sufficiently informative (see figure 3).

3.2   Bottom-Up Budgeting

In accordance with the time-line in figure 2, bottom-up budgeting charges the agent with gathering and reporting information in addition to his effort. While we found that in a top-down setting, the principal may benefit from misreporting unfavorable information as favorable, the contrary is true when the agent reports to the principal under bottom-up budgeting. By erroneously reporting σ_L, the agent causes the principal to undervalue the firm's conditional expected profits; therefore, from the principal's perspective, any profit
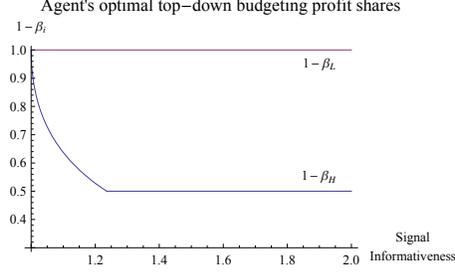
Figure 3: Upon announcing the receipt of the favorable signal, σ_H, the optimal top-down budgeting contract calls for the principal to sell the agent a decreasing share $(1 - \beta_H)$ of the firm profits when signal informativeness is relatively low, and a fixed share once the signal is sufficiently informative. In contrast, the agent is optimally sold the entire stream of firm profits regardless of signal informativeness when the principal reports an unfavorable signal, σ_L.

sharing $(1 - \beta_L) > 0$ will overly compensate the agent relative to his salary concessions. Akin to the principal's problem under top-down budgeting, the optimal bottom-up contract must limit the agent's payoff to erroneously reporting unfavorable news while simultaneously maximizing the principal's expected payoff; i.e., the principal solves:

$$\max_{\alpha_i, \beta_i} \quad p(\alpha_H + \beta_H e_{HH}\hat{\theta}_H) + (1 - p)(\alpha_L + \beta_L e_{LL}\hat{\theta}_L)$$

$$s.t. \quad \alpha_i + \beta_i e_{ii}\hat{\theta}_i \geq 0 \qquad i \in \{H, L\} \tag{7}$$

$$(1 - \beta_i)e_{ii}\hat{\theta}_i - \frac{e_{ii}^2}{2}T - \alpha_i \geq 0 \qquad i \in \{H, L\} \tag{8}$$

$$(1 - \beta_H)e_{HH}\hat{\theta}_H - \frac{e_{HH}^2}{2}T - \alpha_H \geq (1 - \beta_L)e_{HL}\hat{\theta}_H - \frac{e_{HL}^2}{2}T - \alpha_L \tag{9}$$

$$(1 - \beta_L)e_{LL}\hat{\theta}_L - \frac{e_{LL}^2}{2}T - \alpha_L \geq (1 - \beta_H)e_{LH}\hat{\theta}_L - \frac{e_{LH}^2}{2}T - \alpha_H \tag{10}$$

$$e_{ij} \in \arg\max_{e} \quad (1 - \beta_j)\hat{\theta}_i e - \frac{e^2}{2}T - \alpha_j \qquad i, j \in \{H, L\} \tag{11}$$

$$0 \leq \beta_i \leq 1 \qquad i \in \{H, L\}. \tag{12}$$

Constraints (7) and (8) ensure that the principal and agent respectively prefer the principal's proposed contract menu to their reservation payoffs of zero at time $t = 0$. The incentive compatibility constraints, (9) and (10), require that the agent truthfully report his findings

in the budgeting stage at time $t = 2$. If the agent misreports his observation, σ_i as σ_j, then in accordance with equilibrium condition (iii), his choice of effort, e_{ij}, maximizes his resulting payoffs as determined by (11). Finally, (12) prevents the principal from leveraging firm profits just as (6) did in the top-down setting. Unlike the top-down regime, the agent can coordinate his effort to both the realized and reported signal under bottom-up budgeting, therefore his opportunities to profit from misreporting outnumber those facing the principal in the prior subsection. In line with the hierarchy literature (Melumad et al. (1995)), we label this additional flexibility a "control loss" from the principal's perspective. Although the principal cannot ensure that the agent's effort is consistent with his announcement, the agent is indirectly penalized for inconsistencies. To see how, note that the principal reimburses the agent for his equilibrium effort, e_{LL} following a report of σ_L, implying that any effort in excess of e_{LL}; e.g., $e_{HL} - e_{LL}$, goes unreimbursed. In equilibrium, the principal rationally anticipates the agent's strategic reporting and responds with a $t = 0$ contract menu which provides the agent with informational rents should he report truthfully.

Proposition 2. *The optimal bottom-up contract menu is given by:*

$$\alpha_H = \frac{2(1 - 2p)p\hat\theta_H^4\hat\theta_L^2 + p(3p - 2)\hat\theta_H^2\hat\theta_L^4 + p2\hat\theta_H^6 + (p - 1)^2\hat\theta_L^6}{2T\left(p\left(\hat\theta_H^2 - 2\hat\theta_L^2\right) + \hat\theta_L^2\right)^2}$$

$$\alpha_L = \frac{(p - 1)^2\hat\theta_L^6}{2T\left(p\left(\hat\theta_H^2 - 2\hat\theta_L^2\right) + \hat\theta_L^2\right)^2}$$

$$\beta_H = 0$$

$$\beta_L = \frac{p\left(\hat\theta_H - \hat\theta_L\right)\left(\hat\theta_H + \hat\theta_L\right)}{p\left(\hat\theta_H^2 - 2\hat\theta_L^2\right) + \hat\theta_L^2}.$$

To keep the agent from misreporting favorable information, the optimal bottom-up contract menu penalizes unfavorable reports with restricted profit shares $((1 - \beta_L) < 1)$, leading to an inefficiently-low equilibrium effort, e_{LL}. Unlike top-down budgeting, the optimal bottom-up contract menu retains the same functional form over all levels of signal infor-

mativeness. To understand why the two modes differ, recall that inefficient effort is always more destructive in the more productive of the two states. Accordingly, top-down budgeting induces inefficiencies in the state with the most surplus at stake, whereas bottom-up budgets induce analogous inefficiencies when the surplus at stake is relatively low. Because the expected conditional productivity increases (decreases) following favorable (unfavorable) news as the interim signal becomes more informative, responding to changes in signal informativeness with additional effort distortions is always more costly under top-down budgeting relative to bottom-up budgeting. Therefore contrary to the top-down framework where the optimal contract menu exhibits bounded effort distortions, the optimal bottom-up menu counters all increases in signal informativeness with additional effort distortions, as shown in figure 4.
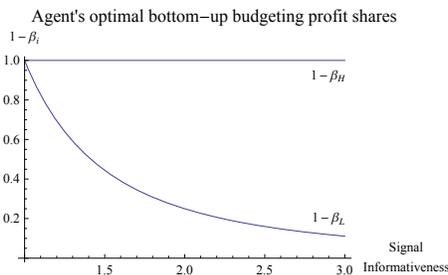

Figure 4: To assuage the agent's incentives to erroneously report σ_L, the optimal bottom-up contract menu decreases the agent's profit share following unfavorable news as signal informativeness increases. Consequently, the agent's equilibrium effort choice, e_{LL} becomes increasingly distorted as signal informativeness rises. Analogous to the top-down regime, the optimal contract exposes the agent to all the firm's profits when he reports his least preferred signal, σ_H.

4 Results

The optimal contract menus motivate truth telling differently across the two budgeting modes: under top-down budgeting the agents efforts are distorted in the productive state whereas his efforts are similarly distorted in the unproductive state under bottom-up budgeting. Pricing theory has shown that a profit maximizing monopolist will avoid imposing

inefficient schedules to her most lucrative customers, suggesting that top-down budgeting may never prove optimal. Although the extent to which the agent's incentives are distorted– and surplus destroyed–differs over the two regimes, the following proposition finds that the classic monopolistic pricing schedule also yields the greatest total surplus in our budgeting construct.

Proposition 3. *Total expected surplus is always greater under bottom-up budgeting.*

While the proposition above speaks to total surplus, satisfying incentive compatibility under each budgeting regime frequently requires that the principal partially forfeit surplus to the agent in the form of rents. Therefore the principal's preferred budgeting regime will not always align with the socially optimal (surplus maximizing) outcome. Within a particular industry, if the principal (agent) is better positioned to gather information than the agent (principal), then one would expect firms therein to be that much more inclined to use a top-down (bottom-up) budgeting paradigm. However, if both parties are equally adept at research, then the following proposition characterizes the circumstances in which the principal favors one paradigm over the other.

Proposition 4. *When the principal and agent can acquire interim information, σ, at identical cost, the principal prefers bottom-up budgeting when the privately obtained signal is sufficiently informative. No additional agency costs are incurred if the principal and agent face differing research costs.*

Our main result above speaks directly to the relative costs of signaling (top-down budgeting) versus screening (bottom-up budgeting). When the signal, σ, is relatively informative (high levels of information or environmental uncertainty), the informed party's payoff to misreporting is relatively large. Therein, the surplus destroyed under top-down budgeting outweighs that forgone under bottom-up budgeting to such an extent, that the additional control loss associated with the latter becomes relatively negligible, and the principal favors bottom-up budgeting. However, as the signal becomes less informative, both the principal's

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and agent's payoff to misreporting private information decreases and the difference in forgone surplus between the two budgeting regimes itself becomes negligible. Consequently, the control loss associated with bottom-up budgeting increases in relevancy, so that when the signal is sufficiently uninformative, the principal favors top-down budgeting instead.

The second part of the proposition speaks to the possibility that the principal and agent face differing research costs. In particular, if the principal's research cost exceeds that of the agent by δ, then the only penalty above and beyond the production and control costs associated with top-down budgeting is given by δ. Put differently, cost differences between the principal and agent do not affect the agent's underlying hidden-effort problem, as research itself is contractible and therefore void of moral hazard.[9]

Both bottom-up and top-down budgeting require that the optimal contract allocate surplus across the principal and agent to satisfy the relevant truth-telling constraints. In the bottom-up regime, the agent is rewarded with informational rents when he truthfully discloses favorable news, whereas in the top-down regime the principal is penalized when she reports favorable news. In both modes, the principal suffers from the informed party's misreporting incentives, largely to the agent's benefit.

Proposition 5. *Using the optimal contract menus, the agent's rents (principal's payoffs) are single peaked ("U" shaped) over signal informativeness in both top-down and bottom-up budgeting. Over the bounded interval of available interim information, $a \in [0,1]$, the principal prefers either the minimal $(a = 0)$ or maximal $(a = 1)$ information availability; whereas the agent always favors a positive level of availability, $a > 0$.*

Under bottom-up budgeting, the agent's rents are initially increasing in signal informativeness because as $\hat{\theta}_H$ diverges from $\hat{\theta}_L$, erroneously reporting unfavorable information allows the agent to further economize on his misreported cost. However, as the signal becomes more informative, the agent is induced to decrease his exerted effort. Therefore, while

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[9]We thank a seminar participant at XXXXXXXXXXX for suggesting that we include a discussion of differing research costs.

the rent earned on every unit of effort increases with signal informativeness, the reduced level of effort induced causes his rents to eventually decrease. Combined, these two tensions leave the agent with unimodal rents under the optimal bottom-up contract menu. Because rents transferred to the agent constitute a zero-sum payment, the principal's payoff is consequently "U" shaped over signal informativeness. Surprisingly, the principal's payoffs are again "U" shaped, and those of the agent are again unimodal over signal informativeness in the top-down budgeting regime. To understand the principal's payoffs under top-down budgeting, recall that the principal benefits from erroneously reporting favorable information by selling the agent an over-valued share of firm. In so doing, the principal reimburses the agent for high levels of effort, regardless of the true signal received. The optimal top-down contract menu ensures that when the principal erroneously reports favorable news, the agent exerts increasingly excessive effort. The marginal penalty associated with this effect initially dominates the principal's marginal benefit to misreporting, though as the signal becomes more informative, the latter marginal benefit overpowers the former marginal penalty, leaving the principal with "U" informational preferences, and the agent with unimodal preferences. We note that Proposition 5 also permits one to compare the relative costs associated with top-down versus bottom-up budgeting should the principal or agent be limited to collecting less interim information than their counterpart.

Although each participant's payoffs are qualitatively identical across the two budgeting regimes, Proposition 4 found that the choice of bottom-up versus top-town budgeting does affect the principal's expected payoff. The intuition behind these seemingly opposing results lies in the fact that the total generated surplus varies across the two budgeting modes as well. Proposition 3 found the bottom-up budgeting surplus to always exceed that generated under top-down budgeting, though we have yet to examine how surplus varies within each regime over signal informativeness.

Proposition 6. *Under both budgeting paradigms, as signal informativeness increases total welfare may decline if the signal is relatively uninformative, though total welfare will always*

rise if the signal is sufficiently informative.

From Lemma 1, greater signal informativeness provides the firm with additional capacity to generate surplus. However, when the signal is privately observed, signal informativeness also affects misreporting payoffs, which in turn determines the level of surplus destruction required to satisfy the relevant truth-telling constraints. Proposition 5 found misreporting payoffs to be unimodal in both budgeting regimes using the optimal contract menus, therefore credible communication requires initially increasing, and later decreasing levels of marginal surplus destruction as signal informativeness increases. As signal informativeness increases, truthful reporting eventually necessities less marginal surplus destruction, though Lemma 1 found that the firm's surplus generating capacity grows convexly. Combining the effects, the marginal profit associated with greater surplus capacity eventually dominate the marginal cost of sustaining truthful reporting, therefore total expected surplus will increase with signal informativeness once the signal becomes sufficiently informative.

4.1 Empirical Discussion

To address the ongoing empirical discussion on participative budgeting, this section provides potential proxies for observable firm attributes resulting from the firm's budgeting choice.

We begin with the most commonly studied firm attribute in the empirical literature, budgetary slack. Young (1985) defines budgetary slack as "the amount by which a subordinate understates his productive capability when given a chance to select a work standard against which his performance will be evaluated", whereas Dunk (1993) measures budgetary slack as the difficulty of attaining budgeted targets. Under bottom-up budgeting, the agent reports his interim information honestly, albeit he collects rents which are equivalent to what he could have obtained had he strategically under-reported his findings. Therefore, within a bottom-up context, Young's definition of budgetary slack agrees most closely with our definition of agent's informational rents. In the top-down framework, the agent collects rents whenever his performance following favorable news exceeds a profit-threshold; therefore, in

light of Dunk's measure, we use the agent's rents as a proxy for budgetary slack in the top-down setting as well.

In an experimental setting, Young (1985) rejects the null hypothesis positively correlating private information with budgetary slack under participative budgeting. Comparing firms that employ participative budgeting with those that do not, Proposition 4 predicts that the former contends with greater information asymmetries than the latter, while Proposition 5 predicts single-peaked rents (budgetary slack) over the level of information asymmetry. Combined, our results suggest a potential increase in budgetary slack following the onset of participative budgeting insomuch as the latter was introduced in response to relatively small increases in information asymmetry. However, because rents were found to eventually decrease over signal informativeness, our model also predicts decreased budgetary slack in response to increases in elevated levels of information asymmetry; with or without participative budgeting. Simply put, Young rejects a monotone relation between slack and information asymmetry under bottom-up budgeting, whereas our model predicts a unimodal relation regardless of the budgeting mode.

Dunk (1993) instead relies on survey data to explain how participative budgeting relates to budgetary slack. Therein, it is reported that budgetary slack varies with the choice of budgetary paradigm in conjunction with two additional firm attributes: the quantity of asymmetric information and the extent to which incentive contracts rely on budgets ("budget emphasis"). Without additional tensions, our model cannot speak to the results in Dunk (1993), as our framework assumes that the level of information asymmetry alone determines budgetary slack. Instead, we address budget emphasis as a consequence of information asymmetry and the optimal choice of budgeting mode. Two measures of budget emphasis commonly found in the empirical literature are incentive strength and expected performance pay which, in our model, translates to $\mathrm{E}[(1 - \beta_i)]$ and $\mathrm{E}[(1 - \beta_i)e_i\hat{\theta}_i]$ respectively. Both measures capture the rate at which the agent is compensated for elevating profits beyond budgeted levels, though incentive strength, $\mathrm{E}[(1-\beta_i)]$, may inadvertently measure aggregate,

rather than net rates. To see why, note that when profits are relatively immune to the agent's efforts; e.g., for sufficiently small values of $e_i \hat{\theta}_i$, incentive strength may register a strong budget emphasis, in spite of the agent rarely receiving compensation for better than budgeted performance. On the other hand, expected performance pay, $\mathrm{E}[(1-\beta_i)e_i\hat{\theta}_i]$, controls for the agent's ability to influence his performance pay, which explains why the two measures behave very differently as signal informativeness varies.

Proposition 7. *As measured by expected performance pay: $E[(1 - \beta_i)e_i\hat{\theta}_i]$, budget emphasis is initially decreasing and later increasing over signal informativeness (information asymmetry). When budget emphasis is alternatively measured via incentive strength, $E[1 - \beta_i]$, it is everywhere decreasing over signal informativeness.*

As the proposition above shows, depending on the measure utilized, budget emphasis may or may not exhibit monotonicity over the level of asymmetric information recorded. The result highlights the importance of carefully considering each measure used to develop proxies in the empirical literature. For instance, the prior literature frequently relies on Cronbach's α to validate firm attribute proxies; however, Cronbach's α cannot address the consistency of any *individual* measure under consideration. Because Cronbach's α only captures set co-variation, without an analytic prediction one may inadvertently mispair measures in forming firm proxies, providing one possible explanation for the mixed empirical results to date.

Another branch in the empirical literature on participative budgeting has studied the relation between the choice of budgeting mode and incentives. For example, Shields and Shields (1998) find that strong incentives are correlated with the use of participative budgeting. As mentioned earlier, incentive strength *per-se*, $E[1 - \beta_i]$, can unravel when the agent's efforts weakly influence profits. To better capture the empirical notion of incentive strength, we propose using either the previously mentioned expected performance pay, or the performance-pay ratio:

$$\frac{E[(1 - \beta_i)e\theta_i]}{E[(1 - \beta_i)e_i - \alpha_i - \frac{e^2}{2T}\theta_i]}.$$

The performance-pay ratio measures the relative share of the agent's non-salary rents versus those resulting from better than budgeted performance. To understand how the performance-pay ratio varies with the choice of budgeting mode, we must first characterize how the proxy behaves over signal informativeness.

Proposition 8. *The performance-pay ratio is eventually increasing in the level of signal informativeness.*

As measured by either expected performance pay or the performance-pay ratio, the agent's incentives may initially decline in response to additional information asymmetry, albeit, both measures will eventually increase. To the extent that firms using bottom-up budgeting contend with significantly greater information asymmetries than those using top-down budgeting (in accordance with Proposition 4), then our model's predictions agree with those of Shields and Shields (1998). In addition to their empirical analysis, Shields and Shields (1998) provides an overview on the empirical budgeting literature to date. Therein, the authors cite the lack of participatory budgeting antecedents in the empirical literature as another possible explanation for the confounding evidence to date. Though we found the choice of budgeting mode to vary at most once over signal informativeness, we also established that most firm attributes vary non-monotonically along the same dimension. Therefore, in accordance with Shields and Shields (1998), our model can substantiate the presence of *any* monotone statistical relation between budgeting mode and arbitrary firm attributes, unless one explicitly controls for what we assume to be the sole participatory budgeting antecedent, the level of information asymmetry.

5 Conclusion

In this paper, we provided a stylized organizational design model encompassing the two primary budgeting distinctions: top-down and bottom-up budgeting. Our framework captures settings where costly private information can be acquired by either the principal *or* agent. Although only one party collects information, both parties stand to gain from the findings: the principal uses the information to minimize the agent's rents, and the agent uses the information to more efficiently select his level of effort. Upon acquiring information, the informed party communicates his or her results through the budgeting mechanism, though they always have the option of strategically misreporting their information as well. When both parties are equally adept at gathering information, we found that the principal will favor bottom-up budgeting over top-down budgeting whenever the privately observed signal is sufficiently informative. Surprisingly, the principal's and agent's information preferences remained qualitatively identical across the two budgeting regimes. Whereas earlier research has warned of potential agency costs exclusive to participative budgeting, our results demonstrate that the top-down alternative need not be more profitable. We then provided proxies for budgetary slack, budget emphasis, and incentive strength over the same parameter space. Our predictions hinge on the implicit assumption that firms jointly select both their budgeting mode and compensation contracts exclusively in response to the availability of interim information and the ex-ante environmental uncertainty. To the extent that firms make organizational design decisions as such, we identified non-monotone relations between the choice of budgeting mode and potential proxies for budgetary slack, budgetary emphasis and incentive strength.

A natural extension to our model would allow both the principal *and* agent to receive private information. Therein, one could again study the efficacy of having the principal signal her information versus screening the agent for his information, and the optimal budgeting sequence if both report their findings. To the extent that either the principal *or* the agent be charged with reporting independent information, we suspect that our results

will remain largely unchanged. However, if the principal's and agent's private information overlap, then the resulting equilibrium will critically depend on off-equilibrium beliefs. To model overlapping, non-contractible information reporting, one must first settle on a set of reasonable punishments if one party reports information which conflicts with that of the other. Unfortunately, the present model cannot tractably support such additional structure without significantly simplifying the present informational structure.

Another venue for future research could extend the present analysis to include unobservable, costly information acquisition. In this framework, the principal would either face a dual-moral hazard signaling problem, or a screening problem with two moral hazard tasks. Earlier work in the Economics literature has considered a similar setting where the agent's private information relies on his unobservable efforts. Lewis and Sappington (1997) find that "extreme reward structures" can satisfy both the moral-hazard and consequential adverse-selection problems. We suspect that the addition of hidden research would contribute relatively similar agency costs across the two budgeting modes, in which case our primary contribution tying information to the choice of budgeting regime would remain largely unchanged.

In closing, we note that our model ignores the many behavioral tensions which undoubtedly contribute to the firm's choice of budgeting mode. We hope that by presenting an economic, unbounded rationality perspective on participative budgeting, future researchers could better delineate how altering perspectives conflict with ours, and ultimately test which of the two perspectives can best explain the observed instances of bottom-up versus top-down budgeting.

6 Appendix

To simplify the exposition let $k = \frac{\hat{\theta}_H}{\hat{\theta}_L}$, our proxy for signal informativeness. Note that k is increasing in both the level of information, a, and with respect to arbitrary mean preserving spreads which enlarge j. Additionally, let Π_P^n denote the principal's expected payoff and Π_A^n that of the agent with $n \in \{FB, BU, TD\}$ denoting the first-best benchmark, bottom-up and top-down regime respectively.

Proof of Lemma 1

The principal maximizes:

$$\max_{e_H, e_L} \quad p \cdot E\left[\theta_H e_H - T\frac{e_H^2}{2} \middle| \sigma_H\right] + (1-p) \cdot E\left[\theta_L e_L - T\frac{e_L^2}{2} \middle| \sigma_L\right],$$

thus the optimal processing efforts are $e_H^{FB} = \frac{\hat{\theta}_H}{T}$ and $e_L^{FB} = \frac{\hat{\theta}_L}{T}$, where the FB superscript denotes the optimal first-best benchmark solution. The principal's first-best profits are hence:

$$\Pi_P^{FB} = \frac{p\hat{\theta}_H^2 + (1-p)\hat{\theta}_L^2}{2T} = \frac{\bar{\theta}^2 + a^2(1-p)p(\theta_H - \theta_L)^2}{2T}.$$

By inspection, Π_P^{FB} is increasing and convex in a. Mean preserving spreads, on the other hand, will only increase the difference $\theta_H - \theta_L$. Because θ_H and θ_L appear only with a positive coefficient via $\theta_H - \theta_L > 0$ in the principal's profits, mean preserving spreads will induce positive and convex gains to the principal's profits as well. The contractibility of the agent's effort allows the principal to pay the agent conditionally on his efforts; therefore, by providing the agent with a menu of contracts consisting of $\alpha_i = \frac{\hat{\theta}_i^2}{2}T$, for $i = H, L$, the agent is indifferent between the each contract and we assume that he resolves his indifference according to either his signal, or in the case of top-down budgeting, the principal's report.

Proof of Proposition 1

The principal solves:

$$\max_{\alpha_H, \alpha_L, \beta_H, \beta_L} \quad p\left(\beta_H e_H \hat{\theta}_H + \alpha_H\right) + (1-p)\left(\beta_L e_L \hat{\theta}_L + \alpha_L\right)$$

$$\text{s.t.} \quad \mathrm{E}[\beta_H e_H \theta + \alpha_H | \sigma_H] \geq \mathrm{E}[\beta_L e_L \theta + \alpha_L | \sigma_H] \qquad (\text{ICP}_H)$$

$$\mathrm{E}[\beta_L e_L \theta + \alpha_L | \sigma_L] \geq \mathrm{E}[\beta_H e_H \theta + \alpha_H | \sigma_L] \qquad (\text{ICP}_L)$$

$$\mathrm{E}[\beta_i e_i \theta + \alpha_i | \sigma_i] \geq 0 \qquad i = H, L \qquad (\text{IRP}_i)$$

$$e_i \in \arg\max_e \quad \mathrm{E}\left[(1-\beta_i)\theta e - \alpha_i - \frac{e^2}{2}T \Big| \sigma_i\right] \quad i = H, L \qquad (\text{ICA}_i)$$

$$\mathrm{E}\left[(1-\beta_i)e_i\theta - \frac{1}{2}Te_i^2 - \alpha_i \Big| \sigma_i\right] \geq 0 \qquad i = H, L \qquad (\text{IRA}_i)$$

$$0 \leq \beta_i \leq 1 \qquad i = H, L$$

In accordance with (ICA_i) the agent exerts effort $e_i = \frac{(1-\beta_i)\hat{\theta}_i}{T}$ for $i = H, L$. We ignore constraints (ICP_H), (IRP_H) and (IRP_L), and later verify that the solution to the relaxed program satisfies these constraints. (IRA_L) will always bind, for otherwise α_L and profits could be increased, while generating slack in (ICP_L). Simplifying (IRA_L) with the agent's optimized processing effort yields:

$$\alpha_L = (1-\beta_L)e_L\hat{\theta}_L - T\frac{e_L^2}{2} = \frac{(1-\beta_L)^2\hat{\theta}_L^2}{2T}. \tag{13}$$

The principal's program thus becomes:

$$\max_{\alpha_H, \beta_H, \beta_L} \quad p \cdot \left(\alpha_H + \frac{\beta_H(1-\beta_H)\hat{\theta}_H^2}{T}\right) + (1-p)\left(\frac{(1-\beta_L^2)\hat{\theta}_L^2}{2T}\right)$$

$$\text{s.t.} \quad \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} \geq \alpha_H \qquad (\text{IRA}_H)$$

$$\frac{(1-\beta_L^2)\hat{\theta}_L^2}{2T} - \frac{\beta_H(1-\beta_H)\hat{\theta}_H\hat{\theta}_L}{T} \geq \alpha_H \qquad (\text{ICP}_L)$$

$$0 \leq \beta_i \leq 1 \qquad i = H, L.$$

Both the objective function and the left hand side (LHS) of constraint (ICP_L) are decreasing

in $|\beta_L|$ while the rest of the program is independent of β_L. To maximize profits, β_L is set to 0, implying, from (13), that $\alpha_L = \frac{\hat{\theta}_L^2}{2T}$. The program thus further reduces to:

$$\max_{\alpha_h,\beta_h} \quad p\left(\alpha_H + \frac{\beta_H(1-\beta_H)\hat{\theta}_H^2}{T}\right) + (1-p)\left(\frac{\hat{\theta}_L^2}{2T}\right) \tag{14}$$

$$s.t. \quad \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} \geq \alpha_H \tag{IRA$_H$}$$

$$\frac{\hat{\theta}_L^2}{2T} - \frac{\beta_H(1-\beta_H)\hat{\theta}_H\hat{\theta}_L}{T} \geq \alpha_H \tag{ICP$_L$}$$

$$0 \leq \beta_H \leq 1$$

We claim that (ICP$_L$) must bind. To see this, note that the principal's profits are increasing in α_H, though α_H is bounded above by either (IRA$_H$) or (ICP$_L$). If the LHS of (ICP$_L$) is less than that of (IRA$_H$), then (ICP$_L$) binds. Otherwise, (IRA$_H$) binds and $\alpha_H = \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T}$. However, if $\alpha_H = \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T}$, then the principal maximizes her profits by lowering β_H until (ICP$_L$) binds. Thus (ICP$_L$) always binds, admitting the equivalent equations:

$$\beta_H(\alpha_H) = \frac{\hat{\theta}_H\hat{\theta}_L \pm \sqrt{\hat{\theta}_H\hat{\theta}_L\left(\hat{\theta}_L\left(\hat{\theta}_H - 2\hat{\theta}_L\right) + 4\alpha_H T\right)}}{2\hat{\theta}_H\hat{\theta}_L} \tag{15}$$

$$\alpha_H(\beta_H) = \frac{\hat{\theta}_L\left(2\left(\beta_H - 1\right)\beta_H\hat{\theta}_H + \hat{\theta}_L\right)}{2T}. \tag{16}$$

The principal's objective function, (14), is maximized when β_H is equal to $1/2$, since substituting (16) into Π_P^{TD} yields:

$$\frac{\partial\Pi_P^{TD}(\beta_H,\alpha_H(\beta_H))}{\partial\beta_H} = \frac{p\left(1 - 2\beta_H\right)\hat{\theta}_H\left(\hat{\theta}_H - \hat{\theta}_L\right)}{T} \geq 0, \tag{17}$$

therefore Π_P^{TD} is concave in β_H. Using $\beta_H = 1/2$ and α_H defined by (16), constraint (ICP$_H$) is satisfied, as it is equivalent to $\frac{\hat{\theta}_H\left(\hat{\theta}_H-\hat{\theta}_L\right)}{4T} \geq 0$. Similar substitution and algebraic manipulation reveal that both (IRP$_H$) and (IRP$_L$) are satisfied with the candidate solution. However, the candidate solution sets (IRA$_H$) to $\frac{(k(k+2)-4)\hat{\theta}_L^2}{8T}$, which is only non-negative for $k \geq \sqrt{5}-1$.

When $k < \sqrt{5} - 1$, the principal will set β_H as large as possible without violating (IRA_H), since her profits are increasing in β_H when $\beta_H < 1/2$. The contract will therefore have a second form when $k < \sqrt{5} - 1$; i.e., when (IRA_H) binds: $\alpha_H = \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T}$, which combined with (15) implies $\beta_H = \frac{\hat{\theta}_H^2 - \hat{\theta}_H\hat{\theta}_L - \sqrt{2}\sqrt{\hat{\theta}_H\hat{\theta}_L^2(\hat{\theta}_H - \hat{\theta}_L)}}{\hat{\theta}_H^2 - 2\hat{\theta}_H\hat{\theta}_L}$. Using these values, (ICP_H) reduces to $\frac{(k-1)\hat{\theta}_L^2}{(k-2)^2 T}\left(2 - k - k^2 + \sqrt{2}\sqrt{(k-1)k}\right)$ which is positive for $k \in (1, \sqrt{5} - 1)$. Similar substitution and algebraic manipulation reveals that the second candidate solution also satisfies both (IRP_H) and (IRP_L).

Proof of Proposition 2

The principal's maximization program is given below, where e_{ij} denotes the agent's processing effort when he observes $\sigma = \sigma_i$ but selects the contract (α_j, β_j) and we denote e_{ii} by e_i:

$$\max_{\alpha_H,\alpha_L,\beta_H,\beta_L} \quad p\left(\alpha_H + \beta_H e_H \hat{\theta}_H\right) + (1-p)\left(\alpha_L + \beta_L e_L \hat{\theta}_L\right)$$

$$s.t. \quad e_{ij} \in \arg\max_e E[(1-\beta_j)\theta e - \alpha_j - T\tfrac{1}{2}e^2|\sigma_i] \qquad i,j \in \{H,L\}^2 \qquad (\mathrm{ICA}_{ij})$$

$$E[(1-\beta_i)e_i\theta - \alpha_i - T\tfrac{1}{2}e_i^2|\sigma_H] \geq 0 \qquad i \in \{H,L\} \qquad (\mathrm{IRA}_i)$$

$$E\left[\left.\begin{array}{c}(1-\beta_H)e_H\theta \\ -\alpha_H - \frac{Te_H^2}{2}\end{array}\right|\sigma_H\right] \geq E\left[\left.\begin{array}{c}(1-\beta_L)e_{HL}\theta \\ -\alpha_L - \frac{Te_{HL}^2}{2}\end{array}\right|\sigma_H\right] \qquad (\mathrm{ICA}_H)$$

$$E\left[\left.\begin{array}{c}(1-\beta_L)e_L\theta \\ -\alpha_L - \frac{Te_L^2}{2}\end{array}\right|\sigma_L\right] \geq E\left[\left.\begin{array}{c}(1-\beta_H)e_{LH}\theta \\ -\alpha_H - \frac{Te_{LH}^2}{2}\end{array}\right|\sigma_L\right] \qquad (\mathrm{ICA}_L)$$

$$E[\beta_i e_i \theta_i + \alpha_i|\sigma_i] \geq 0 \qquad i \in \{H,L\} \qquad (\mathrm{IRP}_i)$$

$$0 \leq \beta_i \leq 1 \qquad i \in \{H,L\}.$$

Applying the first order approach to constraints (ICA_{ij}) yields:

$$e_H = \frac{(1-\beta_H)\hat{\theta}_H}{T} \qquad\qquad e_L = \frac{(1-\beta_L)\hat{\theta}_L}{T}$$

$$e_{LH} = \frac{(1-\beta_H)\hat{\theta}_L}{T} \qquad\qquad e_{LH} = \frac{(1-\beta_L)\hat{\theta}_H}{T}.$$

We again ignore (IRA_H), (ICA_L), (IRP_H) and (IRP_L) and later verify that the solution to the relaxed problem satisfies these constraints. The principal's program thus becomes:

$$\max_{\alpha_H,\alpha_L,\beta_H,\beta_L} \quad p\left(\frac{(1-\beta_H)\,\beta_H\hat{\theta}_H^2}{T} + \alpha_H\right) + (1-p)\left(\frac{(1-\beta_L)\,\beta_L\hat{\theta}_L^2}{T} + \alpha_L\right)$$

$$s.t. \quad \frac{(1-\beta_L)^2\hat{\theta}_L^2}{2T} \geq \alpha_L \qquad\qquad (\text{IRA}_L)$$

$$\frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} - \frac{(1-\beta_L)^2\hat{\theta}_H^2}{2T} + \alpha_L \geq \alpha_H \qquad\qquad (\text{ICA}_H)$$

$$0 \leq \beta_i \leq 1 \qquad i \in \{H,L\}.$$

Increasing α_H until (ICA_H) binds will raise profits without changing (IRA_L), thus we use (ICA_H) to obtain α_H; which yields the principal's new program:

$$\max_{\alpha_L,\beta_H,\beta_L} \quad \frac{-p\hat{\theta}_H^2\left(\beta_H^2 + (\beta_L - 2)\,\beta_L\right) + 2(p-1)\,(\beta_L - 1)\,\beta_L\hat{\theta}_L^2 + 2T\alpha_L}{2T}$$

$$s.t. \quad \frac{(1-\beta_L)^2\hat{\theta}_L^2}{2T} \geq \alpha_L \qquad\qquad (\text{IRA}_L)$$

$$0 \leq \beta_i \leq 1 \qquad i \in \{H,L\}.$$

Profits are increasing in α_L, therefore the principal raises α_L as large as possible while satisfying (IRA_L), resulting in:

$$\max_{\beta_H,\beta_L} \quad \frac{(\beta_L - 1)\,\hat{\theta}_L^2\left((2p-1)\beta_L - 1\right) - p\hat{\theta}_H^2\left(\beta_H^2 + (\beta_L - 2)\,\beta_L\right)}{2T} \qquad (18)$$

$$0 \leq \beta_i \leq 1 \qquad i \in \{H,L\}.$$

The principal's objective function (18) is decreasing in β_H, thus the optimal revenue share is given by $\beta_H = 0$. Optimizing over β_L obtains the menu proposed in the proposition. Substituting the menu from the proposition into (IRA_H), (ICA_L), (IRP_H) and (IRP_L), demonstrates that the previously ignored constraints are satisfied with the candidate solution.

Proof of Proposition 3

When $k \in (1, \sqrt{5}-1)$, subtracting the expected total surplus under top-down budgeting from that obtained under bottom-up budgeting yields: $g(p,k) \times \frac{p\hat{\theta}_L^2}{2T(-2+k)^2(1-2p+k^2p)^2}$, where $g(p,k)$ is quadratic in p and the second function is non-negative. The discriminant of $g(p,k)$ is $-(k-2)^2(k-1)^4(k+1)^3 \left(k \left(k(3k-1) - 8\sqrt{2}\sqrt{(k-1)k} + 8 \right) - 4 \right)$. On the relevant range, it carries the same sign as: $4 - k \left(k(3k-1) - 8\sqrt{2}\sqrt{(k-1)k} + 8 \right)$ which has 3 roots: $k = 1$, $k = 2$ and $k = -2.97$, all of which fall outside of $(1, \sqrt{5}-1)$. Thus, the discriminant does not change in sign over the relevant range and inspection reveals it to be negative, implying that $g(p,k)$ does not admit any real roots and is thus deemed positive by evaluation. Therefore, bottom-up budgeting generates greater expected total surplus when $k \in (1, \sqrt{5}-1)$.

When $k \geq \sqrt{5}-1$, the difference in expected total surplus under top-down and bottom-up budgeting is given by:

$$\frac{p\hat{\theta}_L^2}{8T\left((k^2-2)p+1\right)^2} \left(k^2 + \left(k^6 - 4k^2 + 4\right)p^2 - 2\left(k^4 - 2k^2 + 2\right)p \right). \tag{19}$$

The first part of (19) is positive, while the second part is quadratic in p and has a discriminate $-16(-1+k^2)^3 < 0$, implying that there are no real roots. Evaluation reveals that (19) is always positive, therefore bottom-up budgeting generates the greatest expected surplus.

Proof of Proposition 4

When $k \in (1, \sqrt{5}-1)$, we can write $\Pi_P^{BU} - \Pi_P^{TD}$ as:

$$\begin{pmatrix} k^5 p - k^4 p + k^3 \left(p - 2\sqrt{2}\sqrt{(k-1)k}p \right) - k^2(p-2) \\ +2\left(\sqrt{2}\sqrt{(k-1)k} - 1 \right)k(2p-1) + 4(p-1) \end{pmatrix}$$
$$\times \frac{\theta_L^2(k-1)p((j-1)p+1)^2}{2(k-2)^2 T((k-1)p+1)^2 \left((k^2-2)p+1\right)}.$$

The second term is positive, and therefore any sign variation must emanate from the first term, which is linear in p with a positive leading coefficient, admitting the following necessary

and sufficient condition for the difference in profits to be positive:

$$p > \frac{2}{k(k+1)\left(k + \sqrt{2}\sqrt{(k-1)k} - 1\right) + 2}. \tag{20}$$

If (20) is satisfied, then the principal's profits are greatest under bottom-up budgeting. The RHS of (20) is monotonically decreasing in k (and hence in a or any mean-preserving spread), demonstrating that for a fixed level of p, the difference in profits between the two modes may equal zero at most once.

If we substitute $k = 1$ into (20), then the RHS is equal to 1 and top-down budgeting dominates. Allowing k to increase indefinitely, (20) will eventually hold, and bottom-up budgeting will dominate. However, k, is bounded by either j or leaving the region from which this expression is derived ($k \leq \sqrt{5} - 1$). In the second contract region, with $k > \sqrt{5} - 1$, we have $\Pi_P^{BU} - \Pi_P^{TD} = \frac{(k(k(k+2)p-1)-2p)}{4(k^2-2)p+4}\frac{(k-1)p\hat{\theta}_L^2}{T}$. The second expression and the denominator of the first expression in $\Pi_P^{BU} - \Pi_P^{TD}$ are always positive, so the only possible sign variation will result from variation in $k(k(k+2)p-1) - 2p$. Simplifying, the following condition holds if and only if $\Pi_P^{BU} - \Pi_P^{TD}$ is positive in the region where $k \geq \sqrt{5} - 1$:

$$p > \frac{k}{k^3 + 2k^2 - 2}. \tag{21}$$

If (21) holds, the principal prefers bottom-up budgeting and since the RHS of (21) is decreasing in k (and hence in a or any mean-preserving spread), profits can only cross once j and a vary.

To complete the proof, we must exclude any jump discontinuities in the profit difference as one transitions from one contract form to the other at $k = \sqrt{5} - 1$, though as $k \to \sqrt{5} - 1$, the thresholds on p in (20) and (21) converge, precluding any such discontinuity.

Proof of Proposition 5

To facilitate the exposition, Proposition 5 is proved in five steps. The first two steps concern the principal's profits under each budgeting mode when the level of information, a, is

changing, while the final two steps examine the agent's profits under the two alternative regimes when the level of information, a, is changing. The final step of the proof is to show that the sign of the first and second derivative of the profit function with respect to a for each party carries the same sign as the derivative of the profit function with respect to linear mean-preserving spreads (see (35) below for a formal definition of differentiation with respect to mean preserving spreads).

Principal's profits under top-down budgeting

To show that the principal's profits are "U" shaped over a under top-down budgeting, we first show that profits are decreasing in a when $k < \sqrt{5} - 1$, whereas larger values of a, which induce $k \geq \sqrt{5} - 1$, cause the the principal's profits to initially decrease in a, albeit with a positive second derivative. We begin by differentiating the principal's profits, Π_P^{TD} when $1 < k < \sqrt{5} - 1$, over a:

$$\frac{d\Pi_P^{TD}(\beta_H(a), a)}{da} = \left.\frac{\partial \Pi_P^{TD}(\beta_H, a)}{\partial \beta_H}\right|_{\beta_H = \beta_H(a)} \cdot \frac{\partial \beta_H(a)}{\partial a} + \left.\frac{\partial \Pi_P^{TD}(\beta_H, a)}{\partial a}\right|_{\beta_H = \beta_H(a)}. \qquad (22)$$

We sign the three parts to (22) individually. Note that $\frac{\partial \Pi_P^{TD}(\beta_H, a)}{\partial \beta_H} < 0$ follows directly from expressing $\Pi_P^{TD}(\beta_H, a)$ as:

$$\theta_L^2 \frac{((a-1)(j-1)p-1)^2 - p\left(\begin{array}{c}(\beta_H^2 - 1)\left(a(j-1)(p-1) - jp + p - 1\right)^2 \\ +((a-1)(j-1)p-1)^2\end{array}\right)}{2T},$$

which is decreasing in β_H for $\beta_H \geq 0$. To sign the second term in (22) we compute:

$$\frac{\partial \beta_H(a)}{\partial a} = \frac{\partial \beta_H(a)}{\partial \hat{\theta}_H} \frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial \beta_H(a)}{\partial \hat{\theta}_L} \frac{\partial \hat{\theta}_L}{\partial a} \tag{23}$$

$$\frac{\partial \hat{\theta}_H}{\partial a} = (1-p)(\theta_H - \theta_L) > 0 \tag{24}$$

$$\frac{\partial \hat{\theta}_L}{\partial a} = -p(\theta_H - \theta_L) < 0. \tag{25}$$

$$\frac{\partial \beta_H(a)}{\partial \hat{\theta}_H} = \frac{-1}{k} \left(\frac{2\sqrt{(k-1)k}k + \sqrt{2}((3-2k)k-2)}{2(k-2)2\sqrt{(k-1)k}\hat{\theta}_L} \right) > 0 \tag{26}$$

$$\frac{\partial \beta_H(a)}{\partial \hat{\theta}_L} = \left(\frac{2\sqrt{(k-1)k}k + \sqrt{2}((3-2k)k-2)}{2(k-2)2\sqrt{(k-1)k}\hat{\theta}_L} \right) < 0. \tag{27}$$

From (23) - (27), we can sign the second part of (22), $\frac{\partial \beta_H}{\partial a} \geq 0$. To sign the final term in (22), we substitute $k = \hat{\theta}_H / \hat{\theta}_L$ and the optimal revenue share $\beta_H = \beta_H(a)$ to yield the following expression (an un-ambiguously positive pre-multiplier has been omitted):

$$\left. \frac{\partial \Pi_P^{TD}(\beta_H, a)}{\partial a} \right|_{\beta_H = \beta_H(a)} = -3k + 2\sqrt{2}\sqrt{k-1}\sqrt{k} + 2.$$

On the interval $k \in (1, \sqrt{5}-1)$, the above expression is negative, therefore $\left. \frac{\partial \Pi_P^{TD}(\beta_H, a)}{\partial a} \right|_{\beta_H = \beta_H(a)} < 0$ which combined with $\frac{\partial \beta_H}{\partial a} \geq 0$ and $\frac{\partial \Pi_P^{TD}(\beta_H, a)}{\partial \beta_H} < 0$, establish that Π_P^{TD} is decreasing in a when $1 < k < \sqrt{5} - 1$ under top-down budgeting.

When $k \geq \sqrt{5} - 1$, the principal's profit function takes the form: $\Pi_P^{TD} = \frac{p\hat{\theta}_H(\hat{\theta}_H - \hat{\theta}_L) + 2\hat{\theta}_L^2}{4T}$ implying:

$$\frac{\partial \Pi_P^{TD}}{\partial a} = \frac{1}{4T} \left(2ap(p+1)(\theta_H - \theta_L)^2 + p(\theta_H - \theta_L)(-3p(\theta_H - \theta_L) - 3\theta_L) \right). \tag{28}$$

Since $\frac{\partial \Pi_P^{TD}}{\partial a}$ is linear in a with a positive leading coefficient, the principal's profits are convex on this region, and evaluation demonstrates that $\frac{\partial \Pi_P^{TD}}{\partial a} < 0$ for sufficiently small a. We have therefore shown that the principal's profits are decreasing when $k < \sqrt{5} - 1$ and initially decreasing and convex in a when $k \geq \sqrt{5} - 1$. Since j and a move concomitantly with k the

claim is proven.

Principal's profits under bottom-up budgeting

We now show that the principals' profits are always convex with bottom-up budgeting. With bottom-up budgeting, the principal's profits are given by $\Pi_P^{BU} = \frac{p^2\hat{\theta}_H^4 + (1-2p)p\hat{\theta}_H^2\hat{\theta}_L^2 + (p-1)^2\hat{\theta}_L^4}{2T\left(p\left(\hat{\theta}_H^2 - 2\hat{\theta}_L^2\right) + \hat{\theta}_L^2\right)}$. The numerator of $\frac{\partial^2\Pi_P^{BU}}{\partial a^2}$ carries the sign:

$$-\frac{a^2\left((k^2-2)p+1\right)}{((a-1)(k-1)p+a)^2} \leq 0, \tag{29}$$

whereas the denominator of $\frac{\partial^2\Pi_P^{BT}}{\partial a^2}$ is the product of two functions:

$$\frac{1}{(j-1)^2(p-1)^2p^2\theta_L^2} \times \tag{30}$$

$$\begin{pmatrix} a^6(j-1)^6(p-1)p^2\left(p^2+p-1\right)^2 - 6a^5(j-1)^5p^3\left(p^3-2p+1\right)((j-1)p+1) \\ +3a^4(j-1)^4(p-1)p\left(5p^3-2p+1\right)((j-1)p+1)^2 \\ -4a^3(j-1)^3(p-1)p(5(p-1)p+1)((j-1)p+1)^3 \\ +3a^2(j-1)^2(p-2)(5(p-1)p+1)((j-1)p+1)^4 \\ -6a(j-1)((p-4)p+2)((j-1)p+1)^5 + (p-5)((j-1)p+1)^6 \end{pmatrix}. \tag{31}$$

The first expression, (30), is always positive while the second, (31), is a seventh order polynomial in p, which we label $sp(a, j, p)$. To search for possible roots to $sp(a, j, p)$ with $p \in (0, 1)$, we apply a Mobius transform and complete a Descartes root test[10] by studying the coeffi-

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[10]See Eigenwillig (2007) for details.

cients of $(p+1)^7 sp(a, j, \frac{1}{p+1})$ with respect to p, which are:

$$C_0 = -j^4 \left(3a^2(j-1)^2 - 6aj(j-1) + 4j^2\right)$$

$$C_1 = \begin{pmatrix} -a^6(j-1)^6 + 6a^5 j(j-1)^5 - 12a^4 j^2(j-1)^4 + 4a^3 j^3(j-1)^3 \\ +3a^2(j-4)j^3(j-1)^2 + 30aj^4(j-1) - j^5(5j+24) \end{pmatrix}$$

$$C_2 = \begin{pmatrix} 2a^6(j-1)^6 - 6a^5(j-1)^6 + 3a^4(j-8)j(j-1)^4 - 12a^3 j^2(j-1)^4 \\ +3a^2 j^2(j(5j+4)-6)(j-1)^2 - 12aj^3\left(j^2-5\right)(j-1) - 30j^4(j+2) \end{pmatrix}$$

$$C_3 = \begin{pmatrix} a^6(j-1)^6 - 6a^5(j+1)(j-1)^5 - 3a^4((j-2)j+4)(j-1)^4 \\ +4a^3 j((j-9)j+3)(j-1)^3 - 6a^2 j(j((j-10)j-3)+2)(j-1)^2 \\ -60aj^2(j+1)(j-1)^2 - 5j^3(15j+16) \end{pmatrix}$$

$$C_4 = \begin{pmatrix} -2a^6(j-1)^6 - 6a^5(j-1)^5 - 3a^4(j(j+2)-1)(j-1)^4 + 4a^3(3(j-3)j+1)(j-1)^3 \\ -3a^2(4j+1)(2(j-4)j+1)(j-1)^2 - 30aj(2j-1)(2j+1)(j-1) - 20j^2(5j+3) \end{pmatrix}$$

$$C_5 = \begin{pmatrix} -a^6(j-1)^6 - 3a^4(2j+1)(j-1)^4 + 12a^3(j-1)^4 - 3a^2(4j(3j-5)-1)(j-1)^2 \\ -6a\left(20j^2-1\right)(j-1) - 3j(25j+8) \end{pmatrix}$$

$$C_6 = -a(j-1)(a(j-1)(a(j-1)(3a(j-1)-4)+3(8j-5))+60j)-30j-4$$

$$C_7 = -6a(j-1)(a(j-1)+2)-5.$$

Simplification demonstrates that the coefficients above are always negative when $a \in [0,1]$ and $j > 1$, implying that $sp(a,j,p)$ (and therefore (31)) is constant in sign over the entire interval, $p \in (0,1)$. Evaluation reveals that (31) is negative, therefore $\frac{\partial^2 \Pi_P^{BU}}{\partial a^2} \geq 0$, as both its numerator and denominator are negative.

Agent's rents under top-down budgeting

When $k \leq \sqrt{5} - 1$ both of the agent's individual rationality constraints bind, and he collects no rents. When $k > \sqrt{5}-1$, the agent's rents are given by $\Pi_A^{TD} = p\left(\frac{2\hat{\theta}_H \hat{\theta}_L + \hat{\theta}_H^2 - 4\hat{\theta}_L^2}{8T}\right)$. The agent's rents are initially increasing in a, as $\left.\frac{\partial \Pi_A^{TD}}{\partial a}\right|_{a|k(a)=\sqrt{5}-1} = \frac{(j-1)p\left(\sqrt{5}+\left(5-2\sqrt{5}\right)p\right)((j-1)p+1)\theta_L^2}{4\left(\left(\sqrt{5}-2\right)pT+T\right)} >$

0. However, $\frac{\partial^2 \Pi_A^{TD}}{\partial a^2} = (1 - p(p+4))\frac{(\theta_H - \theta_L)^2}{4T}$, and does not vary in sign over the relevant range, though it can be either uniformly positive or negative over $0 \le a \le 1$; therefore $\frac{\partial \Pi_A^{TD}}{\partial a}$ admits at most a single root, which is attained from above.

Agent's rents under bottom-up budgeting

Under bottom-up budgeting, the agent collects rents given by $\Pi_A^{BU} = \frac{(1-p)^2 p (\hat{\theta}_H - \hat{\theta}_L) \hat{\theta}_L^4 (\hat{\theta}_H + \hat{\theta}_L)}{2T(\hat{\theta}_L^2 + p(\hat{\theta}_H^2 - 2\hat{\theta}_L^2))}$. To prove that his profits are single peaked over a, we show that $\frac{\partial \Pi_A^{BU}(a,j)}{\partial a}\Big|_{a=0} > 0$ and the function $\frac{\partial \Pi_A^{BU}(a,j)}{\partial a}$ has at most one root over the interval $a \in (0,1)$. To this end, we have $\frac{\partial \Pi_A^{BU}(a,j)}{\partial a}\Big|_{a=0} = \frac{p \theta_L^2 (j-1)(1+(j-1)p)}{T} > 0$. To show that the function $\frac{\partial \Pi_A^{BU}(a,j)}{\partial a}$ has at most a single root over $a \in (0,1)$, we begin by noting that its denominator carries the same sign as $-1 - (-3 + a^2(1-j)^2 + 2j)p - (1-a)(j-1)(j-3+a(j-1))p^2 + (1-a)^2(j-1)^2 p^3$. The denominator thus has two roots in a, both of which fall out of the range $(0,1)$ when $0 < p < 1$ and $j > 1$; implying that the denominator has a constant (negative) sign throughout the relevant range. We can express the numerator as: $f(a,p,j)\left((-1+j)(-1+p)^2 p(1+(-1+a+j-aj)p)^3 \theta_L^2\right)$, where $f(a,p,j)$ is a fourth order polynomial in a and the rest of the expression is always positive. We search for possible roots in a over the $(0,1)$ interval by again applying the Descartes test to the Mobius transform on $f(a,p,j)$. In particular, we verify the number of roots of $H(a,j) = (a+1)^4 f(\frac{1}{1+a}, j, p)$, which itself is a fourth degree polynomial over a. The coefficients of a in $H(a,j)$ are negative when $p = 0$, though each coefficient has a unique root over p when $0 < p \le 1$, which we denote R_i, where the index, i, denotes the relevant

order of a:

$$R_0 = \frac{-j^3 + \sqrt{j^6 + 8j^5 - 2j^4 - 14j^3 + j^2 + 6j + 1} - j + 1}{2\left(2j^4 - j^3 - 3j^2 + 2\right)}$$

$$R_1 = \frac{-j^3 - 3j^2 + \sqrt{j^6 + 6j^5 + 99j^4 - 68j^3 - 117j^2 + 54j + 41} - 3j + 3}{2\left(7j^3 - 9j^2 - 6j + 8\right)}$$

$$R_2 = \frac{-j^2 + \sqrt{j^4 + 12j^3 - 10j^2 - 12j + 13} - 2j + 1}{2\left(2j^2 - 5j + 3\right)}$$

$$R_3 = \frac{2j - \sqrt{3j^2 - 2j + 3}}{j - 1}$$

$$R_4 = 1$$

The roots above are ordered, in that for any $j > 1$: $R_0 \leq R_1 \leq R_2 \leq R_3 \leq R_4 = 1$. Thus, for $j > 1$ and $1 > p > 0$, there is at most one sign variation along the ordered coefficients. Since the denominator of $\frac{\partial \Pi_A^{BU}(a,j)}{\partial a}$ does not vary in sign and the numerator has at most a single sign change as a varies, $\frac{\partial \Pi_A^{BU}(a,j)}{\partial a}$ has at most a single root in a over the interval $(0,1)$, as was to be shown.

Profits with respect to linear mean preserving spreads

Under either top-down, bottom-up budgeting, the principals profits (or agent's rents) can be expressed in terms of $\hat{\theta}_H, \hat{\theta}_L, p$ and T, implying that:

$$\frac{d\Pi(\hat{\theta}_H(\theta_H, \theta_L, a, p), \hat{\theta}_L(\theta_H, \theta_L, a, p), T, p)}{da} = \frac{\partial \Pi}{\partial \hat{\theta}_H}\frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial \Pi}{\partial \hat{\theta}_L}\frac{\partial \hat{\theta}_L}{\partial a} \tag{32}$$

$$\frac{d^2\Pi(\hat{\theta}_H(\theta_H, \theta_L, a, p), \hat{\theta}_L(\theta_H, \theta_L, a, p), T, p)}{da^2} =$$
$$\frac{\partial \hat{\theta}_H}{\partial a}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_H^2}\frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_H \hat{\theta}_L}\frac{\partial \hat{\theta}_L}{\partial a}\right) + \frac{\partial \hat{\theta}_L}{\partial a}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_L^2}\frac{\partial \hat{\theta}_L}{\partial a} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_L \hat{\theta}_H}\frac{\partial \hat{\theta}_H}{\partial a}\right), \tag{33}$$

since $\frac{\partial^2 \hat{\theta}_i}{\partial a^2} = 0$. We parameterize a linear mean preserving spread as:

$$\theta_H(\lambda) = \theta_H + \lambda(\theta_H - (p\theta_H + (1-p)\theta_L)) \tag{34}$$

$$\theta_L(\lambda) = \theta_L - \lambda((p\theta_H + (1-p)\theta_L) - \theta_L) \tag{35}$$

where $\lambda > 0$. Using this definition we have that:

$$\frac{\partial \hat{\theta}_H(\theta_H(\lambda), \theta_L(\lambda), a, p)}{\partial \lambda} = a(1-p)(\theta_H - \theta_L) = a\frac{\partial \hat{\theta}_H}{\partial a}$$

$$\frac{\partial^2 \hat{\theta}_H(\theta_H(\lambda), \theta_L(\lambda), a, p)}{\partial \lambda^2} = 0 = \frac{\partial^2 \hat{\theta}_H}{\partial a^2}$$

$$\frac{\partial \hat{\theta}_L(\theta_H(\lambda), \theta_L(\lambda), a, p)}{\partial \lambda} = -ap(\theta_H - \theta_L) = a\frac{\partial \hat{\theta}_L}{\partial a}$$

$$\frac{\partial \hat{\theta}_L(\theta_H(\lambda), \theta_L(\lambda), a, p)}{\partial \lambda} = 0 = \frac{\partial^2 \hat{\theta}_L}{\partial a^2}$$

which implies that:

$$\frac{d\Pi(\hat{\theta}_H(\theta_L, \theta_H, \lambda, a, p), \hat{\theta}_L(\theta_L, \theta_H, \lambda, a, p), T}{d\lambda} = \frac{\partial \Pi}{\partial \hat{\theta}_H}\frac{\partial \hat{\theta}_H}{\partial \lambda} + \frac{\partial \Pi}{\partial \hat{\theta}_L}\frac{\partial \hat{\theta}_L}{\partial \lambda}$$

$$= a\left(\frac{\partial \Pi}{\partial \hat{\theta}_H}\frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial \Pi}{\partial \hat{\theta}_L}\frac{\partial \hat{\theta}_L}{\partial a}\right) \tag{36}$$

$$\frac{d^2\Pi(\hat{\theta}_H(\theta_L, \theta_H, \lambda, a, p), \hat{\theta}_L(\theta_L, \theta_H, \lambda, a, p), T}{d\lambda^2} =$$

$$\frac{\partial \hat{\theta}_H}{\partial \lambda}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_H^2}\frac{\partial \hat{\theta}_H}{\partial \lambda} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_H \hat{\theta}_L}\frac{\partial \hat{\theta}_L}{\partial \lambda}\right) + \frac{\partial \hat{\theta}_L}{\partial \lambda}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_L^2}\frac{\partial \hat{\theta}_L}{\partial \lambda} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_L \hat{\theta}_H}\frac{\partial \hat{\theta}_H}{\partial \lambda}\right)$$

$$= a\frac{\partial \hat{\theta}_H}{\partial a}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_H^2}a\frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_H \hat{\theta}_L}a\frac{\partial \hat{\theta}_L}{\partial a}\right) + a\frac{\partial \hat{\theta}_L}{\partial a}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_L^2}a\frac{\partial \hat{\theta}_L}{\partial a} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_L \hat{\theta}_H}a\frac{\partial \hat{\theta}_H}{\partial a}\right)$$

$$= a^2\left\{\frac{\partial \hat{\theta}_H}{\partial a}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_H^2}\frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_H \hat{\theta}_L}\frac{\partial \hat{\theta}_L}{\partial a}\right) + \frac{\partial \hat{\theta}_L}{\partial a}\left(\frac{\partial^2 \Pi}{\partial \hat{\theta}_L^2}\frac{\partial \hat{\theta}_L}{\partial a} + \frac{\partial^2 \Pi}{\partial \hat{\theta}_L \hat{\theta}_H}\frac{\partial \hat{\theta}_H}{\partial a}\right)\right\} \tag{37}$$

Comparing (32) to (36) and (33) to (37) and noting that $a > 0$, the sign of the both the first and second derivatives of the profit function for the principal with respect to any linear-mean preserving spread is the same as the sign of that same derivative with respect to a. Therefore any statement with respect to first- and second-order behavior with respect to a is equally

valid with respect to linear-mean preserving spreads.

Proof of Proposition 6

Total expected surplus under top-down budgeting when $k < \sqrt{5} - 1$ is simply given by the principal's profits since the agent collects no rents. As shown in Proposition 5, the principal's profits are decreasing with a in this region and therefore total expected surplus is decreasing. When $k \geq \sqrt{5} - 1$, the derivative of the total expected surplus with respect to a is equal to:

$$\frac{a(k-1)(1-p)p\theta_L^2}{4T((a-1)(k-1)p+a)^2}(3k-4). \tag{38}$$

The first part of (38) is always positive while the second is positive when $k \geq 4/3$. In other words, under top-down budgeting, total surplus is increasing with a only when $k \geq 4/3$ and decreasing otherwise. Under bottom-up budgeting, differentiating total expected surplus with respect to a yields an unambiguously positive expression times:

$$1 - 7p + 17p^2 - 17p^3 + 6p^4 + k(-2p + 5p^2 - 3p^3) + k^2(p - 10p^2 + 18p^3 - 9p^4)$$

$$+ k^5(p^3 - p^4) + k^3(-p^2 + p^4) + k^4(3p^2 - 8p^3 + 5p^4) + k^6(p^3 - p^4). \tag{39}$$

For sufficiently large k, the positive k^6 term will dominate and the expression will be positive.

Proof of Proposition 7

When we define budget emphasis as:

$$E[(1 - \beta_i)\theta_i e_i] = p(1 - \beta_H)e_H\hat{\theta}_H + (1 - p)(1 - \beta_L)e_L\hat{\theta}_L, \tag{40}$$

then, under top-down budgeting when $k < \sqrt{5} - 1$, the expression becomes:

$$\frac{\hat{\theta}_L^2 \left(2\left(k^2 + k - 2\right)p - 2\sqrt{2}k\sqrt{(k-1)k}p + (k-2)^2 \right)}{(k-2)^2 T}.$$

Taking the derivative and then examining the limit as a and j go to their minimum values

reveals two negative expression, thus, under top-down budgeting, budget emphasis is initially decreasing. Similar analysis, when $k \geq \sqrt{5}-1$ yields the following conditions for when budget emphasis is increasing:

$$a > \frac{3p}{3p+1} \qquad j > \frac{3ap + a - 3p + 3}{3ap + a - 3p}.$$

In other words, if both a and j are sufficiently large, then budget emphasis is increasing under top-down budgeting. Under bottom-up budgeting (40) becomes:

$$\frac{1}{T} \frac{p\hat{\theta}_H^2 - ((-1+p)^3 \hat{\theta}_L^6}{\hat{\theta}_L^2 + p(\hat{\theta}_H^2 - 2\hat{\theta}_L^2)^2}, \tag{41}$$

the derivative of which, with respect to a yields:

$$-1 - k + (4 + 3(-1+k)k^2)p + (-5 + 3k(2 + k(2 - 4k + k^3)))p^2$$
$$+(2 + k(-6 + k(-3 + 12k - 6k^3 + k^5)))p^3, \tag{42}$$

where a positive pre-multiplier has been removed. Once again we use a Mobius transform and complete a Descrates root test to note that there is, at most a single positive root of this expression on the interval $p \in \{0, 1\}$. Evaluation of (42) at $a = 0$ yields a negative expression. Therefore, under bottom-up budgeting, the budget emphasis is initially decreasing and then, possibly, increasing with respect to a. Note that since (41) can be expressed solely in terms of $\hat{\theta}_L$ and $\hat{\theta}_H$, any shape with respect to a is shared with respect to j. See the discussion at the end of Proposition 4 for details.

Incentive strength, $(1-p)(1-\beta_L) + p(1-\beta_H)$, is weakly decreasing in j and a under top-down budgeting since, Proposition 5 found β_H to be decreasing in k when $k < \sqrt{5} - 1$ and constant thereafter. Under bottom-up budgeting, incentive strength simplifies to:

$$\frac{p\left((k^2 - 1)p - 1\right) + 1}{(k^2 - 2)p + 1},$$

which is decreasing in k.

Proof of Proposition 8

We define the performance-pay ratio as:

$$\frac{E[(1-\beta_i)e\theta_i]}{E[(1-\beta_i)e_i - \alpha_i - \frac{e^2}{2T}\theta_i]},$$

which is equal to:

$$\frac{2\left(p\left(\left(\left(k^2-2\right)kp+k+p\right)\left(\left(k^3-2k-1\right)p+k\right)+3(p-1)\right)+1\right)}{\left(k^2-1\right)(p-1)^2 p} \tag{43}$$

under bottom-up budgeting. Taking the derivative of (43) with respect to k yields:

$$\left(k^2-1\right)^2\left(2k^2-3\right)p^3 + \left(2k^4 - 4k^2 + 1\right)p^2 + 2p - 1, \tag{44}$$

where a positive pre-multiplier has been removed. For a fixed p, the expression above is increasing and unbounded in k, implying that for sufficiently large k, the derivative is increasing. Under top-down budgeting rents are equal to zero when $k < \sqrt{5} - 1$, the measure is therefore undefined therein. When $k \geq \sqrt{5} - 1$ the derivative of the performance-pay ratio with respect to k is given by:

$$-16(1+k) + 4(4+k^2)p,$$

where a positive pre-multiplier has been removed. This expression is positive as long as $k > \frac{2+2\sqrt{1+p-p^2}}{p}$.

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