Participation and unbiased pricing in CS settlement mechanisms.*

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Abstract

Credit default swaps are insurance contracts on default. Currently, there are about 30 trillion USD worth of outstanding CS contracts. These contracts are settled through a centralized mechanism. The design of this market has been criticized for underpricing the asset. In this paper, I take a mechanism design approach and charactrize settlement mechanisms that deliver unbiased price for the asset. A second contribution of my paper is a new notion of the core of a game of incomplete information. This is relevant here because participation in the settlement mechanism cannot be compelled.

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1 Introduction

A Credit Default Swap (CS) is a contract between two agents whose payoff depends on whether some third party will default on a loan. In effect, one of the agents in a CS, called the protection seller, is insuring the other agent, called the protection buyer, against a third party's inability to pay back a loan. The protection buyer pays a fee for an agreed period of time to the protection seller and in return receives loss compensation on a reference entity when a credit event occurs. The first modern CS contract was issued by JP Morgan & Co in 1994. Since then, the CS market has grown enormously. According to data from the International Swaps and Derivatives Dealers Association (ISDA), the notional amount of outstanding CDSs was 31.2 trillion USD in mid-2009.

To understand how this market works, imagine a reference entity has gone bankrupt. Each CS contract corresponds to a reference entity's bond. If AIG, the protection seller, has issued 10 CS contracts on the reference entity's bonds, AIG has to compensate the protection buyers for their loss on ten bonds. For instance, if the recovery rate of the defaulted bond was *known* to be 10 percent and the face value of the bond is $100, AIG has to pay the remaining 90 percent for each contract, which is $9000. The corresponding CS contracts can be settled via either Physical Settlement or Cash Settlement. In the case of cash settlement, the protection seller pays the face value minus the value of the defaulted bond to the protection buyer. In the case of physical settlement, the protection buyer hands the defaulted bond to the protection seller and receives the face value of the bond.

While physical settlement is the natural solution, it is often impossible to physically settle all contracts. First, in most cases the number of outstanding CS contracts is more than the number of bonds.[1] If physical settlement were the only way to settle CS contracts, it would constrain the number of CS contracts to the quantity of outstanding bonds on the reference asset.[2] Second, even if protection buyers could purchase the defaulted bonds for physical settlement, doing so would artificially raise the price of the defaulted bond. For these reasons, an alternative way of settling the contracts by cash transfer has emerged. The challenge for cash settlement is to identify a value for the defaulted bond.

To determine a value for the defaulted bond as well as a quantity of physical and cash settlement, the ISDA introduced a two stage mechanism. In the first stage of the mechanism, only agents with CS contracts participate. This first stage determines the number of defaulted bonds to be bought or sold in the second stage of the mechanism and also a price cap or floor. In the second stage, a uniform price auction determines a price for the defaulted bond. As of 2009, all CS contracts are pegged to the value of the defaulted bond determined by this mechanism, unless both protection buyer and protection seller choose to opt out. The ISDA argues that requiring all parties to a CS to be bound by the results of the mechanism ensures certainty, consistency, enhanced transparency, and liquidity. Regarding this requirement, ISDA states in their website:

"This is a major milestone in the ongoing refinement of practices and processes for the efficient, liquid and transparent conduct of the CS business," said Robert Pickel, Executive Director and Chief Executive Officer, ISDA. "Hardwiring is central to the many improvements ISDA and the industry are making to the CS contract to further ensure that infrastructure and standards for transacting these important risk management instruments are straightforward, secure and widely implemented."[3]

The mechanism used by the ISDA has been the subject of criticism. Gupta and Sundaram (2012) have observed that the defaulted bonds in this mechanism are under priced in the vast majority of

[1] As stated in Summe and Mengle (2011) at the time of Delphi Corporation's bankruptcy it is estimated that there were $28 billion in CDSs outstanding but only $2 billion in defaulted bond bonds.

[2] If short selling was facilitated in this market, in physical settlement agents instead of handing the defaulted bond the protection buyer would short sell the defaulted bond to the protection seller. Since defaulted bonds are traded over the counter short selling the defaulted bonds is impossible or hard.

In a short sale, a trader borrows stock from another agent(broker). She sells the borrowed stock. And when price of the stock drops she buys stock to give back to the broker. These defaulted bonds are traded over the counter. Short sales are not usually facilitated in this market.

[3] http://www.isda.org/press/press031209.html

auctions. Underpricing implies that the protection buyer cannot fully insure against the risk of default by the reference asset. This comes at the cost of efficiency loss. In an efficient allocation, risk neutral agents (protection sellers) should provide full insurance for risk averse agents (protection buyers) against the default risk.

The goal of any design should be to settle the contracts with unbiased prices. A settlement mechanism is unbiased if the cash settlement price is equal to the value of the defaulted bond or if agent's pay-off is equal to the pay-off from physical settlement of all contracts. In this paper, I take a mechanism design approach and look for a settlement mechanism that is unbiased. Moreover, I require four important properties which I enumerate.

1. Ex-post incentive compatible, which means that the mechanism is incentive compatible for all possible agent's belief.
2. Weakly budget balanced, which means that the designer does not have to incur a cost to execute the settlement mechanism.
3. Detail-free, which means that the settlement mechanism does not depend on the functional form of the agent's valuation of the defaulted bond.
4. Robust with respect to agent's participation plan.

The first property does not need any comment. Since there is cash transfer in the mechanism the second property is also standard. The third property is important for practical reasons, since the designer may not know the exact functional form of the valuation function. I allow the designer to ask each agent not only his private information but also his valuation of the defaulted bond given any other agent's private information. For a discussion of this property see section 6.3.

Property four is important since this contract does not compell agents to participate in the ISDA settlement mechanism. Agents may choose to settle some of their contracts outside of the settlement mechanism. I define *participation plan* to describe the agent's decision about how they participate in the central mechanism as well as how they settle contracts outside of the mechanism. Participation plans should satisfy two important criteria. First, when a group of agents make a decision about their participation, it should benefit them. Second, participation decisions should be self confirming. This means that agents update their belief about other agent's types when other agents join a coalition, and given the updated beliefs, agents choose to exit when there is a benefit from doing so. The robustness property listed above is a strong one, which requires the mechanism to be unbiased in all possible participation plans. The main theoretical innovation of this paper is to model how agents choose their level of participation in the mechanism. For more discussion see sections 4.1 and 6.1.

A mechanism that sets the cash settlement price equal to the expected value of the defaulted bond conditional on the designer's information and sets a constant cash settlement quantity[4] satisfies these properties. I show that all mechanisms with these properties must be almost surely of this form; that is, the price must be equal to the expected value of the defaulted bond and the quantity of cash settlement should not depend on agent's reported bids.

As I discussed, participation in this mechanism is voluntary. Any settlement mechanism requires participation by agents, but this cannot be enforced. Choosing not to fully participate harms the transparency of the ISDA settlement procedure and also drives away liquidity from the auction. This is important because liquidity and transparency were among the main reasons offered for a central mechanism to settle CDSs in the first place. I show that the proposed mechanism is almost surely the unique mechanism that ensures full participation and leaves no incentives for agents to sell their contracts to other agents.

I model the environment with the assumption that agents assign a common value to the defaulted bond and that each agent has a signal about the value of the defaulted bond. I propose a property for settlement mechanisms, called unravel-proofness which is related to a novel generalization of the core to the case of incomplete information. I model the case where agents have full information

[4]Cash settlement quantity may vary across agents but does not depend on reported agent's bids.

about each others' private signals, but the designer is uninformed. Also, the case of incomplete information is covered in the paper. I limit attention to detail-free mechanisms because any such mechanism should be robust against different functional forms of the agent's valuation. Detail-free mechanisms do not depend on the functional form of the common value and are ex-post incentive compatible. A similar criterion has been used by Dasgupta and Maskin (2000) and Perry and Reny (2002).

The rest of the paper is organized as follows. In section 2, I review the current literature. The environment model and the leading example are in section 3. Section 4 includes important definitions about settlement mechanisms. In section 5, for the case of complete information, I characterize the set of unbiased settlement mechanisms that insure participation and satisfy properties 1, 2, and 3. The case of incomplete information is in section 6. In section 6, with incomplete information agents, I show the only unbiased settlement mechanism that satisfies properties 1,2,3 and, 4 is almost surely a constant price and quantity mechanism. Section 7 covers possible concluding remarks.

2 Related literature

To the best of my knowledge, there are five papers written about CS contract settlement mechanism, all of which study the current settlement mechanism in use. Gupta and Sundaram (2012) observe that there is a price bias for auctions held in the 2008-2012 period. Similarly, Helwege et al. (2009) compare the mechanism price to the pre and post auction prices of the defaulted bond in a sample of early ten auctions. They find no misspricing in their sample. Coudert and Gex (2010) study the settlement procedure for a number of cases. Their empirical study also reveals the price bias in the auction. Du and Zhu (2010) develop a theoretical model. Taking the first stage of the auction as exogenous, they show how the winner's curse results in price bias in the second period. Abstracting away from an important financial friction, namely inability to short sell, they propose a double auction which correctly prices the defaulted bond. Their mechanism is essentially the physical settlement of all CS contracts. Chernove et al. (2011) document the same price bias as in Gupta and Sundaram (2012). Taking into account multiple financial frictions in the market, they solve for equilibria of the two stage auction, assuming that agents have no private information about the value of the defaulted bond. My paper is the only paper that takes a mechanism design approach. I look for mechanisms that satisfy the attractive properties enumerated in the in the introduction.

A second innovation of my paper is a new notion of the core of a game of incomplete information. I discuss its advantages relative to Myerson (2006), Liu et al. (2012), and Dutta and Vohra (2005). These papers are among the recent papers in the literature. Forges et al. (2002) provide a good survey of the older literature. For a detailed discussion see section 6.1.

3 Leading Example

To illustrate the main theoretical contribution of the paper, I use the following example. The reader may skip reading this example.

Leading Example: *There are three agents, 1, 2, and 3. There is a bond with a face value of* 100. *Assume default has happened and the value of the defaulted bond is* $E[v|s]$*, where* $s = (s_1, s_2, s_3)$ *is the signal profile held by the agents. Agent 1 is a protection seller and agents 2 and 3 are protection buyers. Agents 2 and 3 each may have 10 CS contracts with the protection seller. Assume these homogenouse CS contracts are on the bond. Therefore, there are three possible cases (networks of contracts).*

1. *Agent 2 and 3 have 10 CS contracts with agent 1.*

2. *Agent 2 has 10 CS contracts with agent 1, and agent 3 has no CS contracts.*

3. *Agent 3 has 10 CS contracts with agent 1, and agent 2 has no CS contracts.*

Denote the number of CS contracts that agent i has in case j by n_i^j. Assume $n_i^j > 0$ if agent i is a protection buyer in case j and $n_i^j < 0$ if i is a protection seller. For example, $n_1^1 = -20$ and $n_2^1 = n_3^1 = 10$.

These contracts are settled by either physical settlement or by cash settlement. In the case of physical settlement the protection buyer hands in the defaulted bond to the protection seller and in return receives 100. Therefore, the protection buyer's payoff from the physical settlement of 1 contract is $100 - E[v|s]$, and the protection seller's payoff from the physical settlement is $-(100 - E[v|s])$. In the case of cash settlement the protection seller pays the loss to the protection buyers. Therefore, if p is the price of the defaulted bond, the protection seller pays $100 - p$ to the protection buyer. Let q_i^j be the number of agent i's contracts that are settled through cash settlement and p_i^j be the cash settlement price. In this case, agent is payoff is:

$$(n_i^j - q_i^j)(100 - E[v|s]) + q_i^j(100 - p_i^j).$$

Agent is signal is either 0 or 1, $s_i \in \{0, 1\}$. Assume signals are independently distributed and $s_i = 1$ with probability $\frac{1}{2}$. The expected value of the defaulted bond conditional on signals is as follows:

$$E[v|s] = 21(2s_1 + s_2 + s_3).$$

Assume agents 1 and 2 each possesses nine defaulted bonds. Therefore, some of the contracts must be settled through cash settlement in all cases.

I describe a direct settlement mechanism. A description of a mechanism is a price and a quantity function for each agent in each network. The quantity is the number of CS contracts that are settled by cash settlement and the price is the cash settlement price. Let q_i^j and p_i^j deonote the quantity of cash settlement and cash settlement price for agent i in network j, respectively. Consider the following settlement mechanism:

$$\begin{gathered}
q_1^1(s_1, s_2, s_3) = -6 + 4s_1 - s_2 - s_3, \\
p_1^1(s_1, s_2, s_3) = 28 - 28s_1 + 8s_2 + 8s_3 + 20s_1s_2 + 20s_1s_3 + \tfrac{13}{4}s_2s_3 - \tfrac{27}{4}s_1s_2s_3, \\
q_2^1(s_1, s_2, s_3) = 3 - 2s_1 + s_2, \\
p_2^1(s_1, s_2, s_3) = 28 - 28s_1 - \tfrac{7}{4}s_2 + \tfrac{133}{4}s_1s_2 + 21s_3, \\
q_3^1(s_1, s_2, s_3) = 3 - 2s_1 + s_3, \\
p_3^1(s_1, s_2, s_3) = 28 - 28s_1 - \tfrac{7}{4}s_3 + \tfrac{133}{4}s_1s_3 + 21s_2, \\
q_1^2(s_1, s_2, s_3) = -4,\ q_2^2(s_1, s_2, s_3) = 4,\ q_3^2(s_1, s_2, s_3) = 0, \\
p_1^2(s_1, s_2, s_3) = p_2^2(s_1, s_2, s_3) = 42, \\
q_1^3(s_1, s_2, s_3) = -3.5,\ q_3^3(s_1, s_3) = 3.5,\ q_2^3(s_1, s_2, s_3) = 42, \\
p_1^3(s_1, s_2, s_3) = p_3^3(s_1, s_2, s_3) = 42.
\end{gathered}$$

One can check that these prices and quanitities gaurantee ex-post incentive compatibility. Moreover the following holds:

$$\forall s \in \{0,1\}^3 \text{ and } \forall j \in \{1,2,3\} : \sum_{i=1}^{3} q_i^j(s_1, s_2, s_3) = 0, \tag{1}$$

$$\forall s \in \{0,1\}^3 \text{ and } \forall j \in \{1,2,3\} : \sum_{j=1}^{3} q_i^j(s_1, s_2, s_3)(100 - p_i^j(s_1, s_2, s_3)) = 0. \tag{2}$$

Equation 1 is market clearing condition. Note that $q_i^j(100 - p_i^j)$ is the cash transfer that agent i receives in network j, therefore, equation 2 is the budget balanced condition. In addition to these properties, if $u_i^j(s)$ is agent is payoff from the settlement mechanism in case j, the following holds:

$$E_s[u_i^j(s)] = E_s[n_i^j(100 - E[v|s])].$$

This condition is called unbiased pricing. It means, from an ex-ante point of veiw, all contracts are settled by physical settlement or by cash settlement with the correct price, i.e., $p_i^j(s) = E[v|s]$. This mechanism is not detail-free, the design requires knowledge about the functional form of the value function.

I study agent's incentives to participate in the settlement mechanism when an arbitrary group of agents can form coalitions and settle some of their contracts with an arbitraty blocking mechanism. As an illustration, consider the settlement mechanism which I described above. I model two cases: full information case and incomplete information case.

In the full information case, agents observe each other's signals. Agents one and two may choose to settle all of their contracts outside of the mechanism using another mechanism, called the blocking mechanism. Consider a blocking mechanism in which six contracts are settled physically and four contracts are settled by cash settlement, where the cash settlement prices for agents one and three are 48 and 49, respectively. The spread, the fact that $49 > 48$, gaurantees a positive payoff for the blocking mechanism designer. Set $s_3 = 1$ and consider a game whose players are agents one and two. In this game, players, after observing the signals, simoultaneously choose to exit or stay. If both of them choose to exit, then their pay-off is their pay-off from the blocking mechanism plus the pay-off from the settlement mechanism when there are only two agents present. If at least one of them chooses to stay the coalition is not formed. In this case, their pay-off is only the pay-off from the settlement mechanism when all three agents are present. There is a Nash Equlibrium in which agents one and two both choose the following strategy: exit when the signal is zero, and otherwise stay. In other words, if u_i^e is agent i's pay-off when both agents choose to exit, the following inequalities hold:[5]

$$u_1(0,0,1) \leq u_1^e(0,0,1), \quad u_1(1,0,1) \geq u_1^e(1,0,1), \tag{3}$$
$$u_2(0,0,1) \leq u_2^e(0,0,1), \quad u_2(0,1,1) \geq u_2^e(0,1,1). \tag{4}$$

I interpret equations 3 and 4. Assume signal of agent three is 1. Equation 3 means that when the signal of agent 2 is 0, agent 1 weakly prefers the exist option when his signal is 0 and he prefers the stay option if his signal is 1. Equation 4 means that when the signal of agent 1 is 0, agent 2 weakly prefers the exist option when his signal is 0, and he prefers the stay option if his signal is 1.

In the incomplete information case, I consider a block by agents one and three. In this blocking mechanism, seven contracts are settled physically and three contracts are settled by cash settlement. The cash settlement prices for agents one and three are 30 and 38.5, respectively. The following inequalities hold:[6]

$$E_{s_2}[u_1(0,s_2,0)] \leq E_{s_2}[u_1^e(0,s_2,0)], \quad E_{s_2}[u_1(1,s_2,0)] \geq E_{s_3}[u_1^e(1,s_2,0)],$$
$$E_{s_2}[u_3(0,s_2,0)] \leq E_{s_2}[u_3^e(0,s_2,0)], \quad E_{s_2}[u_3(0,s_2,1)] \geq E_{s_2}[u_3^e(0,s_2,1)].$$

Therefore, there exists a Baysian Nash Equilibrium in which agents 1 and 3 choose the exit option when their signals are 0. In this example, when agents 1 and 3 visit the blocking mechanism, i.e., when $(s_1, s_3) = (0,0)$, the blocking designer's pay-off is $3(38.5 - 30)$.

Given this model of agent's participations, in this paper, I answer the following two questions. First, which settlement mechanism ensures all agents to participate with all of their contracts and is unbiased, budget balanced, and detail-free? Second, if we allow agents to settle a number of their

[5] *See the appendix for the calculations.*

[6] *See the appendix for the calculations.*

contracts with blocking mechanisms and take into account agent's pay-off from blocking mechanisms, which settlement mechanism is unbiased, budget balanced, and detail-free? As I will show, the answer to both questions is a settlement mechanism whose prices and quantities do not depend on the agent's signals.

4 Preliminaries

Without loss of generality, I assume the face value of the defaulted bond is 100. Each **CS contract** has a **protection buyer** and a **protection seller**. In case of a default, the protection buyer should be compensated the loss on the reference asset (bond) by the protection seller. These CS contracts are homogenous and each of them correspond to one bond. I assume that the default has happened, and I consider the contract settlement problem. Let K be the set of all agents. These agents may have CS contracts on the bond between each other. In a **contract matrix** $N = [n_{i,j}]$, agents $i, j \in K$ have net $n_{i,j}$ contracts. Assume $n_{i,j} > 0$ if j is a protection seller and i is a protection buyer: $n_{i,j} = 0$ if they do not have any CS contracts, and $n_{i,j} < 0$ if i is the protection buyer.[7] Throughout this paper, I use the words **network** and contract matrix interchangeably. I assume the number of contracts that any pair of agents have is bounded. Let Ω be the set of all possible contract matrices. It is the set of all contract matrices in which agents have at most $\bar{n} > 0$ contracts, that is,

$$\Omega = \{N = [n_{i,j}] | |n_{i,j}| < \bar{n} \ \forall i, j \in K\}.$$

Let $K(N)$ be the set of agents who have some CS contracts in N and n_i be the net number of contracts that agent $i \in K$ has, formally,

$$K(N) = \{i \in K | n_{i,j} \neq 0 \text{ for some } j \in K\} \text{ and } n_i = \sum_{j \in K(N)} n_{i,j}.$$

Each agent has a number of defaulted bonds; assume agent i has $b_i \geq 0$ defaulted bonds. Each agent has a private signal about the value of the defaulted bond. Agent is signal is drawn from S_i where S_i is a finite subset of real numbers. Given $s \in \prod_{i \in K} S_i$, a profile of agent's signals, the value of the defaulted bond is $E[v|s]$. Let $\mu(s)$ be the probability of observing the signal profile s. If $A \subseteq K$ is a subset of agents, set $S_A = \prod_{i \in A} S_i$. Given $B \subseteq A \subseteq K$ and $s \in S_A$ let $\pi_A(s) \in S_B$ be the projection of s on its B elements. I assume $E[v|s]$ is non-decreasing in agent's signals. That is, given $s_{-i} \in S_{K \setminus \{i\}}$ and $s_i, s'_i \in S_i$:

$$s_i > s'_i \Rightarrow E[v|s_{-i}, s_i] \geq E[v|s_{-i}, s'_i].$$

If q_i of agent is contracts are settled through **cash settlement** with price p_i and the rest, $n_i - q_i$, are settled by **physical settlement**, agent's payoff at signal profile $s \in S_K$ is:[8]

$$(n_i - q_i)(100 - E[v|s]) + q_i(100 - p_i).$$

One can rewrite the pay-off as follows:

$$n_i(100 - E[v|s]) + q_i(E[v|s] - p_i).$$

Where the first term is his pay-off if all of the contracts are physically settled or if the price is equal to the value of the defaulted bond. The second term can be thought of as the bias. An agent is short selling if he leaves the mechanism with net a negative number of defaulted bonds, that is: $b_i < n_i - q_i$. **No short sell** constraint is $q_i \leq n_i - b_i$.

[7] Note that $n_{i,j} + n_{j,i} = 0$.

[8] Agent i gives/gets $n_i - q_i$ of his defaulted bonds and receives/pays the face value of the bond, in addition he receives/pays his loss for the rest of the contracts.

5 Settlement Mechanisms

5.1 Description of a Mechanism

In this environment, a direct settlement mechanism takes the network and the profile of reported signals as inputs and returns a cash settlement quantity and a cash settlement price for each agent. A direct mechanism consists of functions $q_i^N : S_K \to \mathbb{R}$ and $p_i^N : S_K \to \mathbb{R}$ for all $N \in \Omega$ and $i \in K$. The **cash settlement quantity** is q_i^N, and p_i^N is the **cash settlement price** for agent i in the network N. Let $p^N = (p_i^N)_{i\in K(N)}$ and $q^N = (q_i^N)_{i\in K(N)}$ be the profile of price and quantity functions when the network is N and also $p = (p^N)_{N\in\Omega}$ and $q = (q^N)_{N\in\Omega}$ be the price and quantity profiles. Note that I allow agents to have different cash settlement prices; in other words, I am not restricting to $p_i^N = p_j^N$ for $i, j \in K$. This generality covers situations where there is a uniform price and a penalty[9], or if different contracts are settled with different cash settlement prices.

Number of defaulted bonds that are used for physical settlement must clear itself, formally, for all $N \in \Omega$ and $s \in S_K$:

$$\sum_{i\in K}(n_i - q_i^N(s)) = 0.$$

This is equivalent to $\sum_{i\in K} q_i^N(s) = 0$.

This mechanism is **ex-post incentive compatible** if for all $N \in \Omega$, $i \in K$, $s_i, s_i' \in S_i$ and $s_{-i} \in S_{K\setminus\{i\}}$:

$$(n_i - q_i^N(s_{-i}, s_i))(100 - E[v|s_{-i}, s_i]) + q_i^N(s_{-i}, s_i)(100 - p_i^N(s_{-i}, s_i)) \geq$$
$$(n_i - q_i^N(s_{-i}, s_i'))(100 - E[v|s_{-i}, s_i^p]) + q_i^N(s_{-i}, s_i')(100 - p_i^N(s_{-i}, s_i')).$$

This means that agent i with private information s_i should not find it profitable to misreport his signal as s_i', when other agent's signal profile is s_{-i}.

I use the notation (p, q, u) for a settlement mechanism with price, quantity and utility functions p_i^N, q_i^N and u_i^N for all $N \in \Omega$ and $i \in K$. Agent is payoff in the network $N \in \Omega$ when all agents are reporting their signals truthful is

$$u_i^N(s) = n_i(100 - E[v|s]) + q_i^N(s)(E[v|s]) - p_i^N(s)).$$

Since the cash settlement part of the agent's payoff, $q_i^N(s)(E[v|s]) - p_i^N(s))$, is a monetary transfer to the agent, the mechanism is **ex-post budget balanced** if for all $N \in \Omega$ and $s \in S_K$

$$\sum_{i\in K} q_i^N(s)(100 - p_i^N(s)) = 0.$$

It is **ex-post weakly budget balanced** if for all $N \in \Omega$ and $s \in S_K$

$$\sum_{i\in K} q_i^N(s)(100 - p_i^N(s)) \leq 0.$$

It is **ex-ante budget balanced** if, $E[\sum_{i\in K} q_i^N(s)(100 - p_i^N(s))] = 0$ for all $N \in \Omega$. I define **ex-ante weakly budget balanced** mechanisms similarly.

Since for each network $N \in \Omega$ sum of n_is and q_is are zero for all $s \in S_K$,

$$\sum_{i\in K} q_i^N(s)(100 - p_i^N(s)) = \sum_{i\in K} u_i(s).$$

This implies that the mechanism is ex-ante weakly budget balanced if $E[\sum_{i\in K} u_i^N(s)] \leq 0$ for all $N \in \Omega$. I restrict attention to ex-post incentive compatible and ex-ante weakly budget balanced settlement mechanisms. A mechanism has **no short sell** if $q_i(s) \leq n_i - b_i$ for all $i \in K$ and $s \in S_K$.

[9]The current mechanism in use has this form.

5.2 Participation incentives

An agent who does not have any CDS contranct is not obligated to participate in the settlement mechanism, he participates if there is a positive payoff. This motivates the following definition: a mechanism is **ex-post individually rational** if for all $N \in \Omega$, all signal profiles $s \in S_K$, all agents $i \in K$ that satisfy $n_{i,j} = 0 \ \forall j \in K$, the inequality $q_i^N(s)(100 - E[v|s]) \geq 0$ holds. It is **interim individually rational** if for all $N \in \Omega$, all signal profiles $s \in S_K$, all agents $i \in K$ that satisfy $n_{i,j} = 0 \ \forall j \in K$, the inequality $E_{s_{-i}}[q_i^N(s)(100 - E[v|s])] \geq 0$ holds.

I formally model how agents with CDS contracts may settle some of their contracts outside of the settlement mechanism. In standard mechanism design, agents can choose whether or not to participate in the mechanism, they participate when they have a non-negative payoff from participating in the mechanism. Participation in this environment differs in an important way. Agent's outside options are no longer exogenous. The agent's outside options depend on their signals as well as other agent's signals. In other words, if an agent agrees to settle a CS contract though another mechanism, it reveals information about his own private signal. I do not assume the number of contracts that pair of agents have is a private information; rather, agents are legally allowed to either sell some of their contracts prior to settlement mechanism or do not bring a number of contracts they have to the settlement mechanism.

Because of these bilateral decisions that agents can make prior to participating in the mechanism about the number of contracts, the designer may face contract matrices different from the original network of contracts. When the contract matrix is N, if a group of agents choose to settle some of their contracts outside of the settlement mechanism , the designer faces a new contract matrix, namely M. In this case M is a **reduction** of N. Formally, $M = [m_{i,j}]$ is a reduction of $N = [n_{i,j}]$ if, for all $i, j \in K$, $|m_{i,j}| \leq |n_{i,j}|$. I use the notation $M \prec N$, if M is a reduction of N. Let A to be the set of all agents who chose to settle some of their contracts outside of the settlement mechanism. Note that, $A = K(M - N)$. In this sequence, actions of agents in A have **reduced** network N to network M.

A blocking mechanism can be viewed as a settlement mechanism when the set of agents is A and the network of contracts is $N - M$.[10] A blocking mechanism has an important role, it is where all contracts that were not brought to the settlement mechanism are settled.

I present two models, complete information case and the incomplete information case.

5.2.1 Complete information case

Agents in A for a subset of their types **block** the settlement mechanism and **reduce** the network from N to M if there exists a **blocking mechanism** (p', q', u') with the following properties:

- For some prescribed subsets $S'_i \subseteq S_i$, $\forall i \in A$, and some $\bar{s}_{K\setminus A} \in \prod_{i \in K\setminus A} S_i$ the following holds:

$$\text{If } s \in S_K,\ \pi_A(s) \in \prod_{j\in A} S'_j \text{ and } \pi_{K\setminus A}(s) = \bar{s}_{K\setminus A}, \text{ then} \\ u_i^N(s) \leq u'_i(s) + u_i^M(s). \tag{5}$$

Note that $\bar{s}_{K\setminus A}$ is a profile of signals for agents that are not in the coalition. Agents in A join the coalition when their types are in the prescribed subset of types. Inequality 5 means that if all signals of agents A are in the prescribed sets, then the utility of agent $i \in A$ from the settlement mechanism with network N is not larger than his utility from the blocking mechanism plus the utility from the settlement mechanism with network M. This gives agent i incentives to join the coalition when all blocking agent's signals are in the prescribed sets.

The following should also hold:

$$\text{If } s \in S_K,\ \pi_{A\setminus\{i\}}(s) \in \prod_{j\in A\setminus\{i\}} S'_j,\ \pi_{\{i\}}(s) \in S_i \setminus S'_i \text{ and } \pi_{K(N)\setminus A}(s) = \bar{s}_{K(N)\setminus A}, \text{ then} \\ u_i^N(s) \geq u'_i(s) + u_i^M(s). \tag{6}$$

[10]The main differences are that it does not have to be budget balanced and it does not have to clear the number of defaulted bonds used.

Given $i \in A$, inequality 6 means that if agent is signal is not in S_i' and the signal of all other agents in A are in the prescribed sets, then agent is utility from the settlement mechanism with network N is not smaller than his utility from the blocking mechanism plus his utility from the settlement mechanism with network M.

- The blocking designer has a positive pay-off. Formally, the following inequality must hold:

$$E[-\sum_{i\in A}(n_i - m_i - q_i'(s))E[v|s] + q_i'(s)(100 - p_i'(s))|s \in \prod_{j\in A} S_j' \times \{\bar{s}_{K\setminus A}\}] > 0. \tag{7}$$

Inequalities 5 and 6 mean that agents in A, for a subset of their private signals, may form a coalition and settle some of their contracts with the blocking mechanism. These inequalities resemble the competing mechanism design literature.[11] Agents in the coalition, A, choose between (p^N, q^N, u^N) and (p^M, q^M, u^M) plus the blocking mechanism. Consider a game in which agents in A choose to exit the mechanism or stay. The block is formed only when all agents choose to exit. Fix a profile of signals for agents that are not in A; if for some equilibria the agents in the coalition choose the exit option over staying in the settlement mechanism with all of their contracts, then the (p, q, u) mechanism is unraveled.

To understand inequality 7, think of the blocking designer as an agent. Note that in general the blocking mechanism does not have to balance the budget or clear the number of defaulted bonds that are used for physical settlement. Since there may be surplus or deficit in monetary transfer or the number of defaulted bonds, the blocking designer's payoff may not be zero. Inequality 7 says that the blocking designer's expected pay-off conditional on the event that the block is formed must be positive. The first term that appears in the summation is the blocking designer's payoff from defaulted bonds and the second term is his payoff from the monetary transfer.

Proposition 5.1. *Let $M, N \in \Omega$ be a pair of contract matrices where M is a reduction of N. Let A be defined as in the definition of unravel-proofness and (p, q, u) be a settlement mechanism. If the sum of agent's utilities that are in A is higher when the network is N compared to that of network M, inequality 8 holds, then there is no blocking mechanism for which N is reduced to M by agents in A. Moreover, if p_i^M, q_i^M do not depend on s, then the inequality is also a necessary condition.*

$$\sum_{i\in A} u_i^N(s) \geq \sum_{i\in A} u_i^M(s) \text{ for all } s \in \prod_{i\in K} S_i \tag{8}$$

Proof. The sufficient condition's proof is by contradiction. Assume there is a blocking mechanism (p', q', u'). If one adds up inequality 5 in the definition of a blocking mechanism for all $i \in A$, then for all $s \in S_K$ such that $\pi_{K\setminus A}(s) = \bar{s}_{K\setminus A}$:

$$\sum_{i\in A} u_i^N(s) \leq \sum_{i\in A} u_i^M(s) + u_i^{'}(s).$$

Inequality 8 implies,

$$E[\sum u_i'(s)|s \in \prod_{j\in A} S_j' \times \{\bar{s}_{K\setminus A}\}] < 0.$$

Hence, the following holds:

$$\begin{aligned}&\sum_{i\in A} E[u_i^N(s)|s \in \prod_{j\in A} S_j' \times \{\bar{s}_{K\setminus A}\}] \\ &< \sum_{i\in A} E[u_i^M(s)|s \in \prod_{j\in A} S_j' \times \{\bar{s}_{K\setminus A}\}].\end{aligned}$$

[11] See Pai (2012).

Therefore for some $s \in S_K$,

$$\sum_{i \in A} u_i^N(s) < \sum_{i \in A} u_i^M(s).$$

This contradicts the assumption in the proposition.

To establish the necessary part of the proposition, let (q, p, u) be a settlement mechanism which does not satisfy (4), i.e., for some $s^* = (s_A^*, s_{K\setminus A}^*) \in S_K$:

$$\sum_{i \in A} u_i^N(s^*) < \sum_{i \in A} u_i^M(s^*). \tag{9}$$

I construct the blocking mechanism (q', p', u'). For all $i \in A$ and $s_A \in S_A$, set

$$q'{}_i(s_A) = q_i^N(s^*) - q_i^M(s^*).$$

and $p_i'(s_A) = p_i'$ where p_i' is the unique solution to

$$u_i^N(s^*) = u^M(s^*) + (n_i - m_i - q_i'(s_A))(100 - E[v|s^*]) + q_i'(s_A)(100 - p_i'). \tag{10}$$

This equation has a unique solution since the right hand side is linear in p_i'.

Set $S_i' = \{s_i'\}$ for all $i \in A$. The mechanism (p', q', u') is ex-post incentive compatible since for all $i \in A$, p_i' and q_i' do not depend on s_i. Inequality 5 is satisfied by construction. To check inequality 6, let $i \in A$ and $s_i'' \neq s_i'$. Let $s_0 \in S$ be such that $\pi_{\{i\}}(s_0) = s_i''$ and for all $j \in A \setminus \{i\}$, $\pi_{\{j\}}(s_0) = s_j'$ and $\pi_{K\setminus A}(s_0) = s_{K\setminus A}'$. If $(s_i'', s_{K\setminus\{i\}}) \in S_K$ and $s_j = s_j'$ for all $j \in I$ & $j \neq i$, incentive compatibility of the settlement mechanism and construction of the (p', q', u') mechanism imply:

$$\begin{aligned}
&u_i^N(s_0) \geq u_i^N(s') + q_i^N(s')(E[v|s_0] - E[v|s'])\\
&= u_i^M(s') + u_i'(s') + q_i^N(s')(E[v|s_0] - E[v|s'])\\
&\geq u_i^M(s_0) + u_i'(s_0) +\\
&(-q_i^M(s') - q_i'(s') + q_i^N(s'))(E[v|s_0] - E[v|s'])\\
&= u_i^M(s_0) + u_i'(s_0)
\end{aligned}$$

Because of 9 this mechanism has a budget deficit at signal profile s'.

□

A settlement mechanism (p, q, u) is **unravel-proof** if for any pair of contract matrices $M, N \in \Omega$ and $A \subseteq K$, where agents in A reduced N to M, agents in A can not form a block for a subset of their types.

I show a corollary of proposition 5.1. The following corollary ensures that the set of unravel-proof mechanisms is not empty.

Corollary 5.2. *Settlement mechanism (p, q, u) for which $p_i^N = E[v]$ and $q_i^N = n_i$ is unravel-proof.*[12]

Proof. See the appendix for the proof.

□

Proposition 4.1 implies a weak necessary condition for unravel-proof mechanisms.

The novelty in this paper is that I consider the possibility that the outside option of an agent depends on the set of other agents who exercise their outside option. This is because the agents who choose to 'exit' may decide to band together and settle some of their contracts among themselves through a different mechanism. Such a possibility was first considered in the literature on cooperative games, culminating in the notion of the core. The notion of unraveling presented in this paper is

[12] This is a mechanism that settles all of the contracts by physical settlement.

related to the block in matching theory and the block in cooperative game theory. The difference between blocking in matching theory and the notion of unraveling is that in my set up the network is predetermined and only price and quantity of cash settlement is chosen though a mechanism. Unravel-proofness is a property for a mechanism that is defined over networks but stability is defined over a possible match. Unlike similar concepts in corporate game theory my generalization can be naturally extended to environments with incomplete information. A generalization of the unravel-proof notion to environments with incomplete information is presented in section 6.

5.2.2 Incomplete information case

I extend the blocking mechanism definition to environments where agents do not know each others signals but share a prior. When agents make decisions about whether or not to join the blocking mechanism they update their belief upon observing other agent's decisions. An agent's choice to participate in a blocking mechanism reveals that his private signal is in some subset of types. Other agents take this into account when making their decisions.

For all $i \in A$, subsets $\emptyset \neq S'_i \subseteq S_i$ are called the prescribed sets. Let event E^i be defined as $E^i = \{s \in S_K | \pi_{A\setminus\{i\}}(s) \in \prod_{j \in A \setminus \{i\}} S'_j \,, \pi_{\{i\}}(s) \in S_i \setminus S'_i\}$. Define event E as $E = \{s \in S_K | \pi_A(s) \in \prod_{j \in A} S'_j\}$. Event E^i is the event that all agents in As private signals are in the prescribed sets except for agent i. Event E is the event that all agents in As private signals are in the prescribed sets. The inequalities in the blocking mechanism definition, inequalities 5 and 6, change to the following inequalities:

$$E_{s_{-i}}[u_i^N(s)|E] \leq E_{s_{-i}}[u'_i(s) + u_i^M(s)|E], \tag{11}$$

$$E_{s_{-i}}[u_i^N(s)|E^i] \geq E_{s_{-i}}[u'_i(s) + u_i^M(s)|E^i]. \tag{12}$$

To interpret 11 and 12, imagine a game whose players are agents in A. These agents after observing their private signals choose whether or not to participate in the blocking mechanism. If all of these agents decide to participate in the blocking mechanism, their payoff is their payoff from the blocking mechanism plus their payoff from the settlement mechanism when the network is M. If some of them decide not to participate in the blocking mechanism, their payoff is only the payoff from the settlement mechanism when the network is N. The mechanism is unraveled if this game has a Bayesian Nash Equilibrium in which agents in A for a subset of their types choose the blocking mechanism. With the new definition of a block, unravel-proof mechanism is naturally redefined. I call this notion of unraveling **interim unravel-proof**.

The notion of unravel-proofness under incomplete information can be interpreted as a stability condition. The set of mechanisms that satisfy the property can be interpreted as the core of the underlying game of incomplete information. This notion of a core differs from other notions introduced in the literature in three important ways.

1. Unlike Liu et al. (2012), I have a 2-sided set up and I do not assume that stability is common knowledge among the agents.

2. Unlike Dutta and Vohra (2005), agents do not rely on a common knowledge event to make an inference about other agent's types.

3. Unlike Dutta and Vohra (2005), and Myerson (2007) the blocking mechanism is incentive compatible.

I use the following example to elaborate the differences between unravel-proofness and the existing literature on cores with incomplete information.

Example 1 *Consider the set up as in the leading example. If I take the approach in Wilson (1978), the mechanism is not in the fine core, since*

$$u_1(0,0,1) \leq u_1^e(0,0,1) \text{ and } u_2(0,0,1) \leq u_2^e(0,0,1). \tag{13}$$

Since $E[u_i(s)] = n_i(100 - E[v])$, the sum of any pair of agent's payoffs from the blocking mechanism when all agents with all of their contracts are present is the same as the case where these two agents are present with a subset of their contracts. Therefore, if both agents, from an ex-ante perspective, choose the exit option over staying, the designer's pay-off is not positive. Hence, all unbiased mechanisms are in the coarse core.

Myerson (2007)s approach is similar to the definition of unravel-proofness in the case of incomplete information. The main difference is that the blocking mechanism does not have to be incentive compatible. Therefore, any incentive compatible blocking mechanism in the incomplete information case of my setup is serves as a block for Myerson (2007).

To understand Dutta and Vohra (2005)s model consider the following modification to the leading example. Assume signals are not independent. Let $p(s_1, s_2, s_3)$ is the probability of observing signal profile (s_1, s_2, s_3). Assume $p(0, 0, 1) > 0$ and

$$p(0, s_2, s_3) = 0 \ \forall (s_2, s_3) \neq (0, 1), \tag{14}$$

$$p(s_1, 0, s_3) = 0 \ \forall (s_1, s_3) \neq (0, 1), \tag{15}$$

$$p(s_1, s_2, 1) = 0 \ \forall (s_1, s_2) \neq (0, 0). \tag{16}$$

Let E be an event that $(s_1, s_2, s_3) = (0, 0, 1)$. Equations 14, 15, and 16 guarantee that E is a common knowledge event. In other words, agents 1 and 2 are able to verify whether event E has occurred or not. Inequality 13 implies that it is a block given the Dutta and Vohra (2005)s approach. Dutta and Vohra (2005)s model requires agents to rely on a common knowledge event. However, in my setup, since the signals are independent, there is no common knowledge event. Therefore, all settlement mechanisms are in the coarse core defined by Dutta and Vohra (2005).

5.3 Unbiased mechanisms

As I mentioned in section 2, several authors have criticized the current settlement mechanism in use for underpricing the bond. I look for mechanisms that overcome this issue. Since agents hold private information about the value of the defaulted bond ex-post correct pricing is not possible. However, I consider a weaker condition, I look for mechanisms that are unbiased from the ex-ante perspective. I define unbiassedly in two ways.

5.3.1 Weakly-unbiased

A **weakly-unbiased mechanism** is the one that does not over or under price the defaulted bond in expectation. Formally, Given the designer's belief, ρ, mechanism (p, q, u) is weakly unbiased if, for all $N \in \Omega$ and $i \in K$:

$$E[u_i^N] = [n_i(100 - E[v|s])]. \tag{17}$$

Equation 17 means that from an ex-ante perspective the agent's payoff from the settlement mechanism is the same as their payoff from physical settlement of all contracts or cash settlement with the price equal to the value of the defaulted bond. Note that since both price and quantity may depend on the signal profile, this condition is **not** equivalent to $E[p_i^N] = E[v]$.

Observation: *If $E[u_i^N] = n_i(100 - E[v])$, then $E[\sum_{i \in K} u_i^N] = (\sum_{i \in K} n_i)(100 - E[v]) = 0$. Therefore, a mechanism is weakly ex-ante budget balanced, if it is weakly-unbiased.*

5.3.2 Strictly-Unbiased

Since agents may settle some of their contracts outside of the settlement mechanism, agent's total payoff is not only the payoff from the settlement mechanism, rather, it should include the payoff from the blocking mechanisms. A mechanism is **strictly-unbiased,** if the agents total payoff, including the payoff from the settlement mechanism and the blocking mechanisms, from an ax-ante perspective, is equal to the agent's payoff from physical settlement of all contracts or cash settlement with the correct price.

To formally define strictly-unbiased mechanisms, I first define **participation-choice**. For some networks a group of agents may find it profitable to settle some of their contracts with a blocking mechanism. Using the techniques from the definition of unravel-proofness, I allow agents to take these actions, a mechanism is strictly-unbiased if it is unbiased regardless of these actions.

Consider an ex-post incentive compatible settlement mechanism, (p, q, u) which may not be unravel-proof. Participation-choice is a collection of sets $(P_N)_{N\in\Omega}$ where elements of P_N capture the sub networks that join a coalition. For all $N \in \Omega$ each element of $c \in P_N$ has a network $c_{Ne} \in \Omega$, a set of agents $c_A \subseteq K(c_{Ne})$, a set of type profiles $c_t = \prod_{i\in c_A} S_i^c$, where $S_i^c \subseteq S_i$, and a blocking mechanism for the c_{Ne} network,$(\hat{p}^c, \hat{q}^c, \hat{u}^c)$. This participation-choice should satisfy three conditions.

- Let $A_N(s)$ be the set of all coalitions that contain the signal profile $s \in S_{K(N)}$. Formally, $A_N(s) = \{c \in A_N | \pi_{c_A}(s) \in c_t\}$. Let $\bar{N}(s) = \sum_{c\in A_N(s)} c_{Ne}$, it must be that $N - \bar{N}(s) \prec N$. This means that whatever is left after coalitions are formed is a reduction of N.

- Agent i' payoff from participation-choice A_N is as follows:

$$u_i^{A_N}(s) = \sum_{c\in A_N(s):i\in c_A} \hat{u}_i^c(s) + u_i^{N-\bar{N}(s)}(s).$$

  Joining the coalitions for the prescribed types must be a Bayesian Nash Equilibrium. Formally, for all $N \in \omega$ and $c \in A_N$ let events E_c and E_c^i be defined as:

$$E_c = \{s \in S_{K(N)} | \pi_{A_c}(s) \in \prod_{j\in K(c_N)} S_j^c\},$$
$$E_c^i = \{s \in S_{K(N)} | \pi_{A_c}(s) \in \prod_{j\in K(c_N)-\{i\}} S_j^c \ , \pi_{\{i\}}(s) \in S_i \setminus S_i^c\}.$$

  For all $i \in c_A$, the following inequalities should hold:

$$E_{s_{-i}}[u_i^{A_N}(s)|E_c] \geq E_{s_{K(c_N)-\{i\}}}[u_i^{A_N\setminus\{c\}}(s)|E_c],$$
$$E_{s_{-i}}[u_i^{A_N}(s)|E_c^i] \leq E_{s_{-i}}[u_i^{A_N\setminus\{c\}}(s)|E_c^i].$$

- Given a coalition $c \in A_N$, when the signal profile is $s \in S_{K(N)}$, agent $i \in K(c_{Ne})$ enters the coalition c if $\pi_{c_A}(s) \in c_t$. The blocking designer's expected payoff from the blocking mechanism should be positive conditional on the event that all agents in c_A join the blocking mechanism. This is similar to inequality 7 which insures that the blocking mechanisms are self-sustaining.

Given a participation plan for each functional form and agent's prior, agent is ex-ante payoff is defined as $E[u_i^{A_N}(s)]$, for all $s \in S_K$. The mechanism is strictly-unbiased**,** if $E[u_i^{A_N}(s_{K(N)})] = E[n_i(100-E[v|s_{K(N)}])]$ for all $N \in \Omega$ and all participation plans. The mechanism is **weakly budget balanced regardless of agent's participation choice** if, for all $N \in \Omega$ and all participation plans the following holds:

$$E[\sum_{i\in K(N)} u_i^{N-\bar{N}(s_{K(N)})}(s_{K(N)})] \leq 0.$$

5.4 Other properties

In this section, I introduce and motivate three properties.

We do not want the mechanism to depend on the functional form of $E[v|s]$ or the prior μ. This is because the designer may not know the functional form. I assume that the functional form is common knowledge among the agents but the designer does not know the exact functional form and agent's priors. My interpretation of Wilson doctrine is that the designer has a belief about the functional form and agent's priors, μ. Let κ be the support of designer's belief. It consists of pairs of functional form and agent's priors, in other words, elements of κ are of the form of $(\mu, E[v|s])$. Let ρ be the designer's prior about the pair of agent's prior and value function.

In a settlement mechanism, agents, after observing their signals, report messages. A mechanism is **detail-free** if whenever the ex-post value of the defaulted bond is the same, then the prices and quantities are the same.

Here is a formal definition. Let $m(s)$ be the profile of messages when the signal profile is s. A mechanism is detail-free if for all pair of signal profiles $s^1, s^2 \in K$, all networks $N \in \Omega$, all agents $i \in K$, and all pair of valuation functions $E[v|s]$ and $E[v'|s]$, that satisfy $E[v^1|s^1] = E[v^2|s^2]$, the equalities $q_i^N(m(s^1)) = q_i^N(m(s^2))$ and $p_i^N(m(s^1)) = p_i^N(m(s^2))$ hold.

Observation: *Since ex-post the only payoff relevant object is the ex-post value of the object, namely $E[v|s]$, therefore, all full information equilibria of any settlement mechanism can be implemented by a detail free mechanism.*

Given this assumption, the property of weakly-unbiased, strictly unbiased, and weakly budget balanced are redefined naturally. Simply one should replace E with $E_\rho E_\mu$ in all this assumptions. This is because in the ex-ante perspective is the designer's perspective.

The current mechanism that is in use only takes the net number of contracts as an input and not the detail of the network of contract. This motivates the property of **robust with respect to network.** Here is the formal definition, a mechanism is robust with respect to the network if, for all pair of networks, $M = [m_{i,j}]$ and $N = [n_{i,j}]$, that satisfy, $\sum_{j \in K} m_{i,j} = \sum_{j \in K} n_{i,j}$ for all $i \in K$, the following equality holds: $\forall i \in K$, $q_i^N(s) = q_i^M(s)$ and $p_i^N(s) = p_j^M(s)$.

The last attractive property for a settlement mechanism that I introduce is **anonymity.** A settlement mechanism is anonymous if for all pair of networks $N^1 = [n_{i,j}^1], N_2 = [n_{i,j}^2] \in \Omega$, such that there exists a onto and one to one mapping $f : K \to K$ that satisfy $n_{i,j}^1 = n_{f(i),f(j)}^2$, the equalities $q_i^{N^1}(s) = q_{f(i)}^{N^2}(s)$ and $p_i^{N^2}(s) = p_{f(i)}^{N^2}(s)$ hold.

6 Results

I look for mechanisms that satisfy a number of properties that I have introduced in the previous section. Before presenting the characterization result, I introduce a class of mechanisms called the **trivial mechanisms**. For all number of contracts n, that satisfy$|n| < \bar{n}$, let $q^n \in \mathbb{R}$ be real number that satisfy two conditions: (i) $q^{-n} = -q^n$ and $q^0 = 0$. Let $q_i^N(s) = \sum_{j \in K} q^{n_{i,j}}$ and $p_i^N(s) = E_\rho E_\mu[v]$. In this class of settlement mechanisms prices and quantities do not depend on the reported signals. If $q^n = 0$ for all n, then this settlement mechanism is the physical settlement of all contracts. No short sell constraint can be easily imposed on this class of settlement mechanisms. It must be that $\forall N \in \Omega$ and $i \in K(N)$, $\sum_{j \in K(N)} q^{n_{i,j}} \leq n_i - b_i$.

Theorem 1. *If (p, q, u) is an unravel-proof, detail-free, robust with respect to network, and weakly-unbiased settlement mechanism then, almost surely it is in the class of trivial mechanisms. Moreover, all trivial mechanisms satisfy these properties.*

Proof. A Sketch of the proof is presented, for the complete proof see the appendix. If price and quantities do not depend on the signals, the mechanism is detail-free. Since for all $N \in \Omega$ and $i, j \in K$ $p_{i,j}^N = E_\rho E_\mu[v|s]$ and the q_i^N does not depend on the reported message profile, the mechanism is unbiased. To check unravel-proof condition, one needs to check the sufficient conditions in proposition 4.1. To show the reverse, I use induction. First, I establish the case where there are only two agents. For the case of two agents, I use budget balancedness and detail-freeness properties. To do the inductive step, I use the following lemma.

Lemma 6.1. *Consider the assumptions and the set up in proposition 5.1, if the settlement mechanism is unbiased and p_i^M and q_i^M do not depend on s, then almost surely for all $s \in S_K$:*

$$\sum_{i \in A} u_i^N(s) = \sum_{i \in A} u_i^M(s).$$

□

The unravel-proof property in theorem 1 is the full information notion. The same theorem still holds with the imperfect information model if I make the following assumption about the designer's belief.

Assumption 6.2. ***Full Rank Beleif*** *Let $\kappa^0 \subset \kappa$ be a zero probability set. Given the functional form $E[v|s]$ and signal s_i for agent i*

$$\text{if } \sum_{s_{-i}} \mu(s_{-i}|s_i)x(s_{-i}) = 0 \ \forall (E[v|s], \mu) \in \kappa - \kappa^0 \text{ then } x(s_{-i}) = 0 \text{ for all } s_{-i}$$

This assumption resembles the assumption in Cremer and Mclean (1988) but it is different.

Theorem 2. *Under assumption 6.2, if (p, q, u) is an interim unravel-proof, detail-free, robust with respect to network, and weakly-unbiased settlement mechanism then, almost surely it is in the class of trivial mechanisms.*

Proof. See the appendix for the proof.

□

The motivation for unravel-proofness is that participation in the mechanism ensures liquidity and transparency. However, this property can be replaced with a stronger version of the unbiasedness, strictly unbiasedness property.

Theorem 3. *Under assumption 6.2, if (p, q, u) is an interim unravel-proof, detail-free, robust with respect to network, and strictly-unbiased settlement mechanism then, almost surely it is in the class of trivial mechanisms.*

Proof. See the appendix for the proof.

□

7 Discussion

In this section I discuss the importance of some the properties listed in the theorems.

7.1 Importance of Detail-free and Unbiased Conditions

Detail-freeness property and assumption 6.2 were used only for the case of two agents. I provide a counter example without these assumptions.

Example 2: *Assume there are two agents and $\bar{n} = 1$. Agent 1 has signals $\{G_1, B_1\}$ and agent two has signals $\{G_2, B_2\}$, signals are independent with equal probability. Assume $E[v|B_1, B_2] = 1$,$E[V|B_1, G_2] = 2$,$E[v|B_2, G_1] = 3$ and $E[v|G_1, G_2] = 4$. Consider the mechanism in which $q(B_1, B_2) = 5$, $p(B_1, B_2) = 1.05$, $q(B_1, G_2) = 4$, $p(B_1, G_2) = 2.5626$, $q(B_2, G_1) = 6$, $p(B_2, G_1) = \frac{15.25}{6}$,$q(G_1, G_2) = 5$ and $p(G_1, G_2) = 4.05$. This mechanism is unbiased, unravel-proof and weakly budget balanced.*

Even though the mechanism in the example 2 has attractive properties, the design requires knowledge about the exact functional form.

From the proof of the lemma 6.1, it is easy to see that one can generalize the weakly-unbiased condition to **ex-ante uniform price** condition which requires from an ex-ante prespective the cash settlement price be the same for all agents. Formally, for some price function $p : S \to \mathbb{R}$, all $N \in \Omega$, and all $i \in K$:

$$E_\rho E_\mu[u_i^N(s)] = E_\rho E_\mu[n_i(100 - p(s))].$$

If the unbiased condition is not satisfied, I present a counter example.

Example 3: *Assume there are three agents, i, j and, k. Consider a settlement mechanism from the class of trivial mechanisms, let v_l^N be the payoff function. Design a settlement mechanism with utility function $u_l^N = v_l^N + \epsilon_l^N$. Where $(\epsilon_l^N)_{l\in\{i,j,k\}}$ is a profile of monetary transfers to agents that is uniquely identified by these equalities*

$$\epsilon_i^N + \epsilon_j^N + \epsilon_k^N = 0, \epsilon_k^N = \epsilon_j^N, \text{ and } \epsilon_i^N = \text{ the number of agents that } i \text{has contract with in } N.$$

Note that this settlement mechanism does not satisfy the ex-ante uniform price property, but it is unravel proof, detail-free, and budget balanced.

7.2 Selling CDS contracts

Agents are legally allowed to both settle some of their contracts with a settlement mechanism other than the defaulted mechanism and sell some of their settlement mechanism. This motivates to change the definition of unravel-proofness to make sure agents do not have incentives to take these two actions. Also, the definition of strictly-unbiasedness can be changed to accommodate both actions. I change the definition of network reduction as follows:

Formally, M is a reduction of N if there exists a sequence $(M^t = [m_{i,j}^t]_{i,j\in K})_{t=0}^{t=\tau}$ such that $M^0 = N$,M^1,M^2,...,$M^\tau = M$ and given M^t for $0 \leq t \leq \tau - 1$, M^{t+1} satisfies one of the following two cases:

1. Contract matrix $M^{t+1} \neq M^t$ is such that for all $i, j \in K$ $|m_{i,j}^t| \geq |m_{i,j}^{t+1}|$ and if $m_{i,j}^t \neq 0$, then $m_{i,j}^t$ and $m_{i,j}^{t+1}$ have the same sign. In this case, set of agents who took this action are all agents $i \in K$ such that $m_{i,j}^t \neq m_{i,j}^{t+1}$ for some agent $j \in K$.

2. Contract matrix $M^{t+1} \in \Omega$ is constructed from M^t by removing some contracts that are between two agents $i \in K$ and $j_1 \in K$ and adding these contracts to contracts between i and another agent, $j_2 \in K$. It must be that $m_{i,j_1}^{t+1} + m_{i,j_2}^{t+1} = m_{i,j_1}^t + m_{i,j_2}^t$. No other contract is removed or added. This is the case where agent $j_1$buys some of the contracts that are between j_2 and i from agent j_2. In this case, agents j_1 and j_2 took an action.

With this modification strictly-unbiasedness and unravel-proofness are redefined.

Theorem 4. *Given the new definitions, the property of robust with respect to the network in theorem 1,2,3 can be dropped.*

Proof. See the appendix. □

8 Concluding Remarks

I took a mechanism design approach to address the design of CS contract settlement problem. The design would have been trivial if short selling was possible, because one could settle all contracts physically. Physical settlement of all contracts would result in an ex-post unbiased and detail-free settlement mechanism. Inability to short sell makes the design problem non-trivial. An important issue considered in this paper, neglected by other authors, is participation. Any settlement mechanism should take into account this issue when making predictions regarding the settlement price and agents payoffs. The main result of the paper is that any settlement mechanism that is detail-free and from designer's ex-ante stand point sets an unbiased price is a trivial mechanism. This mechanism sets price equal to the expected value of the defaulted bond conditional on designer's information. Moreover, this mechanism is almost surely the unique mechanism that satisfy enumerated properties in the introduction. Moreover it guarantees participation by all agents.

The proposed mechanism requires the price to be equal to the value conditional on the designer's information (the publicly available information). There is no non-trivial robust mechanism that would deliver unbiased prices. The lesson to be drawn is that one can not produce unbiased prices with standard auction designs in which the price is sensetive to agent's bids.

The tool that was developed in this paper was a new approach to extend core to the case of incomplete information. I considered the 'exit game' before joining the blocking mechanism. This way of modeling can be applied to other problems such as Dark Pools. In general this can be applied to mechanism design problems in which one of designer's goals is to ensure participation and agents are allowed to get together and use another mechanism for their purpuse.

9 Appendix

9.1 Proof of inequalities in the leading example:

One can rewrite the utility that agent i receives when q_i contracts are settled physically with price p_i as $u_i = n_i(100 - E[v|s]) + q_i(E[v|s] - p_i)$. Therefore,

$$u_1(0,0,1) = n_1(100 - E[v|s_1 = 0, s_2 = 0, s_3 = 1]) + q_1(0,0,1)(E[v|s_1 = 0, s_2 = 0, s_3 = 1] - p_1(0,0,1))$$
$$= -20(100 - 21) - 7(21 - 36) = -1475$$
$$u_1(1,0,1) = n_1(100 - E[v|s_1 = 1, s_2 = 0, s_3 = 1]) + q_1(1,0,1)(E[v|s_1 = 1, s_2 = 0, s_3 = 1] - p_1(1,0,1))$$
$$= -20(100 - 63) - 3(63 - 28) = -845$$
$$u_1^e(0,0,1) = -10(100 - 21) - 4(21 - 42) - 10(100 - 21) - 3(21 - 48) = -1415$$
$$u_1^e(1,0,1) = -10(100 - 63) - 4(63 - 42) - 10(100 - 63) - 3(63 - 48) = -869$$
$$u_2(0,0,1) = 10(100 - 21) + 3(21 - 49) = 706$$
$$u_2(0,1,1) = 10(100 - 42) + 4(42 - \frac{189}{4}) = 559$$
$$u_2^e(0,0,1) = 10(100 - 21) + 3(21 - 49) = 706$$
$$u_2^e(0,1,1) = 10(100 - 42) + 3(42 - 49) = 559$$

The first term in u_i^e is agent is utility from the settlement mechanism when only agents two and three are present, and the second term is agent is utility from the blocking mechanism. For the incomplete information case I have

$$E[u_1(0,s_2,0)] = \frac{1}{2}(E[u_1(0,0,0) + E[u_1(0,1,0)]) = -20(100 - E[v|s_1 = 0, s_3 = 0]) + \frac{1}{2}(168 + 105) =$$
$$-20(100 - E[v|s_1 = 0, s_3 = 0]) + 136.5$$
$$E[u_1(1,s_2,0)] = \frac{1}{2}(E[u_1(1,0,0) + u_1(1,1,0)] = -20(100 - E[v|s_1 = 1, s_3 = 0]) + \frac{1}{2}(-84 - 105) =$$
$$-20(100 - E[v|s_1 = 1, s_3 = 0]) - 94.5$$
$$E[u_1^e(0,s_2,0)] = -20(100 - E[v|s_1 = 0, s_3 = 0]) - 3.5(10.5 - 42) - 3(10.5 - 30) =$$
$$-20(100 - E[v|s_1 = 0, s_3 = 0]) + 168.75$$
$$E[u_1^e(1,s_2,0)] = -20(100 - E[v|s_1 = 1, s_3 = 0]) - 3.5(52.5 - 42) - 3(52.5 - 30) =$$
$$-20(100 - E[v|s_1 = 1, s_3 = 0]) - 104.25$$
$$E[u_3(0,s_2,0)] = 10(100 - E[v|s_1 = 0, s_3 = 0]) - 84$$
$$E[u_3(0,s_2,1)] = 10(100 - E[v|s_1 = 0, s_3 = 1]) - 21$$
$$E[u_3^e(0,s_2,0)] = 10(100 - E[v|s_1 = 0, s_3 = 0]) - 84$$
$$E[u_3^e(0,s_2,1)] = 10(100 - E[v|s_1 = 0, s_3 = 1]) - 21$$

9.2 Proof of Corollary 5.2:

Since p_i^N and q_i^N do not depend on s, all I need to show is that the settlement mechanism satisfies the sufficient conditions in proposition 4.1. Given N and a reduced contract matrix M:

$$\sum_{i \in A} u_i^N(s) = \sum_{i \in A} n_i(100 - E[v]) =$$
$$= (100 - E[v]) \sum_{i \in A} n_i = (100 - E[v])(\sum_{i \in K} n_i - \sum_{i \in K \setminus A} n_i) = -(100 - E[v]) \sum_{i \in K \setminus A} n_i$$

By construction of A, if $i \notin A$ then $n_i = m_i$, therefore

$$
\begin{aligned}
&-(100 - E[v]) \sum_{i \in K \setminus A} n_i = -(100 - E[v]) \sum_{i \in K \setminus A} m_i \\
&= \sum_{i \in A} u_i^M(s)
\end{aligned}
$$

9.3 Proof of Theorem 1

Proof. Any mechanism in the class of trivial mechanisms satisfies the sufficient condition in proposition 5.1. Therefore, mechanisms with this form are unravel-proof.

To prove the reverse, I show the following lemma:

Lemma 9.1. *If mechanism (p, q, u) is ex-post individually rational, unravel-proof, anonymous, and ex-ante weakly budget balanced, then (i) $u_i^N(s) = 0$ for all signal profiles $s \in S_K$ and agents $i \in K$ who do not have any CDS contract in network N and (ii) the mechanism is ex-post budget balanced.*

Proof. Consider the contract matrix in which no agent has a CDS contract, namely the 0 contract matrix. Since the mechanism is anonymous, $q_i^0(s) = q_j^0(s)$ for all $i, j \in K$ and $s \in S_K$. Also since $\sum_{i \in K} q_i^0(s) = 0$, I conclude $q_i^0(s) = 0$ for all $i \in K$ and $s \in S_K$. Given $N \in \Omega$, consider a block by all agents who have CDS contracts with all of their contracts. If the mechanism is unravel-proof proposition 5.1. implies $\sum_{i \in K(N)} u_i^N(s) \geq 0$ for all $s \in S_K$. Ex-post budget balancedness implies for all $i \in K \setminus K(N)$, $u_i^N(s) \geq 0$ for all $s \in S_K$. Since the mechanism ex-ante weakly budget balanced $E_\rho E_\mu[\sum_{i \in K} u_i^N(s)] \leq 0$. The last three inequalities implies almost surely for all $i \in K \setminus K(N)$ and $s \in S_K$, $u_i^N(s) = 0$ and also $\sum_{i \in K(N)} u_i^N(s) = 0$.Therefore, $\sum_{i \in K} u_i^N(s) = 0$ for all $i \in K$.

□

I establish the induction base. If $|K(N)| = 2$, I prove u_i^N has the form described in the theorem. Assume N is a network in which agents $i, j \in K$ have n CDS contracts in which i is a protection seller and j is a protection buyer. Lemma 9.1 implies $\forall\ s \in S_K$: $u_i^N(s) + u_j^N(s) = 0$. Moreover since the mechanism is budget balanced, the following holds: $q_i^N(s)(p_i^N(s)) + q_j^N(s)(p_j^N(s)) = 0$. Therefore, $q_i^N(s) + q_j^N(s) = 0$ and $p_i^N(s) = p_j^N(s) = 0$. Let N' be a contract network in which i and j have n CDS contracts where j is a protection seller and i is a protection buyer. Since the mechanism is anonymous, the following holds.

$$
(q_i^N(s), p_i^N(s)) = (q_j^{N'}(s), p_j^{N'}(s)) \text{ and } (q_j^N(s), p_j^N(s)) = (q_i^{N'}(s), p_i^{N'}(s)). \tag{18}
$$

Let $s_i, s_i' \in S_i$ and $s_{-i} \in S_{K \setminus \{i\}}$. Ex-post incentive compatibility implies:

$$
\begin{aligned}
&q_i^N(s_i, s_{-i})(E[v|s_i, s_{-i}] - p_i^N(s_i, s_{-i})) \geq q_i^N(s_i', s_{-i})(E[v|s_i, s_{-i}] - p_i^N(s_i', s_{-i})) \text{ and} \\
&q_i^{N'}(s_i, s_{-i})(E[v|s_i, s_{-i}] - p_i^{N'}(s_i, s_{-i})) \geq q_i^{N'}(s_i', s_{-i})(E[v|s_i, s_{-i}] - p_i^{N'}(s_i', s_{-i})).
\end{aligned} \tag{19}
$$

I add up the inequalities in 19 and apply 18 and conclude:

$$
0 = [q_i^N(s_i, s_{-i}) + q_j^N(s_i, s_{-i})](E[v|s_i, s_{-i}] - p_i^N(s_i, s_{-i})) \geq [q_i^N(s_i', s_{-i}) + q_j^N(s_i', s_{-i})](E[v|s_i, s_{-i}] - p_i^N(s_i', s_{-i})) = 0 \tag{20}
$$

Equation 20 implies that inequalities in 19 hold with equality. I show $q_i^N(s_i, s_j, s_{K \setminus \{i,j\}}) = q_i^N(s_i', s_j', s_{K \setminus \{i,j\}})$, for all $s_i, s_i' \in S_i$, $s_j, s_j' \in S_j$, and $s_{K \setminus \{i,j\}} \in S_{K \setminus \{i,j\}}$. Since all incentive inequalities hold with equality I have:

$$
q_i^N(s_i, s_j', s_{K \setminus \{i,j\}}) = q_i^N(s_i', s_j', s_{K \setminus \{i,j\}}) \text{ and } q_j^N(s_i, s_j', s_{K \setminus \{i,j\}}) = q_j^N(s_i, s_j, s_{K \setminus \{i,j\}}). \tag{21}
$$

Since $q_i^N(s) + q_j^N(s) = 0$, 21 implies that $q_i^N(s_i, s_j, s_{K\setminus\{i,j\}}) = q_i^N(s'_i, s'_j, s_{K\setminus\{i,j\}})$. Therefore, q_i^N does not depend on s_i and s_j. Since the mechanism is anonymous, it does not depend on s_j for all $j \in K$.

I prove lemma 6.1.

Proof. Taking expectation from the conclusion of proposition 4.1 implies

$$\sum_{i\in A} E_\mu[u_i^N(s)] \geq \sum_{i\in A} E_\mu[u_i^M(s)] \text{ for all } s \in S. \tag{22}$$

Since the mechanism is unbiased,

$$\sum_{i\in A} E_\rho E_\mu[u_i^N(s)] = \sum_{i\in A} E_\rho E_\mu[u_i^M(s)]. \tag{23}$$

23 and the conclusion of proposition 5.1. imply that almost surely for all $s \in S_K$:

$$\sum_{i\in A} u_i^N(s) = \sum_{i\in A} u_i^M(s).$$

Note that since there are a finite number of signals, almost surely means for almost all functional forms and agent's priors.

□

Let $\kappa_{N,M} \subseteq \kappa$ be the set of functional forms and agent's priors that satisfy the lemma. I prove the theorem for $\bigcap_{M\prec N}\kappa_{M,N}$. Since there are a finite combination of $M, N \in \Omega$ and $\kappa - \kappa_N$ is of measure zero for all $N \in \Omega$, the compliment of $\cap_{M,N\in\Omega}\kappa_{M,N}$ is a measure zero set. Let $N \in \Omega$ and $N(i)$ be the set of agents that $i\in K$ has contracts with, $N(i) = \{j \in K | n_{i,j} \neq 0\}$. Let $f_N = \sum_{i\in K} |N(i)|$, I prove by induction on $\min\{f_N, |K(N)|\}$. So far, I have established the result for $f_N \leq 2$ and $|K| \leq 2$. Therefore the result is true for all $N \in \Omega$ such that $\min\{f_N, |K(N)|\} \leq 2$. Given $N \in \Omega$, where $\min\{f_N, |K(N)|\} > 2$, and $i \in K(N)$ I show u_i^N has the form described in the theorem. There are four cases:

1. $N(i)$=$\{j\}$ for some $j \in K$ and j has contracts with some other agents.
   Proof. *I construct contract matrix N^0 from N by removing all contracts except for contracts that are between agents i and j. Induction base implies $u_i^{N^0}(s) = q^{n_{i,j}}(E_\mu[v|s] - E_\rho E_\mu[v])$. Lemma 6.1 implies $u_j^{N^0}(s) = \sum_{l\in K\setminus\{i\}} u_l^N(s)$ for all $s \in S_{K(N)}$. Corollary 4.3 implies $\sum_{i\in K} u_i^N(s) = 0$ for all $s \in S_K$, therefore $u_i^N(s) = u_i^{N^0}(s)$= $q^{n_{i,j}}(E_\mu[v|s] - E_\rho E_\mu[v])$ for all $s \in S_K$. This proves the result for this case.*

2. $N(i)$=$\{j\}$ for some $j \in K$ and j has no contract with other agents and there is no agent.
   Proof. *Without loss of generality assume j is a protection seller. Since $|K(N)| > 2$, assume agents $l, l' \in K(N)\setminus\{i,j\}$ have a number of CDS contracts. Starting from the contract network N, I construct a network N' as follows: remove the contracts between i and j, add $n_{i,j}$ contracts between i and l and $n_{i,j}$ contracts between l and j, where i and l are protection sellers. Since the mechanism is robust with respect to the network of contracts, $u_i^N(s) = u_i^{N'}(s)$ for all $s \in S_K$. I Apply step 1 for the network N' and complete the proof of this step.*

3. Agent i has contracts with more than one agent, i.e., $|N(i)| > 1$.
   Proof. *Let $j, l \in N(i)$. I remove contracts between i and j and name the new network N^3. Network N^4 is constructed from N by removing the contracts between i and l. Remove contracts between i & j and i & l to construct network N^5. Induction hypothesis implies that the theorem is true for N^3, N^4 and N^5. Since j has the same contracts in N^3 and N^5 and l has the same contracts in N^4 and N^5, for all $s \in K(N)$, I have:*

$$u_j^{N^2}(s) = u_j^{N^4}(s), \tag{24}$$

$$u_l^{N^3}(s) = u_l^{N^4}(s). \tag{25}$$

Lemma 6.1 for N^3,N^4, and N^5 implies for all $s \in S_K$:

$$u_i^N(s) + u_j^N(s) = u_i^{N^3}(s) + u_j^{N^3}(s), \tag{26}$$

$$u_i^N(s) + u_l^N(s) = u_i^{N^4}(s) + u_l^{N^4}(s), \tag{27}$$

$$u_i^N(s) + u_l^N(s) + u_j^N(s) = u_i^{N^5}(s) + u_l^{N^5}(s) + u_j^{N^5}(s), \tag{28}$$

$$u_i^{N^3}(s) = \sum_{k \in K \setminus \{j\}} q^{n_{i,k}}(E_\mu[v|s] - E_\rho E_\mu[v]), \tag{29}$$

$$u_i^{N^4}(s) = \sum_{k \in K \setminus \{l\}} q^{n_{i,k}}(E_\mu[v|s] - E_\rho E_\mu[v]), \tag{30}$$

$$u_i^{N^5}(s) = \sum_{k \in K \setminus \{j,l\}} q^{n_{i,k}}(E_\mu[v|s] - E_\rho E_\mu[v]). \tag{31}$$

Add 26 and 27 and subtract 28 and Apply 29, 30, and 31 to imply:

$$u_i^N(s) = \sum_{k \in K} q^{n_{i,k}}(E_\mu[v|s] - E_\rho E_\mu[v]).$$

This proves the result for this case.

□

9.4 Proof of Theorem 2

I establish the result of lemma 6.1 for the case of interim unravel-proof mechanisms. Let $M, N \in \Omega$, $M \prec N$ and assume p_i^M and q_i^M do not depend on the reported message. If for some $\bar{s}_A \in S_A$

$$E[\sum_{i \in A} u_i^N(s)|\pi_A(s) = \bar{s}_A] < E[\sum_{i \in A} u_i^M(s)|\pi_A(s) = \bar{s}_A],$$

one can design a blocking mechanism similar to proposition 4.1 . This implies for all $\bar{s} \in S_K$

$$E[\sum_{i \in A} u_i^N(s)|\pi_A(s) = \pi_A(\bar{s})] \geq E[\sum_{i \in A} u_i^M(s)|\pi_A(s) = \pi_A(\bar{s})].$$

Since the mechanism is unbiased, for all $\bar{s} \in S_K$

$$E[\sum_{i \in A} u_i^N(s)|\pi_A(s) = \pi_A(\bar{s})] \geq E[\sum_{i \in A} u_i^M(s)|\pi_A(s) = \pi_A(\bar{s})].$$

Assumption 6.2 implies almost surely for all $s \in S_K$

$$\sum_{i \in A} u_i^N(s) = \sum_{i \in A} u_i^M(s).$$

The rest of the proof is almost identical to the proof of theorem 1.

9.5 Proof of Theorem 3

- Step 1: First I show the mechanism is almost surely ex-post budget balanced. Let $N \in \Omega$, for all $s_{K(N)} \in S_{K(N)}$ that satisfy $\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) < 0$ consider the blocking mechanism as in proposition 4.3. In this participation plan the designer observes the network N for all $s_{K(N)} \in S_{K(N)}$ that satisfy $\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) \geq 0$. Since the mechanism is weakly budget balanced it must be that for all functional forms and almost all agent's priors $\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) = 0$.

- Step 2: Let $M, N \in \Omega$ where $M \prec N$, assume $p_i^M$and q_i^M do not depend on $s_{K(M)}$ and the mechanism is ex-post budget balanced and unbiased for the M network. Consider the blocking mechanism as in proposition 4.3 for all $s_{K(N)} \in S_{K(N)}$that satisfy

$$\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) < \sum_{i \in K(N-M)} u_i^M(s_{K(N)})$$

These coalitions generate a participation plan. Let $\bar{S}_{K(N)}$be the set of all signal profiles that satisfy $\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) < \sum_{i \in K(N-M)} u_i^M(s_{K(N)})$. Since the mechanism is unbiased, for all $i \in K(N)$

$$E_\rho E_\mu[u_i^M(s_{K(N)}) I_{\{s_{K(N)} \in \bar{S}_{K(N)}\}} + u_i^N(s_{K(N)}) I_{\{s_{K(N)} \notin \bar{S}_{K(N)}\}}] = E_\rho E_\mu[n_i(100 - E[v|s_{K(N)}])]$$

Summing up theses equalities for all $i \in K(N-M)$ and definition of $\bar{S}_{K(N)}$ implies:

$$E_\rho E_\mu[\sum_{i \in K(N-M)} u_i^M(s_{K(N)})] \geq E_\rho E_\mu[\sum_{i \in K(N-M)} n_i(100 - E[v|s_{K(N)}])]$$
$$= E_\rho E_\mu[\sum_{i \in K(N-M)} m_i(100 - E[v|s_{K(N)}])$$

Since in the participation plan the M network does not unravel, unbiasedness implies the inequality must be equality. Hence, for all $s_{K(N)} \notin \bar{S}_{K(N)}$,

$$\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) = \sum_{i \in K(N-M)} u_i^M(s_{K(N)})$$

Assume $\bar{S}_{K(N)}$ is non-empty for a positive measure of designer's belief support. I have:

$$E_\rho E_\mu[\sum_{i \in K(N-M)} u_i^N(s_{K(N)})] < E_\rho E_\mu[\sum_{i \in K(N-M)} n_i(100 - E[v|s_{K(N)}])]$$

This contradicts with unbiasedness of the participation plan in which the N network does not unravel. Therefore almost surely for all $s_{K(N)} \in S_{K(N)}$

$$\sum_{i \in K(N-M)} u_i^N(s_{K(N)}) = \sum_{i \in K(N-M)} u_i^M(s_{K(N)})$$

Using step 1 and 2 and replicating the proof of theorem 1, I conclude that the mechanism is almost surely of the form described in theorem 1.

9.6 Proof of theorem 4

Since the new notions of strictly-unbiasedness and unravel-proofness are stronger and step 2 of proof of theorem 1 is the only part that uses the property of robust with respect to the network of contracts, I all I need to do is to replace this step. Here is the replacement.

1. $N(i)$=$\{j\}$ for some $j \in K$ and j has no contract with other agents and there is an agent who has contracts with at least two agents.
   Proof. *Lemma 5.2 for N and N^0 implies $u_i^N(s)+u_j^N(s) = 0$ for all $s \in S_K$.*[13] *Let $l \in K \setminus \{i, j\}$ be the agent who has contracts with two other agents. Assume $l' \in K \setminus \{i, j\}$ has contracts with agent l. Consider a coalition in which agent l' sells all of his contracts that he has with l to agent j and all other contracts except for those between i and j are settled in the blocking mechanism. Let N^1 be the reduced network. Note that $K(N^1) = \{i, j, l\}$. I prove that the result*

[13] *Note that the first induction base implies $u_i^{N^0}(s) + u_j^{N^0}(s) = 0$.*

is valid for N^1. Given N^1, remove contracts that i and j have. Induction base and lemma 5.2 imply $u_i^{N^1}(s)+u_j^{N^1}(s) = q^{n^1_{j,l}}(E_\mu[v|s]-E_\rho E_\mu[v])$. Similarly, $u_j^{N^1}(s)+u_l^{N^1}(s) = q^{n^1_{i,j}}(E_\mu[v|s]-E_\rho E_\mu[v])$. Corollary 4.3 implies $u_i^{N^1}(s)+u_j^{N^1}(s)+u_l^{N^1}(s) = 0$. Last three equalities imply the following equalities: $u_j^{N^1}(s) = q^{n^1_{j,l}}(E_\mu[v|s]-E_\rho E_\mu[v]) + q^{n^1_{j,i}}(E_\mu[v|s]-E_\rho E_\mu[v])$, $u_i^{N^1}(s) = -q^{n^1_{j,i}}(E_\mu[v|s]-E_\rho E_\mu[v]) = q^{n^1_{i,j}}(E_\mu[v|s]-E_\rho E_\mu[v])$, and $u_l^{N^1}(s) = -q^{n^1_{j,l}}(E_\mu[v|s]-E_\rho E_\mu[v]) = q^{n^1_{l,j}}(E_\mu[v|s]-E_\rho E_\mu[v]])$. Therefore, the result is true for N^1. I apply lemma 5.2 for N and N^1 and conclude:

$$\sum_{k\in K\setminus\{i\}} u_k^N(s) = \sum_{k\in K\setminus\{i\}} u_i^{N^1}(s) \text{ for all } s\in S_K.$$

Proposition 4.3 implies for all $s \in S_K$:

$$u_i^N(s) = u_i^{N^1}(s) = q^{n^1_{i,j}}(E_\mu[v|s]-E_\rho E_\mu[v]).$$

This proves the result for this case.

2. $N(i)$=$\{j\}$ for some $j\in K$ and j has no contract with other agents and there is no agent who has contracts with at least two agents.
Proof. *Since $|K(N)|>2$, assume agents $l, l' \in K(N)\setminus\{i,j\}$ have a number of CDS contracts. Construct N^1 as in case 2 and N^2 similar to case N^1 with the difference that agent l sells all of his contracts to agent j. Similar argument as in case two shows the result is valid for N^2. Lemma 5.2 implies for all $s\in S_{K(N)}$*

$$\sum_{k\in K\setminus\{i,l\}} u_k^N(s) = \sum_{k\in K\setminus\{i,l\}} u_i^{N^1}(s),$$
$$\sum_{k\in K\setminus\{j,l\}} u_k^N(s) = \sum_{k\in K\setminus\{j,l\}} u_i^{N^2}(s).$$

Since unravel-proofness implies budget balancedness,

$$\sum_{k\in\{i,l\}} u_k^N(s) = \sum_{k\in\{i,l\}} u_i^{N^1}(s), \tag{32}$$
$$\sum_{k\in\{j,l\}} u_k^N(s) = \sum_{k\in\{j,l\}} u_i^{N^2}(s). \tag{33}$$

Subtracting (12) from (11) and applying $u_i^N(s)+u_j^N(s)=0$ implies

$$2u_i^N(s) = \sum_{k\in\{i,l\}} u_i^{N^1}(s) - \sum_{k\in\{j,l\}} u_i^{N^2}(s). \tag{34}$$

Since agent l has the same number of contracts in N^1 and N^2, $u_l^{N^1}(s) = u_l^{N^2}(s)$. Since the result is true for N^1 and N^2,

$$u_i^{N^1}(s) = q^{n_{i,j}}(E_\mu[v|s]-E_\rho E_\mu[v]) \text{ and } u_j^{N^2} = q^{n_{j,i}}(E_\mu[v|s]-E_\rho E_\mu[v]).$$

Hence, $u_i^{N^1}(s) = q^{n_{i,j}}(E_\mu[v|s]-E_\rho E_\mu[v])$.

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