Facility Network Design with Equilibrium Outcomes

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1 Introduction

This article will contribute to the literature on facility network design by considering the role of competition in the evaluation of capacity investment decisions. As firms have turned to global supply networks to satisfy multiple regional demands, strategic decisions concerning the location and centralization of production must be made to balance a number of important factors: differentials in the cost of labor and inputs, proximity to key markets, and uncertainty in economic and regulatory conditions all play a critical role. Furthermore, responsive pricing and transhipment of products have enhanced the economic linkages between regional markets, making them increasingly difficult to analyze in isolation.

We are motivated by problems of network design in capacity-intensive industries, such as chemical processing, technology and auto industries. In such industries, capacity investment represents a large portion of costs, and the economics dictate that resources, once brought online, must be used at a high rate. Such industries are especially vulnerable to market uncertainties, and over-investment in capacity can spur intense competition, with the subsequent drop in prices making up-front investments difficult to recoup. Such price depressions have famously been observed in the paper, hard-drive, and auto industries, among others.

Globalization offers one measure of protection against such uncertainties by facilitating the pooling of production capacity for use in serving multiple markets, and thus reducing the associated risks. Dong et al. (2010) cite the example of the chemical manufacturer Solutia, who, after making investments for its U.S. and European markets, uses transhipment between the two markets to balance out its mismatched demand. If the cost of transhipment is not too dear, then this flexibility will reduce risks and further enable the centralization of production to where it can be done most profitably.

While there has been much focus on flexibility in the design of supply networks (see Kouvelis and Su (2005) for an overview of various design frameworks), it remains to incorporate the effect of such flexible production strategies on global market conditions. When one considers that optimal plant locations and capacity investment decisions are linked directly to the relative profitability of regional markets, such analysis becomes crucial. There are two important dependencies here to address:

i. A firm's own investments in production and distribution capacity can alter the equilibrium prices in the markets that it serves.

ii. The ability to redistribute resources between markets introduces supply-side links that affect the prevailing prices and elasticities in regional markets.

Despite these considerations, it is common practice in the literature on facility network design to either assume a deterministic set of market demands and aim exclusively at minimizing costs, or assumes a fixed price in each market while addressing the producer's ability to respond to uncertainty in relative market sizes.

Our study asks the question of how the nature of competition in regional markets, and the flexibility of competitors' production capacity impacts decisions of facility location and production centralization. We study a stylized example with a foreign (offshore) and domestic (onshore) market. As in the literature with monopolies, transportation costs and the differentials in production costs and market size will play a critical role. However, our framework allows us to consider additional factors relating to the relative competitive response across markets. Most importantly, we consider the role of the competitor's location in our investment decisions, as this impacts equilibrium outcomes directly. Furthermore, as in the literature on strategic technology choice (see Goyal and Netessine (2007) and Anand and Girotra (2007)), there is potentially an advantage to inflexibility in that it allows one to improve its competitive position by committing to production in its preferred market. Lastly, we revisit the effects of variability to see how the competitor's ability to respond to uncertainty impacts on a firm's optimal network configuration.

2 Literature Review

Our model of capacity investment is intended to integrate three heretofore loosely related areas of study: *facility network design*, *production flexibility*, and *multi-market oligopoly*. We categorize the existing literature on the basis of its approach to these three topics. Along with some seminal works in each category, we cite several studies that succeed in contributing across two of the three areas. In each case, we discuss the way in which a fully integrated approach can provide additional insights.

2.1 Facility Network Design

The rise of global supply chains has brought academic attention to various aspects of facility network design. As practical interests have evolved from a strictly cost-driven focus on offshoring to encompass integrated global logistics networks, the literature has similarly expanded its focus.

A popular framework proposed by Ferdows (1997), categorizes a firm's foreign factories by their strategic roles, and describes the way in which these roles may evolve as new competencies are developed within the plant and the organization as a whole. A foreign factories's value may be derived from low-cost production, as well as proximity to markets and access to technical expertise. While the discussion is centered on individual factories, some network-wide properties - logistics costs and protection against regulation and exchange-rate uncertainty - factor into the analysis. A more recent framework, proposed by Kouvelis and Munson (2004), takes a process-oriented view of the network design, reflective of the tightly integrated structures that have come to define global supply chains. The manufacturing process is divided into sub-assemblies, which may each be distributed across multiple geographies.

The integrated view of global supply chains, as a single facility network whose design can be optimized, lends itself naturally to analytical and computational investigation. A seminal work in the application of mathematical programming to facility network problems is the model of Cohen and Lee (1989). The authors optimize material flows through sourcing, production, and distribution across geographies. A mixed-integer linear programming approach models both the choice of facilities to open, and selection of optimal processing rates. The model of Kouvelis et al. (2010) extends previous mathematical programming approaches to capture some of the key drivers of location decisions, in the form of regional trade agreements, subsidized financing, and tax incentives from governments eager to spur investment.

We will suggest computational techniques for approaching larger scale competitive models, but the focus in this article is on strategic insight extracted from a stylized example of a global network. In this respect, our work is closest to that of Lu and Van Mieghem (2009), Dong et al. (2010), and Allon and Van Mieghem (2010). Lu and Van Mieghem (2009) study the role of uncertainty in offshoring decisions. Capacity investments are made prior to the realization of uncertainty market sizes. The authors find that, in contrast to a deterministic setting, it may be optimal to centralize production in the more costly onshore market to minimize the costs of ex-post transshipment. We employ a network structure similar to that used there, although prices in their paper are exogenous and fixed. Dong et al. (2010) study a capacity investment model with ex-post transhipment, while incorporating price-sensitive demand and the ability also to adjust prices after observing market uncertainty. Responsive pricing is found to limit sensitivity to asymmetries in cost and market size. Allon and Van Mieghem (2010) use a queuing analysis to asses the benefit of responsiveness that comes from incorporating a near-shoring capability into an offshore production strategy.

Each of the papers in this last group is important for infusing notions of production flexibility into the network design problem. We build on this literature by modeling the additional element of competition, but remain within the context of the flexible network design problem.

2.2 Production Flexibility

Beyond those papers dealing explicitly with facility location, there is a broader literature on production flexibility, with which we share some common themes. The early literature developed in the 1980's with the advent of computer-controlled flexible and manufacturer systems, and moreover, an admission at the time that American manufacturers were struggling to compete with their more nimble Japanese counterparts. The survey by Gerwin (1993) chronicles the thinking of the time, which emphasized flexibility as a strategic weapon, allowing one to enter, or threaten to enter, new markets quickly.

Fine and Freund (1990) developed a two-stage model in which a monopolist chooses investment levels in n dedicated technologies, and one fully flexible technology, prior to learning their true revenue function. The flexible technology is beneficial in handling the variability, but this benefit is balanced against additional costs. Another approach, adopted by Fine and Pappu (1990) and Roller and Tombak (1990) was to justify flexibility not through variability, but purely as a competitive weapon. Here, the value is found to be more dubious. Fine and Pappu presented the investment decision as a prisoner's dilemma, highlighting cases where neither competitor uses its flexible technology, but both are compelled to invest in flexibility as a deterrent. Roller and Tombak study two-stage games where the a technology choice is followed by Cournot competition. Here also, the investment game was characterized as a prisoner's dilemma, leading to the conclusion that firms can be better off without the *option* to invest in flexibility.

An idea that is closely related to flexibility is that of postponement. In a single product, stochastic environment, Van Mieghem and Dada (1999) look at various postponement strategies, characterized by the ordering and time-separation of capacity, quantity, and pricing decisions. In terms of this timing, we study models with upfront capacity investment, and further postponement of pricing, since the Cournot game dictates that commitments are made to production quantities before pricing. This type of model is developed further by Chod and Rudi (2005) who extend the Van Mieghem and Dada setup into a two-product setting.

Closest to our research are papers that work within the Chod and Rudi setup, or similar, but now incorporate competition, as was done by Roller and Tombak (1990) for the deterministic setting. The competitive model in Goyal and Netessine (2007) simultaneously models the benefit of flexibility under uncertainty and the strategic weakness that comes from flexibility under competition. The authors describe a tradeoff underlying the firm's technology choice, depending largely on the degree of variability. A similar phenomenon is described by Anand and Girotra (2007) in the context of "delayed differentiation". The firms we study face a similar tradeoff, but may also consider a broader set of actions, such as the ability to shape costs by choosing the location of their investment, and to impose various levels of *process flexibility*, consistent with a hybrid network design.

Given this broader focus, it is worth linking with some additional research on process flexibility. Seminal work is this area was done by Jordan and Graves (1995), who show that a process can achieve nearly all the flexibility that comes with fully flexible machines (producing all variants) through, for example, a chain of dual-purpose machines. This has been followed with similar results in a number of settings, such as that of Graves and Tomlin (2003) and Bassamboo et al. (2010). When selecting capacity levels endogenously, the *newsvendor network* approach provides a general methodology for designing an optimally flexible network. This framework, detailed in Van Mieghem and Rudi (2002), involves a set of upper-level network capacity decisions, followed by a constrained resource allocation problem to be solved after uncertainty is resolved. The problem we present is a competitive variant of a newsvendor network problem, since multiple firms allocate resources simultaneously, following the initial investment phase.

2.3 Multi-market Oligopoly

Another area of work that is related to ours is the literature on multi-market oligopoly. This research, including a significant literature involving variational inequality models, is concerned with characterization and computation of equilibrium outcomes for firms that compete across multiple, interconnected markets. We draw a distinction between our work and the papers discussed here, owing to our central focus on capacity investment decisions. In particular, we are interested in settings where there is a temporal separation between investment decisions and decisions on production and pricing, so that the equilibria we consider play out only in a second-stage of the problem. For this reason, our results bear more resemblance qualitatively to those of the previous two sections, even where that literature may not involve competition.

Nevertheless, we share much with this literature in terms of our technical approach. The book by Okuguchi and Szidarovsky (1990) studies oligopoly in a number of quite general settings. In particular, the analysis of multi-product linear Cournot games can be extended to the setting of our lower-level production game. Another series of papers, notably Miller et al. (1992) and Miller et al. (1995), use sensitivity analysis of variational inequalities to study competitive facility location problems specifically. An example in Miller et al. (1995) illustrates the impact of competitor responses on location decisions, comparing computed solutions against those of a firm who ignores competition. Additionally, Harker (1986) discusses models of spatial competition, with competition for market demands as well as transportation resources, that relate to our problem setting.

3 Model

We study a model of capacity investment for a firm competing in both a domestic market and a foreign market. The firm decides the location and capacity of production facilities, which are subsequently used in production for one or both of these markets. We look at the problem of selecting resource capacities from the perspective of a single firm, subject to the fixed capacity decisions of a competitor. After the capacities are selected, both firms compete in a Cournot fashion by simultaneously selecting product quantities to offer in each market, subject to their respective resource constraints. The model reflects the differences in timing among investment and production decisions. Investments in capacity are typically made in advance of more specific production decisions, such as the allocation of resources between product types and transshipment between regional markets.

The decision structure we have outlined is similar to that employed in the newsvendor networks of Van Mieghem and Rudi (2002). The firm invests in capacities prior to the resolution of uncertainty, and then selects production quantities, subjects to capacity constraints, in a second stage.

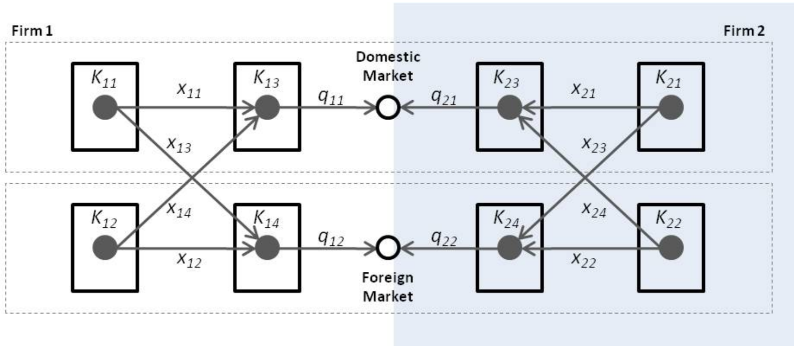

Figure 1: Network structure for two firms ($i \in \{1, 2\}$) with decision variables K_i, q_i, and x_i.

Dong et al. (2010) extend the model to the case of price-sensitive demand, and allow responsive ex-post pricing. The major distinction between our model and these setups is the introduction of competition in the ex-post portion of the model. We consider several alternative network structures, all of which fit generally into a model with four resources (see Figure 1), as employed in Lu and Van Mieghem (2009). We present the notation for this model, and subsequently specialize by placing exogenous restrictions on the use of particular resources.

For each firm $i \in \{1, 2\}$, define:

* resource capacities: $K_i \in \mathbb{R}^4_+$,
* activities: $x_i \in \mathbb{R}^4_+$,
* production quantities: $q_i \in \mathbb{R}^2_+$,
* marginal production costs: $c_i \in \mathbb{R}^4_+$.

For firm 1, capacities are selected subject to capacity costs $C_K \in \mathbb{R}^4_+$. Production costs are decomposed into:

* offshore production costs: $c_P \in \mathbb{R}_+$,
* production cost differential: $\Delta_P \in \mathbb{R}_+$,
* transportation costs: $c_T \in \mathbb{R}_+$,

so that c_1 consists of:

$$c_{11} := c_P + \Delta_P, \qquad c_{12} := c_P, \qquad c_{13} := c_P + \Delta_P + c_T \qquad c_{14} := c_P + c_T. \tag{1}$$

For firm 2, K_2 and c_2 are fixed exogenously.

We assume a linear price function in each market $m \in \{1, 2\}$. We define:

* market "size" vector: $\alpha \in \mathbb{R}^2_+$,

so that the expected market price is $E[p_m] = \alpha_m - q_{1m} - q_{2m}$.

Finally, we allow uncertainty in both prices and production costs through the additive shifters:

* price shifter: $\epsilon \in \mathbb{R}^2_+$, with $\epsilon \sim F_p(\epsilon)$ and $E[\epsilon] = 0$,
* cost shifter: $\omega \in \mathbb{R}^4_+$, with $\omega \sim F_c(\omega)$ and $E[\omega] = 0$,

for general joint probability distributions F_p and F_c respectively. We allow for a single cost shifter to interact with both firms' production costs, with the assumption that these terms represent industry wide patterns in input and transportation costs.

The network design problem we consider for firm 1 is:

$$
\begin{aligned}
\Pi^* = \max_{K_1 \in \mathbb{R}^4_+} \; & \mathbb{E}[\pi_1(K_1, \epsilon, \omega)] - C_K' K_1 \\
& S\,K_1 = \mathbf{0}.
\end{aligned}
\tag{2}
$$

where firms simultaneously solve:

$$
\begin{aligned}
\pi_i(K_1, \epsilon, \omega) = & \max_{q_i \in \mathbb{R}^2_+,\, x_i \in \mathbb{R}^4_+} \; q_i'\,(\alpha - q_1 - q_2 + \epsilon) - x_i'\,(c_i + \omega) \\
\text{subject to:} \quad & A\,x_i \le K_i \\
& R\,x_i = q_i.
\end{aligned}
\tag{3}
$$

where, as in Lu and Van Mieghem (2009):

$$
A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}
$$

and S allows for structure to be imposed exogenously on the network (by default S is a 4 x 4 matrix of zeros). The matrix A maps production activities x_i to the quantities of resource capacity that they require, and the matrix R maps each activity to a specific product output.

Within this general form, we will use the matrix S to reflect alternative production strategies, regarding the location and potential pooling of production (see Figure 2). For a *market-focused* (MF) production strategy, we require $K_{i1} = K_{i3}$ and $K_{i2} = K_{i4}$, so in order to serve both markets, the firm must invest in both of facilities 1 and 2. For a *centralized* production strategy at location $l \in \{1, 2\}$, we require $K_{i,-l} = 0$, so that all production occurs at facility l. When production is centralized, we can further distinguish between a *centralized-flexible* (CF) strategy and a *centralized-dedicated* (CD) strategy. For CF, the condition $K_{i3} = K_{i4} = K_{il}$ assures that any portion of the production at facility l can be used in either market. For CD, the condition $K_{i3} + K_{i4} = K_{il}$ requires that all production is earmarked for a specific market. Because capacity in a centralized

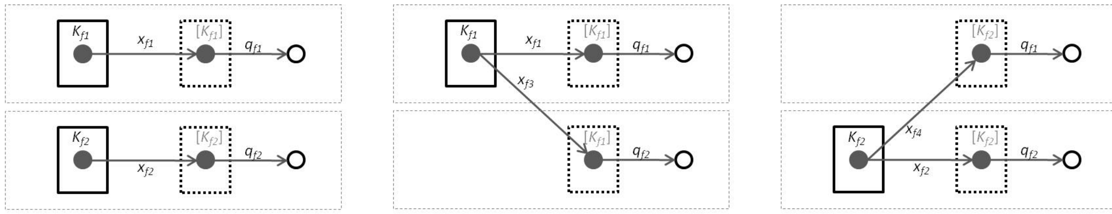

Figure 2: Network configurations for the MF, CF-1 and CF-2 strategies.

setting is characterized by a single element of K_i, we will sometimes abuse notation by referring to this element, actually K_{il}, as simply K_i.

For any finite choice of K_1, the resulting production game in (2) yields an equilibrium, with a unique choice of $q = \{q_i\}_{i \in \{1,2\}}$. This follows from reformulating (2) as an equivalent variational inequality problem, as is done for the multi-market Cournot game in Okuguchi and Szidarovsky (1990).

Proposition 1. *For any K_1, an equilibrium exists in the production game. The equilibrium is unique by q, and the second-stage profit function $\pi_1(K_1, \epsilon, \omega)$ is well-defined.*

The result in Proposition 1 applies to somewhat more general demand systems than the one we can consider. In particular, substitutions between products can be allowed by introducing a slope matrix β, so that the price vector p is determined by $p = \alpha - \beta(q_1 + q_2)$. The result will hold when β is positive definite. For the remainder, we ignore substitutions, choosing instead to focus on geographically separated markets for a single good. The more general model will be useful in extending our results to a multi-product setting.

3.1 Analysis of Production Game

In this section, we will look at the competitive quantity-setting game, subject to fixed capacities for both firms. We consider both the CF and MF configurations for firm 1, while firm 2 employs a CF configuration in both settings. Thus, we characterize the (CF,CF) and (MF,CF) strategy profiles.

For both the CF and MF market configurations, x_i is uniquely determined by q_i, so we choose to deal with q_i directly. Accordingly, the notation of costs is reduced to a 2-vector $\bar{c}_i$, so that total production and transportation costs for firm i are $\sum_{m \in \{1,2\}} q_{im} \bar{c}_{im}$. The makeup of $\bar{c}_i$ is then as follows:

$$\bar{c}_i|_{CF,\, l=1} = (c_{i1}, c_{i3}), \quad \bar{c}_i|_{CF,\, l=2} = (c_{i4}, c_{i2}), \quad \bar{c}_i|_{MF} = (c_{i1}, c_{i2}) \tag{4}$$

Accordingly, we will reduce notation for cost shifters ω to a 2-vector $\bar{\omega}$. Later on, we will want to consider the causes of $\bar{\omega}$ explicitly, but unless noted otherwise, we can restrict all stochastic effects to ϵ alone. This simplifies notation and can be done without loss of generality by considering ϵ_m to effect the margin $p_m - \bar{c}_{im}$. The objective π_i is then equal to $q_i'(\alpha - q_1 - q_2 + \epsilon)$.

The following assumption simplifies our analysis by eliminating those cases where the firm chooses not to serve one market. This is justified by observing that before investing in capacity to serve a market, a firm will typically have at least a minimal amount of information necessary to ensure a positive production quantity.

Assumption 1. *All firms produce positive quantities for both the domestic and foreign market. That is, q_{11}, q_{12}, q_{21}, and q_{22} are positive for all realizations of ϵ and ω.*

This is essentially an assumption on the symmetry of marginal production costs, providing a lower bound on the size of profit margins with respect to the production cost differential.

It will be helpful to define the quantities (for notational purposes only):

$$q_{im}^M = \frac{\alpha_m - c_{im} + \epsilon_m}{2}, \qquad q_{im}^C = \frac{\alpha_m - 2c_{im} + c_{-i,m} + \epsilon_m}{3}, \tag{5}$$

which refer, respectively, to the monopoly and Cournot equilibrium quantities that would result if capacities are ignored. Note that while these terms are independent of any of the decisions that we model, they do depend on knowledge of a specific realization of ϵ.

For a given K, and ϵ, the production game allows a closed-form solution, with the appropriate expressions depending on whether each firm's capacity constraint is binding at equilibrium. To account for all possible types of equilibria, the results are presented for regions: Ω_1 (neither firm at capacity), Ω_2 (firm 1 only at capacity), Ω_3 (firm 2 only at capacity), Ω_4 (both firms at capacity). We partition the domain of ϵ by the region of the resulting equilibrium.

Proposition 2. *The outcomes of the production game for a (CF,CF) strategy profile are as shown in Table 1.*

Region	Firm 1 production: $q_{1m}(K,\epsilon)$	Firm 2 production: $q_{2m}(K,\epsilon)$
Ω_1	q_{1m}^C	q_{2m}^C
Ω_2	$\frac{K_1}{2} + \frac{(q_{1m}^C - q_{1,-m}^C)}{2}$	$q_{2m}^M - \frac{K_1}{4} + \frac{(q_{1m}^C - q_{1,-m}^C)}{4}$
Ω_3	$q_{1m}^M - \frac{K_2}{4} + \frac{(q_{2m}^C - q_{2,-m}^C)}{4}$	$\frac{K_2}{2} + \frac{(q_{2m}^C - q_{2,-m}^C)}{2}$
Ω_4	$\frac{K_1}{2} + \frac{(q_{1m}^C - q_{1,-m}^C)}{2}$	$\frac{K_2}{2} + \frac{(q_{2m}^C - q_{2,-m}^C)}{2}$

Region	Conditions on ϵ
Ω_1	$(\epsilon_1+\epsilon_2) \leq 3(K_1 - E[q_{11}^C] - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \leq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$
Ω_2	$(\epsilon_1+\epsilon_2) \geq 3(K_1 - E[q_{11}^C] - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \leq 2(K_2 - E[q_{21}^M] - E[q_{22}^M]) + K_1$
Ω_3	$(\epsilon_1+\epsilon_2) \leq 2(K_1 - E[q_{11}^M] - E[q_{12}^M]) + K_2$ $(\epsilon_1+\epsilon_2) \geq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$
Ω_4	$(\epsilon_1+\epsilon_2) \geq 3(K_1 - E[q_{11}^C] - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \geq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$

Table 1: Characterization of the (CF,CF) production game.

We note that for any choice of K_1, either $P(\Omega_1) = 0$ or $P(\Omega_2) = 0$. If $K_1 + (\bar{c}_{11} + \bar{c}_{12}) \geq (\leq)$ $K_2 + (\bar{c}_{21} + \bar{c}_{22})$, then firm 1 (firm 2) produces to capacity only when both firms do.

For an MF strategy, the partition of Ω_2 is refined to consist of $\Omega_{2.1}$ (K_{11} is tight), $\Omega_{2.2}$ (K_{12} is tight), and $\Omega_{2.3}$ (both K_{11} and K_{12} are tight). The same applies to Ω_4.

Proposition 3. *The outcomes of the production game for a (MF,CF) strategy profile are as follows in Table 2.*

For $\epsilon \in \Omega_n$, firm 1 secures second-stage profits of

$$\pi(K_1, \epsilon)|_{\Omega=\Omega_n} = \sum_m [q_{1m}(K, \epsilon)(\alpha_m + \epsilon_m - \bar{c}_{1m} - q_{1m}(K, \epsilon) - q_{2m}(K, \epsilon))]_{\Omega=\Omega_n}$$

In general, the optimal investment solves the equation:

$$\Pi^* = \max_{K_1} \sum_n \int_{\Omega_n} \pi(K_1, \epsilon)|_{\Omega=\Omega_n} \, dF(\epsilon) \tag{6}$$

whose form will depend on the nature of uncertainty in demand. To simplify going forward, we employ the *clearance* assumption. This is used by Goyal and Netessine (2007), who justify it on the grounds of pre-commitments to produce that exist in certain industries. This is one possible justification, but more generally, we are interested in cases where the capacity constraints bite. In these cases, even if holdback is possible, at the optimal level of investment, there will be a very low probability of it being invoked. Thus, the clearance assumption will not effect our analysis of the optimal network configuration substantially.

Assumption 2. *Both firms produce to capacity. That is $Ax_i = K_i$.*

Assumption 2 simplifies the analysis by restricting results to region Ω_4 in the (CF,CF) case, and $\Omega_4.3$ in the (MF,CF) case. We will examine the effects of this assumption by comparing with a restriction to region Ω_1, and through numerical study of the unrestricted problem. In contrast to case of clearance, the restriction to Ω_1 is justified in cases where capacity is inexpensive in comparison to marginal costs of inputs and production, and so capacity is unlikely to bind at the optimal capacity. Assumption 2 remains in force unless noted otherwise.

4 Optimal Investments

We now focus on industries that can be modeled by Assumption 2. This simplification enables a closed-form solution of the optimal capacity investments, which can then be used to compare the performance of alternative network configurations.

We begin with the optimal investments for the (CF,CF) strategy profile. The location of firm 2 is known, but is of interest only through its effect on $\bar{c}_2$, which is known and will remain fixed throughout. Similarly, we describe the investment decision for firm 1 in terms of a fixed $\bar{c}_1$, which will be linked subsequently to a choice of location for firm 1's centralized production facility. Finally, we assume $C_K = (c_K, c_K, 0, 0)$. The optimal investments and profits follow.

Region	Firm 1 production: $q_{1m}(K,\epsilon)$	Firm 2 production: $q_{2m}(K,\epsilon)$
Ω_1	q_{1m}^C	q_{2m}^C
$\Omega_{2.1}$	$(\ K_{11}\ ,\ q_{12}^C\ )$	$(\ q_{21}^M - \frac{K_{11}}{2}\ ,\ q_{22}^C\ )$
$\Omega_{2.2}$	$(\ q_{12}^C\ ,\ K_{12}\ )$	$(\ q_{21}^C\ ,\ q_{22}^M - \frac{K_{12}}{2}\ )$
$\Omega_{2.3}$	K_{1m}	$q_{2m}^M - \frac{K_{1m}}{2}$
Ω_3	$q_{1m}^M - \frac{K_2}{4} + \frac{(q_{2m}^C - q_{2,-m}^C)}{4}$	$\frac{K_2}{2} + \frac{(q_{2m}^C - q_{2,-m}^C)}{2}$
$\Omega_{4.1}$	$\left(\ K_{11}\ ,\ \frac{(4q_{12}^M+3q_{12}^C+2q_{21}^M-K_{11}-2K_2)}{7}\ \right)$	$\left(\ \frac{3K_2}{7}+\Gamma_1\ ,\ \frac{4K_2}{7}-\Gamma_1\ \right)$
$\Omega_{4.2}$	$\left(\ \frac{(4q_{11}^M+3q_{11}^C+2q_{22}^M-K_{12}-2K_2)}{7}\ ,\ K_{12}\ \right)$	$\left(\ \frac{4K_2}{7}-\Gamma_2\ ,\ \frac{3K_2}{7}+\Gamma_2\ \right)$
$\Omega_{4.3}$	K_{1m}	$\frac{K_2}{2} + \frac{2q_{2m}^M - K_{1m}}{4} - \frac{2q_{2,-m}^M - K_{1,-m}}{4}$

Here, $\Gamma_m = \frac{(4q_{2m}^M - 2K_{1m} - 3q_{2,-m}^C)}{7}$.

Region	Conditions on ϵ
Ω_1	$\epsilon_1 \leq 3(K_{11} - E[q_{11}^C])$ $\epsilon_2 \leq 3(K_{12} - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \leq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$
$\Omega_{2.1}$	$\epsilon_1 \geq 3(K_{11} - E[q_{11}^C])$ $\epsilon_2 \leq 3(K_{12} - E[q_{12}^C])$ $(\frac{\epsilon_1}{2} + \frac{\epsilon_2)}{3} \leq K_2 - E[q_{21}^M] + \frac{K_{11}}{2} - E[q_{22}^C]$
$\Omega_{2.2}$	$\epsilon_1 \leq 3(K_{11} - E[q_{11}^C])$ $\epsilon_2 \geq 3(K_{12} - E[q_{12}^C])$ $(\frac{\epsilon_1}{3} + \frac{\epsilon_2)}{2} \leq K_2 - E[q_{22}^M] + \frac{K_{12}}{2} - E[q_{21}^C]$
$\Omega_{2.3}$	$\epsilon_1 \geq 3(K_{11} - E[q_{11}^C])$ $\epsilon_2 \geq 3(K_{12} - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \leq 2(K_2 - E[q_{21}^M] - E[q_{22}^M]) + K_{11} + K_{12}$
Ω_3	$\epsilon_1 \leq 2(K_{11} - E[q_{11}^M]) + \frac{K_2}{2}$ $\epsilon_2 \leq 2(K_{12} - E[q_{12}^M]) + \frac{K_2}{2}$ $(\epsilon_1+\epsilon_2) \geq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$
$\Omega_{4.1}$	$\epsilon_1 \geq 3(K_{11} - E[q_{11}^C])$ $(\epsilon_1+3\epsilon_2) \leq 7K_{12} - (4q_{12}^M + 3q_{12}^C + 2q_{21}^M - K_{11} - 2K_2)$ $(\epsilon_1+\epsilon_2) \geq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$
$\Omega_{4.2}$	$(3\epsilon_1+\epsilon_2) \leq 7K_{11} - (4q_{11}^M + 3q_{11}^C + 2q_{22}^M - K_{12} - 2K_2)$ $\epsilon_2 \geq 3(K_{12} - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \geq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$
$\Omega_{4.3}$	$\epsilon_1 \geq 3(K_{11} - E[q_{11}^C])$ $\epsilon_2 \geq 3(K_{12} - E[q_{12}^C])$ $(\epsilon_1+\epsilon_2) \geq 3(K_2 - E[q_{21}^C] - E[q_{22}^C])$

Table 2: Characterization of the (MF,CF) production game.

Proposition 4. *For the (CF,CF) strategy profile, the optimal investment for firm 1 is:*

$$K_1^{CF} = E[q_{11}^M] + E[q_{12}^M] - \frac{K_2}{2} - c_K \tag{7}$$

The total expected profits that result are:

$$\Pi_{CF}^* = \frac{(E[q_{11}^M] + E[q_{12}^M] - \frac{K_2}{2} - c_K)^2}{2} + \frac{E[(q_{11}^C - q_{12}^C)^2]}{2} \tag{8}$$

The form of (7) and (8) reveals an underlying separation between the aggregate capacity investment and the allocation of production to specific markets. The investment level K_1^{CF} is independent of the competitor's productions costs. In setting this investment level, competition is felt only through the aggregate capacity K_2. The profits in (8) decompose into a first term that depends only on this aggregate investment level, and a $\frac{E[(q_{11}^C - q_{12}^C)^2]}{2}$ term that incorporates the relative size, and competitiveness, of the two markets. As used here, the Cournot quantities, q_{1m}^C, give a competition-adjusted measure of the two market sizes. The second profit term then indicates that the firm benefits from asymmetry in the market sizes. Holding the aggregate market size constant, it is most profitable to control a single, large market than to split production equitably cross two. As long as production is centralized, this property will be seen to motivate a natural dispersion of firm locations, as each firm seeks to define a highly profitable "home" market.

Next, and again making use of exogenous costs $\bar{c}_1$, we look at the case where capacity is committed to markets in advance as in the (MF,CF) strategy. In this case, the optimal outcomes are as follows in the proposition.

Proposition 5. *For the (MF,CF) strategy profile, the optimal investment for firm 1 is:*

$$K_{1m}^{MF} = \left(E[q_{11}^M] - \frac{K_2}{4} - \frac{c_K}{2}\right) + \frac{E[q_{11}^M] - E[q_{12}^M]}{2} - \frac{E[q_{21}^M] - E[q_{22}^M]}{2} \tag{9}$$

The total expected profits that result are:

$$\Pi_{MF}^* = \frac{(E[q_{11}^M] + E[q_{12}^M] - \frac{K_2}{2} - c_K)^2}{2} + \frac{9}{16}\left(E[q_{11}^C] - E[q_{12}^C]\right)^2 \tag{10}$$

In the MF case, capacity investments are specified for the individual market. So, in contrast to the results in Proposition 4, we see that competitor costs enter into (10) through the q_2^M terms. However, we note that the aggregate investment, given here by $K_{11}^{MF} + K_{12}^{MF}$, remains in precisely the same form as for the CF case. Thus, overall capacity investment remains independent of competitor costs, regardless of the choice of production location or centralization decision.

Denote the optimal aggregate investment level by $K_1^* := E[q_{11}^M] + E[q_{12}^M] - \frac{K_2}{2} - c_K$. This allows a convenient side-by-side comparison of the profits in the centralized and market-focused regimes:

$$\Pi_{CF}^* = \underbrace{(K_1^*)^2/2}_{\text{Aggregate Investment}} + \underbrace{E^2[q_1^C - q_2^C]/2}_{\text{Ex-Ante Allocation}} + \underbrace{Var(q_1^C - q_2^C)/2}_{\text{Ex-Post Allocation}} \tag{11}$$

$$\Pi_{MF}^* = \underbrace{(K_1^*)^2/2}_{\text{Aggregate Investment}} + \underbrace{E^2[q_1^C - q_2^C]/2}_{\text{Ex-Ante Allocation}} + \underbrace{E^2[q_1^C - q_2^C]/16}_{\text{Value of Commitment}} - c_{K_o} \tag{12}$$

Here we have divided profits according to the forces driving each portion[1]. In both settings there is a profit component that depends on the size of the optimal aggregate investment. The additional terms then reflect on how effectively a firm with that particular market structure is able to allocate resources to the markets that are most profitable.

For the CF case, we have divided the remaining profits into *ex-ante* and *ex-post* allocation terms. While all allocation in the CF case is committed only after resolving market uncertainty, the ex-ante term is indicative if the average quantities that the firm commits to each market in the CF setting. This portion of profits could be achieved without using any flexibility. The last term, the value of ex-post allocation, then reflects the additional profits that are achieved by delaying the allocation decision to the production stage. This term is proportional to the variance in the asymmetry between the markets. Here, greater variance creates more opportunities to shift resources ex-post into a more attractive market.

On the other hand, with an MF structure, all capacity is committed to markets at the investment stage. With no ability to reallocate, variation in profit margins is "averaged out" of the expected profit function. There is, however, additional value to the MF firm that comes through a strategic first-mover advantage. By committing capacity to individual markets in an earlier stage, the MF firm takes up a stronger position, and is able to achieve a more favorable ex-ante allocation of resources. The resulting profit appears in 12 as the *value of commitment*. The final term c_{K_o} reflects some economies of scale by assigning an additional cost to building and operating facilities in two distinct locations.

The various profit drivers just defined are at the heart of the network design decisions that we now discuss.

4.1 Location Decision

We now focus on the centralized production configuration, and analyze the firm's decision to locate production in either the domestic ($l = 1$), or foreign ($l = 2$) market. We use the expected profits obtained in (8), and plug in for the costs $\bar{c}_1|_{l=\hat{l}}$ associated with each potential location $\hat{l}$. The relative profits will then depend critically on the magnitude of the individual components that make up overall marginal production cost; i.e. the cost savings Δ_P from offshore production and the transportation cost c_T for transshipment between markets.

To define the impact of Δ_P and c_T, we will first need to summarize the relevant market characteristics. We capture the impact of consumer demand and the competitive landscape through two factors:

* $\lambda_1 := K_1^*|_{c_1=(c_P,c_P)}$, a measure of the *aggregate market size* (note that the overall investment level, a proxy here for market size, is the same with both CF and MF strategies, modulo location-specific costs).

[1] Note that the terms K_1^*, q_1^C, and q_2^C, while defined equivalently in both settings, will vary according to differences in $\bar{c}_1$ between the CF and MF cases. The respective costs, $\bar{c}_i|_{CF,\ l}$ and $\bar{c}_i|_{MF}$ are defined in (4).

* $\delta_1 := E[q_{11}^C|_{c_1=(c_P,c_P)}] - E[q_{12}^C|_{c_1=(c_P,c_P)}]$, a measure of the (competition-adjusted) *market size differential*.

Describing the market as such, the difference in profits between onshore and offshore production can be written as:

$$\Pi^*_{CF}|_{l=1} - \Pi^*_{CF}|_{l=2} = \left(\frac{1}{2}\right)\left((\Delta_P)^2 + \Delta_P(c_T - 2\lambda_1) + \left(\frac{8}{3}\right)\delta_1 c_T\right). \tag{13}$$

We represent the optimal location decision by a threshold $\bar{\Delta}$ for the production cost differential, calculated so that the optimal strategy for locating a centralized production facility is to locate onshore ($l = 1$) when Δ_P is less than $\bar{\Delta}$.

Proposition 6. *For a (CF,CF) strategy profile, firm 1 locates production in market 1 for $\Delta_P < \bar{\Delta}$ and in market 2 for $\Delta_P > \bar{\Delta}$, where:*

$$\bar{\Delta} = \left(\frac{2\lambda_1 - c_T}{2}\right) - \sqrt{\left(\frac{2\lambda_1 - c_T}{2}\right)^2 - \left(\frac{8}{3}\right)\delta_1 c_T} \tag{14}$$

Assuming that δ_1 is positive (i.e. the onshore market is more attractive, otherwise it is always optimal to locate offshore), the threshold is increasing in c_T. This is intuitive, as a large transportation cost makes it difficult for an offshore firm to serve it's target market. The effect is enhanced as δ_1 increases and so the threshold increases in δ_1 as well. On the other hand, the threshold is decreasing in λ_1. The insight here is that as overall production increases, the impact of production costs savings from locating offshore will factor more heavily.

As presented, none of the terms appearing in (14) depend on the variance of ϵ. This is the case when the distribution of ϵ is not itself allowed to depend on the location choice. Indeed, if the distribution of ϵ is fixed, then the ex-post allocation value is the same for either production location. Such behavior is sure to arise when the impact of cost shifters, ω, is negligible, leaving ϵ to act only on prices. Some interesting behavior arises when relaxing this restriction to allow for location-specific cost shifters. For example, if both firms are subjected to the same fluctuations in transportation costs, and firm 2 is similarly located in one of the two markets considered, then we replace δ_1 with $(\delta_1 + I\frac{\sigma^2}{3})$, where $I = 1$ when firm 2 locates offshore, and $I = -1$ when firm 2 locates onshore. In this case, variability in costs motivates a location opposite that of the competing firm.

4.2 Centralization of Production

We now examine the conditions under which either a centralized or market-focused strategy emerge as the optimal network configuration. To gain intuition regarding the optimal decision, we will need to distinguish between several effects that in concert account for the differences in outcome between the CF and MF strategies.

We begin with location held fixed, isolating the profit difference that results purely from the distinction between dedicated and flexible capacity. This distinction was illustrated in the equations

(8) and (10), in which a tradeoff is established between the value of early commitment in the MF setting and of ex-post production allocation in the CF setting. Here we express the net value of dedicated capacity in terms of c_T, δ_P, and market characteristics. The effect, which depends on the location considered for centralized production, is then:

$$V_{DED}|_{l=1} = (\Pi^*_{CD}|_{l=1} + c_{K_o}) - \Pi^*_{CF}|_{l=1} = \underbrace{\frac{(\delta_1 + (2/3)c_T)^2}{16}}_{\text{Value of Commitment}} - \underbrace{\frac{\sigma^2_{\{\epsilon_1 - \epsilon_2\}}}{18}}_{\text{Ex-Post Allocation}} \tag{15}$$

$$V_{DED}|_{l=2} = (\Pi^*_{CD}|_{l=2} + c_{K_o}) - \Pi^*_{CF}|_{l=2} = \underbrace{\frac{(\delta_1 - (2/3)c_T)^2}{16}}_{\text{Value of Commitment}} - \underbrace{\frac{\sigma^2_{\{\epsilon_1 - \epsilon_2\}}}{18}}_{\text{Ex-Post Allocation}} \tag{16}$$

As expected, the value decreases with both the degree of variability and the fixed cost of operating a second facility. The value of commitment increases with the degree of asymmetry between markets. Assuming $\delta_1 > (2/3)c_T > 0$, the effective asymmetry increases with c_T for an onshore firm, while it decreases in c_T for a firm located in the foreign market.

The second effect to consider in choosing network structure is the *localization* value inherent in a system with two production facilities. The MF setup allows for both markets to be served without incurring transportation costs, which generates additional revenue and cost savings associated with the MF strategy. This is strictly an advantage when comparing to onshore centralization. When compared to offshore centralization, advantages are mitigated by the production cost savings, Δ_P, that the MF firm gives up in its domestic production facility.

Fixing again the location of centralized production, this value of localization is:

$$\begin{aligned} V_{LOC}|_{l=1} &= \Pi^*_{MF} - \Pi^*_{CD}|_{l=1} = \frac{(\Delta_P + c_T)}{8}\left[4\lambda_1 - 6\delta_1 - 3c_T - \Delta_P\right] && (17) \\ V_{LOC}|_{l=2} &= \Pi^*_{MF} - \Pi^*_{CD}|_{l=2} = \frac{(\Delta_P - c_T)}{8}\left[-4\lambda_1 - 6\delta_1 + 3(\Delta_P + c_T)\right] && (18) \end{aligned}$$

The onshore term $V_{LOC}|_{l=1}$ will generally be positive and increasing in the amount of transhipment to the foreign market that is expected when centralizing onshore. The directional relationships in $V_{LOC}|_{l=2}$ depend ultimately on the sign of $(\Delta_P - c_T)$, which determines the network structure that can most efficiently serve the onshore market.

Combining these effects, and the threshold from Proposition 6, yields a full characterization of the optimal network configuration, in terms of δ_P and c_{K_0}.

Proposition 7. *If $\Delta_P < \bar{\Delta}$ it is optimal to localize production for $c_{K_0} < \bar{c}_{ON} := V_{DED}|_{l=1} + V_{LOC}|_{l=1}$, and to centralize production in the domestic market otherwise. If $\Delta_P > \bar{\Delta}$ it is optimal to localize production for $c_{K_0} < \bar{c}_{OFF} := V_{LOC}|_{l=2} + V_{COM}|_{l=2}$, and to centralize production in the foreign market otherwise.*

In terms of the decision to centralize, we note that spatial aspect of the problem adds a dimension to the decision that does not factor into related decisions about production flexibility that occur within the context of a fixed location. One consideration is the localization value defined above to reflect changes in cost structure. In addition, we consider that in comparison to the MF firm, a firm choosing to "centralize" may still choose its optimal location endogenously. This shrinks the optimality region for dedicated capacity below what either $\bar{c}_{ON}$ or $\bar{c}_{OFF}$ suggest, since the dedicated profits must exceed those of *both* centralized locations to justify the decentralized approach.

Figure (4) presents a stylized rendering of the decision space defined in Proposition 6. Note that when variability is large enough (and negatively correlated between markets), it may be the case that $\bar{c}_{ON}$ or $\bar{c}_{OFF}$ intersect below the Δ_P axis, indicating that centralization is preferred even when there are no economies of scale in using fewer facilities. In the remainder of the paper, we discuss factors affecting the shape of these optimality regions, defined by the three thresholds $\bar{c}_{ON}$, $\bar{c}_{OFF}$, and $\bar{\Delta}_P$.

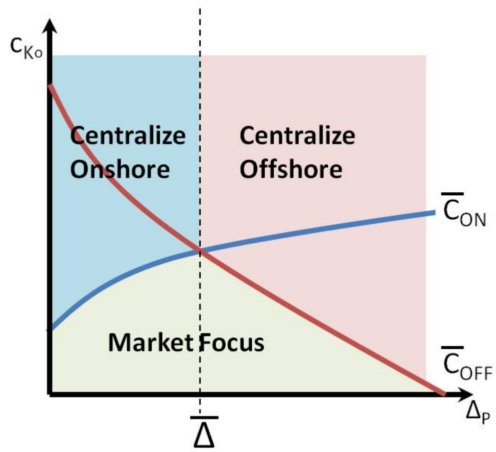

Figure 3: Optimal network configurations, determined by production cost differential and facility setup cost.

5 Impact of Competition

We have so far described the basic thresholds determining optimal network configurations, and addressed their sensitivity to underlying market characteristics. We will now characterize the way that competition may impact these thresholds, and by extension, influence the optimal choice of network configuration. There are three different types of effects that need to be considered when competition is introduced into market. These are, respectively:

- ∗ introduction of additional production capacity for supplying global markets in aggregate;
- ∗ skewing of relative market profitability due to asymmetry in competitor's cost structure;

* competitive influence on product allocation/transshipment (i.e., the distinction between the equilibrium and monopoly allocation of resources)

In some instances, these forces are aligned, making it simple to understand the impact of additional competition. Often, however, these forces will conflict, with the dominant force depending on the particulars of a firm's market position. As such, care is required in drawing broad conclusions about the effect of competition on network design.

In this section, we characterize the sensitivity of location and centralization choices to each of the these three effects. The third effect, that of the equilibrium allocation, can be considered a "market-neutral" effect of adding competition, in the sense that the measures of aggregate (λ) and relative (δ) profitability are held constant. The other two effects allow a more nuanced consideration of competition by accounting for the resulting changes to indicators of aggregate and relative market profitability. A full view should integrate these effects. For instance, while purely allocation-based effects will encourage an offshore location for centralized production, there is tension between the changes in aggregate industry capacity that can result from the entrance of a large competitor. In the face of such a large competitor, the dominant effect may be that which encourages on onshore facility, making it best to cultivate a competitive advantage in a key market at the expense of head-to-head competition on a broader global scale.

	ON → OFF ($\bar{\Delta}$)	MF → ON ($\bar{c}_{ON}$)	MF → OFF† ($\bar{c}_{OFF}$)
Aggregate Capacity: K_2	$\Uparrow$	$\Downarrow$	$\Delta_P > c_T$: $\Uparrow$ $\Delta_P < c_T$: $\Downarrow$
Cost Differential: $(c_{22} - c_{21})$	$\Uparrow$	$\Updownarrow$	$\Delta_P > c_T$: $\Updownarrow$ $\Delta_P < c_T$: $\Uparrow$
Competitive Allocation*	$\Downarrow$	$\Uparrow$	$\Delta_P > c_T$: $\Uparrow$ $\Delta_P < c_T$: $\Updownarrow$

**compared to monopoly with same λ_1, δ_1*
† assumes $q_1^C|_{OFF} > q_2^C|_{OFF}$

Table 3: Impact of competition on network configuration thresholds.

5.1 Competitor's Aggregate Capacity

In terms of the threshold values - $\bar{\Delta}$, $\bar{c}|_{ON}$, and $\bar{c}|_{OFF}$ - introduced prior, the impact of a competitor's aggregate capacity is felt through the aggregate investment term, λ_1. Recall from (7) and the definition of λ_1 that in increase in K_2 leads to a decrease in λ_1.

For the location threshold $\bar{\Delta}$, Table 3 indicates that λ_1 and δ_1 have opposing effects. When the aggregate term dominates, offshore investment becomes optimal for a wide range of Δ_P. When the asymmetry term dominates, the opposite holds. The net result is that we find $\bar{\Delta}$ to increase with K_2.

More generally, we find that a larger aggregate investment on the part of firm 1 motivates a configuration with the lowest combined production cost, defined as the sum of the costs involved in serving the onshore and the offshore market. As a result, the centralization threshold when $\Delta_P < \bar{\Delta}$, $\bar{c}|_{ON}$, is decreasing in K_2. For $\Delta_P > \bar{\Delta}$, we must divide the analysis into two cases, as the network structure having the lowest combined production costs is dependent on the relative size of the transportation and production cost differentials Δ_P and c_T.

5.2 Competitor's Cost Differential

We next look at the competitor cost differential $\bar{c}_{22} - \bar{c}_{21}$, which measures the size of the (possibly negative) cost advantage that firm 2 has in serving the onshore market. Here the impact on the location threshold is felt through the δ_1 term, rather than through the aggregate term, λ_1. The overall impact is to encourage firm 1 to locate where the costs are larger for firm 2. To see this, recall the convexity of the profit function with respect to the vector of competition-adjusted market sizes. This property dictates that firms can profit more when they are differentiated so that each dominates a particular market than they can when competing on equal footing in each.

The impact of cost differentials on the centralization threshold is more complicated. Recalling Proposition 7, we see that δ_1 appears in one term representing location advantages, and a second term related to the value of pre-commitment in a competitive setting. When these two effects are at odds, we can see either effect dominate, and so both outcomes are possible.

The "localization effect" is such that a larger cost differential motivates firm 1 to assume the network structure from which the onshore market can be served most cheaply. The resulting effects are to decrease $\bar{c}_{ON}$, decrease $\bar{c}_{OFF}$ for $\Delta_P > c_T$, and increase $\bar{c}_{OFF}$ for $\Delta_P < c_T$.

The commitment effect is included to account for those additional profits that can be achieved from a market-focused approach, owing to the ability of firm 1 to pre-commit its desired production allocation. This benefit is largest when there is a substantial asymmetry between markets, since by locking in a large production quantity in the more attractive onshore market, the firm discourages competitors and helps to insure itself a large share of these profitable sales. Thus, the commitment effect related to an increase in δ_1 is to raise both centralization thresholds, $\bar{c}_{ON}$ and $\bar{c}_{OFF}$, enlarging the optimal region for the MF configuration.

5.3 Competitive Influence on Allocation

The effects of adding competition (note that similar effects apply when a competitor's dedicated capacity is replaced with a flexible setup) are also multifaceted. To illustrate, we compare the equations below, partitioning profits for a monopolist firm,

$$\Pi^*_{CF,M} = \underbrace{(K_1^*)^2/2}_{\text{Aggregate Investment}} + \underbrace{E^2[q_1^M - q_2^M]/2}_{\text{Ex-Ante Allocation}} + \underbrace{Var(q_1^M - q_2^M)/2}_{\text{Ex-Post Allocation}} \tag{19}$$

$$\Pi^*_{MF,M} = \underbrace{(K_1^*)^2/2}_{\text{Aggregate Investment}} + \underbrace{E^2[q_1^M - q_2^M]/2}_{\text{Ex-Ante Allocation}} - c_{K_o}, \tag{20}$$

to those of a competitive firm, displayed previously in (11) and (12).

The first effect that we note relates to the switch between q_m^C and q_m^M in the allocation terms. The differences between these terms is two-fold. Note that we can write $(q_1^C - q_2^C) = \frac{2}{3}(q_1^M - q_2^M) + \frac{c_{22}-c_{21}}{3}$. The $\frac{c_{22}-c_{21}}{3}$ term represents the impact of competitor's costs, discussed in the previous subsection. However the second effect of competition is a rescaling of $(q_1^M - q_2^M)$ so that firm 1's profit margins factor less into the allocation terms under competition. In either setting, competitive or monopoly, firm 1 can increase its profits by locating closer to the more attractive (we assume this is onshore) market, thus producing more asymmetric profit margins. However, competition diminishes this effect. As gains from allocation are dependent on market asymmetries, this implies that this "allocation effect" of adding competition is to shift thresholds in the directions opposing those of the "localization effect", which was discussed above for an increase in δ. That is, since the addition of competition makes it more difficult for the firm to achieve its best allocation of production, the firm under competition invests as it would for a more symmetric pair of markets.

There is a second effect on the allocation that is again related to the commitment value of the market-focused approach. Regardless of the value of δ, there is a non-negative commitment value which increases all centralization thresholds when competition is added. When $\Delta_P < c_T$, this effect will go opposite the first for $\bar{c}_{OFF}$, and so the net effect is undetermined.

6 Numerical Results

In the above, we have simplified certain aspects of the network design problem to facilitate a closed-form analysis. The following numerical study will look at what happens when some of these restrictions are relaxed. The results put our assumptions into perspective, and provide some additional insight on how firms may plan for uncertainty with their network design.

6.1 Location without Clearance

Throughout Sections 4 and 5, we have held Assumption 2 in force, guaranteeing that the solution of the production game falls in region Ω_4 of tables (1) and (2). The results above are thus best-suited for the case where investments are costly enough that neither firm is able to hold much spare capacity.

To relax this assumption, we begin by taking as counterpoint the location threshold in cases where marginal production costs dominate investments costs, so that both firms essentially produce as if unconstrained. This corresponds to the area denoted as region Ω_1 in figures (1) and (2).

Under this alternative assumption, the difference in profits between onshore and offshore production is:

$$\Pi^*_{CF}|_{l=1} - \Pi^*_{CF}|_{l=2} = \left(\frac{8}{9}\right)\left((\Delta_P)^2 - \left(\frac{3}{2}\right)\Delta_P E[q_{11}^C|_{l=2} + q_{12}^C|_{l=2} - c_K] + \left(\frac{3}{2}\right)\delta_1 c_T\right). \quad (21)$$

The threshold on costs differences for locating centralized production, $\bar{\Delta}|_{\Omega_1}$, is defined here by the following proposition:

Proposition 8. *For a (CF,CF) strategy profile, firm 1 locates production in market 1 for $\Delta_P < \bar{\Delta}$ and in market 2 for $\Delta_P > \bar{\Delta}$, where:*

$$\bar{\Delta} = \left(\frac{3}{4}\right)\left(E[q_{11}^C|_{l=2} + q_{12}^C|_{l=2} - c_K] - \sqrt{E^2[q_{11}^C|_{l=2} + q_{12}^C|_{l=2} - c_K] - \left(\frac{8}{3}\right)\delta_1 c_T}\right). \quad (22)$$

With the exception of sensitivity to K_2, which no longer factors into the threshold, it can be seen that the sensitivity results for this threshold are directionally the same as those in the first column of Table 3. The high level conclusions garnered in Section 4 thus appear to be robust to the ratio of investment and production costs.

Of course, for decisions made under uncertainty, we cannot expect the ex-post solution to the production game to fall into any single region for all scenarios. Rather, the capacity level will be chosen endogenously by firm, with regards to its cost profile so as to maximize expected profits. We analyze this decision numerically for a set of problem instances, while adjusting the competitor capacity, K_2, and the capacity investment cost c_K to induce a range of optimal policies with regards to the chosen probability of having constraints bind ex-post.

The parameters employed are $\alpha = (50, 40)$, $c_2 = (5, 5)$, $c_T = 4$, and $c_P = 2$. These were chosen to satisfy Assumption 1 while allowing for significant gaps in both marginal production costs and potential revenues between the two markets. We employed 30 demand scenarios, which were generated by simulated draws from a uniform distribution. Demand shocks were positively correlated between the two markets. The results are displayed in Figure 4.

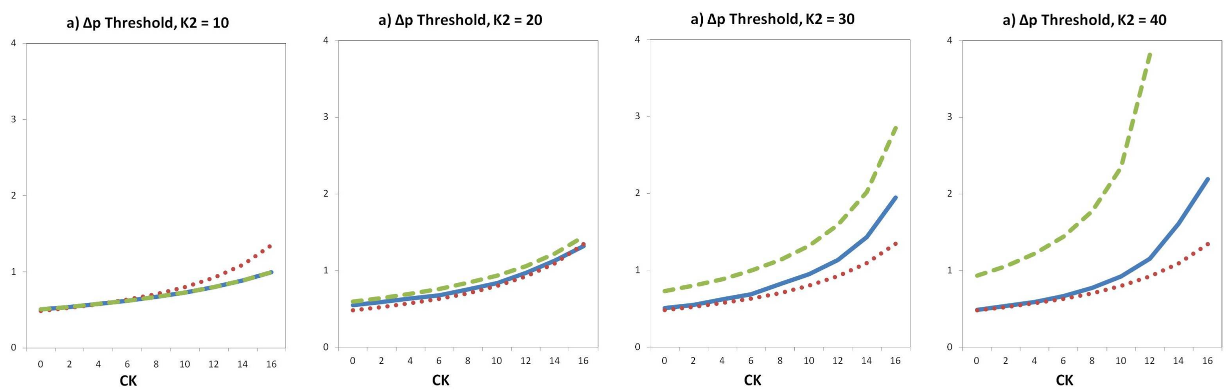

Figure 4: Thresholds ($\bar{\Delta}_P$) for location decision, graphed by capacity cost (c_K). Shown for $K_2 = \{10, 20, 30, 40\}$. (Solid lines show actual numerical threshold, dashes show Ω_4 threshold, dots show Ω_1 threshold)

It is seen that with $c_K = 0$, the actual threshold is exactly as for the approximation in Ω_1. Meanwhile, the calculated threshold continues to move closer to that of region Ω_4 as we adjust c_K upwards. Finally, we note that the actual threshold matches that of Ω_4 exactly for the case of low competitor capacity, $K_2 = 10$. In this paper, we are most interest in the case of high-cost capacity industries, so assumption 2 is likely to be appropriate for most cases. When this is combined correlates with tight capacity industry-wide, the assumption of clearance will be quite accurate.

7 Conclusion

To summarize, we have presented a model of capacity investment by firms that anticipate the actions of their competitors. The competitive interactions are represented by a lower-level game of constrained multi-market Cournot competition. We characterize the optimal capacity investments, and develop thresholds that describe the conditions under which the firm should invest in a domestic facility, a foreign facility, or dedicated facilities in both markets.

We detail the various effects that competition can have on the optimal choice of network configuration. This is shown to decompose into three distinct effects, which at times can act counter each other. The result is that the effect of competition is difficult to summarize in a one-size-fits all proposition. Rather, the impact can be highly specific to a firm's particular strengths and likely market niche. Our work suggests that as firms around the globe rethink the design of their global networks; abandoning standard offshoring practices for more customized approaches, it is unlikely to lead all firms down the same path. In fact, competitive pressure is likely to push firms down divergent paths. For some, the answer may be near-shoring and a more focused approach, while others will expand their global reach towards increasingly integrated global networks.

There are a number of interesting ways in which this work can be extended. One idea that was touched on only briefly was the scenario where the location of firms can play a role in determining correlations between each firm's random cost shifters. We gave the example of transportation cost shocks, but there are a number of other location-specific costs shocks, e.g. those relating to currency values and input prices, that may be worth studying in more detail. We have also noted that firms in our formulation prefer asymmetric markets, with negative correlations. One may wonder how our results will differ for a risk-averse firm that prefers more level profits. In particular, such preferences may incent firms to cluster, thus ensuring that any random shocks effect them equally. Lastly, we are involved in some parallel work to evaluate efficient methods of solving these capacity investment problems on a large scale.

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A Proof of Statements

Proof of Proposition 1. To address the second-stage production game, we adopt the technique of Theorem 3.4.3 in Okuguchi and Szidarovsky (1990) and convert the equilibrium problem to a quadratic program with the desired properties. Since each firm solves a convex problem in the second stage, an outcome (q_1, x_1, q_2, x_2) is an equilibrium if and only if there exist multipliers $(\lambda_1 \in \mathbb{R}^4_+,\ \mu_1 \in \mathbb{R}^4_+,\ \lambda_2 \in \mathbb{R}^4_+,\ \mu_2 \in \mathbb{R}^4_+)$, such that:

$$\begin{aligned} \begin{bmatrix} \alpha+\epsilon \\ \alpha+\epsilon \end{bmatrix} - c_1 - \begin{bmatrix} q_{-i}+2q_i \\ q_{-i}+2q_i \end{bmatrix} + \mu_i - A'\lambda_i &= 0 \quad (i \in \{1,2\}) \\ x_i &\perp \mu_i, \quad (i \in \{1,2\}) \\ (K_i - q_{i1} - q_{i2}) &\perp \lambda_i, \quad (i \in \{1,2\}) \\ Rx_i &= q_i, \quad (i \in \{1,2\}). \end{aligned} \tag{23}$$

Now, formulate the following quadratic optimization problem, taking (q_1, x_1, q_2, x_2) simultaneously as decision variables:

$$\begin{aligned} \max_{q_1\in\mathbb{R}^2_+,\, x_1\in\mathbb{R}^4_+, q_2\in\mathbb{R}^2_+,\, x_2\in\mathbb{R}^4_+} \quad & \textstyle\sum_{i\in\{1,2\}} \left[q_i'(\alpha - q_i - \frac{q_{-i}}{2} + \epsilon) - x_i'(c_i+\omega)\right] \\ \text{subject to:} \quad & Ax_i \le K_i, \quad (i\in\{1,2\}) \\ & Rx_i = q_i, \quad (i \in \{1,2\}). \end{aligned} \tag{24}$$

Note that the first order conditions for (24) are precisely the conditions in (23). Furthermore, since the objective of (24) is concave, the conditions are necessary and sufficient, so the set of equilibria are exactly the set of optimal solutions to (24). Since (24) is a quadratic program, with strictly concave objective, the equilibrium is both guaranteed to exist and unique for any choice of K, ϵ, and ω. □

Proof of Proposition 2. To construct the production table, we first impose the first order conditions for the optimization problem in (3), simultaneously for both firms 1 and 2. We introduce lagrange multipliers λ_i for firm i's capacity constraint. In all regions, the conditions:

$$\begin{aligned} \alpha_m + \epsilon_m - \bar{c}_{im} - q_{-i,m}(K,\epsilon) - 2q_{im}(K,\epsilon) &= \lambda_i, \quad (m\in\{1,2\}, i \in\{1,2\}) \\ (K_i - q_{i1}(K,\epsilon) - q_{i2}(K,\epsilon)) &\perp \lambda_i, \quad (i\in\{1,2\}) \end{aligned}$$

are in force. We then add the conditions:

$$\begin{aligned} \lambda_1 &= 0, \quad \text{in regions } \Omega_1 \text{ and } \Omega_3 \\ \lambda_2 &= 0, \quad \text{in regions } \Omega_1 \text{ and } \Omega_2 \\ q_{11}(K,\epsilon)+q_{12}(K,\epsilon) &= K_1, \quad \text{in regions } \Omega_2 \text{ and } \Omega_4 \\ q_{21}(K,\epsilon)+q_{22}(K,\epsilon) &= K_2, \quad \text{in regions } \Omega_3 \text{ and } \Omega_4 \end{aligned}$$

Solving the resulting systems of equations for each region yields the given production quantities. For conditions on ϵ, observe that, under Assumption 1, the conditions in (25) are necessary and sufficient to describe a second-stage equilibrium. The equilibrium is then in Ω_j for any ϵ which admits a solution $\{q_1(K,\epsilon), \lambda_1, q_2(K,\epsilon), \lambda_2\}$ that satisfies both (25) and the two additional constraints corresponding to region Ω_j. The six equality constraints in the resulting system are sufficient to define $\{q_1(K,\epsilon), \lambda_1, q_2(K,\epsilon), \lambda_2\}$ as displayed in the production table. After applying the conditions associated with Ω_j, we are left with two inequality constraints that are needed to ensure the complementarity conditions in (25) are satisfied. After plugging in $\{q_1(K,\epsilon), q_2(K,\epsilon)\}$ from the production table, these inequalities provide bounds on those ϵ that produce an optimum in region Ω_j. $\square$

Proof of Proposition 3. The procedure here is as in Proposition 3. In this case, K_1 and λ_1 are 2-vectors. The first order conditions for (3) in the (MF,CF) game are:

$$\begin{aligned}
\alpha_m + \epsilon_m - \bar{c}_{im} - q_{-i,m}(K,\epsilon) - 2q_{im}(K,\epsilon) &= \lambda_i, \quad (m \in \{1,2\}, i \in \{1,2\}) \\
(K_{1m} - q_{1m}(K,\epsilon)) &\perp \lambda_{1m}, \quad (m \in \{1,2\}) \\
(K_2 - q_{21}(K,\epsilon) - q_{22}(K,\epsilon)) &\perp \lambda_2,
\end{aligned}$$

The additional constraints by regions are then:

$$\begin{aligned}
\lambda_{11} &= 0, \quad \text{in regions } \Omega_1 \text{ and } \Omega_3 \text{ and } \Omega_{2.2} \text{ and } \Omega_{4.2} \\
\lambda_{12} &= 0, \quad \text{in regions } \Omega_1 \text{ and } \Omega_3 \text{ and } \Omega_{2.1} \text{ and } \Omega_{4.1} \\
\lambda_2 &= 0, \quad \text{in regions } \Omega_1 \text{ and } \Omega_2 \\
q_{11}(K,\epsilon) &= K_{11}, \quad \text{in regions } \Omega_{2.1} \text{ and } \Omega_{2.3} \text{ and } \Omega_{4.1} \text{ and } \Omega_{4.3} \\
q_{12}(K,\epsilon) &= K_{12}, \quad \text{in regions } \Omega_{2.2} \text{ and } \Omega_{2.3} \text{ and } \Omega_{4.2} \text{ and } \Omega_{4.3} \\
q_{21}(K,\epsilon) + q_{22}(K,\epsilon) &= K_2, \quad \text{in regions } \Omega_3 \text{ and } \Omega_4
\end{aligned}$$

We can now solve for production quantities and bounds on ϵ. $\square$

Proof of Proposition 4. The assumptions in place indicate that the optimal solutions comes from region Ω_4 for all ϵ. According to Table 1, the second-stage production quantities are $q_{1m}(K,\epsilon) = \frac{K_1}{2} + \frac{(q^C_{1m} - q^C_{1,-m})}{2}$ and $q_{2m}(K,\epsilon) = \frac{K_2}{2} + \frac{(q^C_{2m} - q^C_{2,-m})}{2}$. The resulting profits are $\pi(K_1,\epsilon)|_{\Omega=\Omega_4} = (K_1)\left[q^M_{11} + q^M_{12} - \frac{K_1+K_2}{2} - c_K\right] + \frac{E[(q^C_{11}-q^C_{12})^2]}{2}$. After taking expectations, we get that K_1 is chosen to maximize $E[\pi(K_1,\epsilon)|_{\Omega=\Omega_4}] = (K_1)\left(E[q^M_{11}] + E[q^M_{12}] - \frac{K_1+K_2}{2} - c_K\right) + \left(\frac{E[(q^C_{11}-q^C_{12})^2]}{2}\right)$. The function is concave in K_1 and maximized at $K_1^{CF} = E[q^M_{11}] + E[q^M_{12}] - \frac{K_2}{2} - c_K$. Plugging this into the expected profit function yields Π^*_{CF}. $\square$

Proof of Proposition 5. The assumptions in place indicate that the optimal solutions comes from region Ω_4 for all ϵ. According to Table 2, the second-stage production quantities are $q_{1m}(K,\epsilon) = K_{1m}$ and $q_{2m}(K,\epsilon) = \frac{K_2}{2} + \frac{2q^M_{2m} - K_{1m}}{4} - \frac{2q^M_{2,-m} - K_{1,-m}}{4}$. The resulting profits can be written as: $\pi(K_1,\epsilon)|_{\Omega=\Omega_4} = \sum_{m\in\{1,2\}}\left[(K_{1m})\left(E[q^M_{1m}] - \frac{K_2}{4} - \frac{c_K}{2} - \frac{E[q^M_{2m}] - E[q^M_{2,-m}]}{4} + \frac{(3\epsilon_1+\epsilon_2)}{2}\right)\right]$. All stochastic terms enter linearly, dropping out when expectations are taken. The expected profit function is separable and concave in K_{11} and K_{12}. We optimize each investment separately, yielding the solution in (9). Plugging this into the expected profit function yields Π^*_{MF}. $\square$

Proof of Proposition 6. The threshold is obtained by setting the profit differential in (13) to zero, and solving the resulting quadratic equation for Δ_p. Note that $K_1^{CF}|_{l=2} = \lambda_1 - \frac{c_T}{2}$ and

$K_1^{CF}|_{l=1} = \lambda_1 - \frac{c_T}{2} - \Delta_P$, and that both terms must be positive for Assumption 1 to hold. It is clear from (13) that $\Pi^*_{CF}|_{l=1} - \Pi^*_{CF}|_{l=2} > 0$ at $\Delta_P = 0$. Furthermore, $\frac{\partial\left[\Pi^*_{CF}|_{l=1} - \Pi^*_{CF}|_{l=2}\right]}{\partial \Delta_P} = \Delta_P + \frac{c_T}{2} - \lambda_1$ is negative under Assumption 1. It follows that onshore production is favored for Δ_P less than the smaller root of (13), and offshore production is favored elsewhere in the admissible space of parameters. □

Proof of Proposition 7. The proposition follows directly from equations (15)-(18). □

Proof of Proposition 8. The threshold is obtained by setting the profit differential in (21) to zero, and solving the resulting quadratic equation for Δ_p. As done for Proposition 6, we note that $\frac{\partial\left[\Pi^*_{CF}|_{l=1} - \Pi^*_{CF}|_{l=2}\right]}{\partial \Delta_P} = K_1^{CF}|_{l=1}$, which is negative by Assumption (1). The threshold $\bar{\Delta}$ must then equal the smaller root of (21). □